CHAPTER

13 Probability

13.1 Introduction

Kumar and Sudha were walking together to play a carroms match:

Kumar : Do you think we will win?

Sudha : There are 50 percent chances of that. We may win.

Kumar : How do you say 50 percent?

Do you think Sudha is right in her statement?

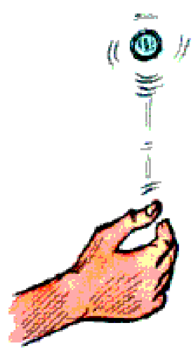
Is her chance of wining 50%?

In this chapter, we study about such questions. We also discuss words the like ‘probably’, ‘likely’, ‘possibly’, etc. and how to quantify these. In class IX we studied about events that are extremely likely and in fact, are almost certain and those that are extremely unlikely and hence almost impossible. We also talked about chance, luck and the fact that an event occurs one particular time does not mean that it would happen each time. In this chapter, we try to learn how the likelihood of an event can be quantified.

This quantificatioin into a numerical measure is referred to as finding 'Probability'.

13.1.1 What is Probability

Consider an experiment: A normal coin was tossed 1000 times. Head turned up 455 times and tail turned up 545 times. If we try to find the likelihood of getting heads we may say it is 455 out of 1000 or $\frac{455}{1000}$ or 0.455.



This estimation of probability is based on the results of an actual experiment of tossing a coin 1000 times. These estimates are called experimental or empirical probabilities. In fact, all experimental probabilities are based on the results of actual experiments and an adequate recording of what happens in each of the events. These probabilities are only 'estimations'. If we perform the same experiment for another 1000 times, we may get slightly different data, giving different probability estimate.

Many other persons from different parts of the world have done this kind of experiment and recorded the number of heads that turned up.

For example, the eighteenth century French naturalist Comte de Buffon, tossed a coin 4040 times and got 2048 heads. The experimental probability of getting a head, in this case, was $\frac{2048}{4040}$ i.e., 0.507.

J.E. Kerrich, from Britain, recorded 5067 heads in 10000 tosses of a coin. The experimental probability of getting a head, in this case, was $\frac{5067}{10000} = 0.5067$. Statistician Karl Pearson spent some more time, making 24000 tosses of a coin. He got 12012 heads, and thus, the experimental probability of a head obtained by him was 0.5005.

Now, suppose we ask, 'What will be the experimental probability of getting a head, if the experiment is carried on up to, say, one million times? Or 10 million times? You would intuitively feel that as the number of tosses increases, the experimental probability of a head (or a tail) may settle down closer and closer to the number 0.5 , i.e., $\frac{1}{2}$. This matches the theoretical probability of getting a head (or getting a tail), we will learn how to find the theoretical probability.

This chapter is an introduction to the theoretical (also called classical) probability of an event, Now we discuss simple problems based on this concept.

13.2 Probability - A Theoretical Approach

Let us consider the following situation: Suppose a ‘fair’ coin is tossed at random.

When we speak of a coin, we assume it to be 'fair', that is, it is symmetrical so that there is no reason for it to come down more often on one side than the other. We call this property of the coin as being 'unbiased'. By the phrase 'random toss', we mean that the coin is allowed to fall freely without any bias or interference. (Here we dismiss the possibility of its 'landing' on its edge, which may be possible, for example, if it falls on sand). We refer to this by saying that the outcomes, head and tail, are equally likely.

For basic understanding of probability, in this chapter, we will assume that all the experiments have equally likely outcomes.

Now, we know that the experimental or empirical probability P(E) of an event E is

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$$

Do This

a. Outcomes of which of the following experiments are equally likely?

1. Getting a digit 1, 2, 3, 4, 5 or 6 while throwing a die.
2. Picking a different colour ball from a bag of 5 red balls, 4 blue balls and 1 black ball.
3. Winning in a game of carrom.
4. Units place of a two digit number selected may be 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9.
5. Picking a different colour ball from a bag of 10 red balls, 10 blue balls and 10 black balls.
6. Raining on a particular day of July.

b. Are the outcomes of every experiment equally likely?

c. Give examples of 5 experiments that have equally likely outcomes and five more examples that do not have equally likely outcomes.

Activity

(i) Take any coin, toss it, 50 times, 100 times, 150 times and count the number of times a head and a tail come up seperately. Record your observations in the following table:-

S. No.	Number of experiments	Number of heads	Probability of head	Number of tails	Probability of tails
1.	50				
2.	100				
3.	150				

What do you observe? Obviously, as the number of experiments are more and more, probability of head or tail reaches 50% or $\frac{1}{2}$. This empirical interpretation of probability can be applied to every event associated with an experiment that can be repeated a large number of times.

Probability and Modelling

The requirement of repeating an experiment has some limitations, as it may be very expensive or unfeasible in many situations. Of course, it worked well in coin tossing or die throwing experiments. But how about repeating the experiment of launching a satellite in order to compute the empirical probability of its failure during launching, or the repetition of the phenomenon of an earthquake to compute the empirical probability of a multi-storeyed building getting destroyed in an earthquake? For finding these probabilities we calculate models of behaviour and use them to estimate behaviour and likely outcomes. Such models are complex and are validated by predictions and outcomes. Forecast of weather, result of an election, population demography, earthquakes, crop production etc. are all based on such models and their predictions.

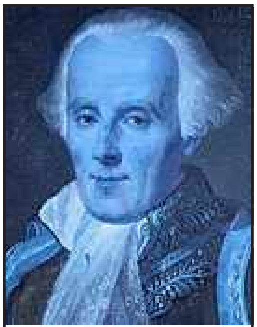
"The assumption of equally likely outcomes" (which is valid in many experiments, as in two of the examples seen, of a coin and of a die) is one of the assumption that leads us to the following definition of probability of an event.

The theoretical probability (also called classical probability) of an event T, written as P(T), is defined as

$$P(T) = \frac{\text{Number of outcomes favourable to T}}{\text{Number of all possible outcomes of the experiment}}$$

where we assume that the outcomes of the experiment are equally likely. We usually simply refer to theoretical probability as Probability.

The definition of probability was given by Pierre Simon Laplace in 1795.


Pierre Simon Laplace (1749 – 1827)

Probability theory had its origin in the 16^{th} century when an Italian physician and mathematician J. Cardan wrote the first book on the subject, The Book on Games of Chance. James Bernoulli (1654 -1705), A. De Moivre (1667-1754), and Pierre Simon Laplace are among those who made significant contributions to this field. In recent years, probability has been used extensively in many areas such as biology, economics, genetics, physics, sociology etc.

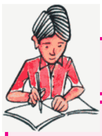
13.3 Mutually Exclusive Events

If a coin is tossed, we get head or tail, but not both. Similarly, if we select a student of a high school that student may belong to one of either 6, 7, 8, 9 or 10 class, but not to any two or more classes. In both these examples, occurrence of an event prevents the occurrence of other events. Such events are called mutually exclusive events.

Two or more events of an experiment, where occurance of an event prevents occurances of all other events, are called **Mutually Exclusive Events.** We will discuss this in more detail later in the chapter.

13.4.1 Finding Probability

How do we find the probability of events that are equally likely? We consider the tossing of a coin as an event associated with experiments where the equally likely assumption holds. In order to proceed, we recall that there are two possible outcomes each time. This set of outcomes is called the sample space. We can say that the sample space of one toss is {H, T}. For the experiement of drawing out a ball from a bag containing red, blues, yellow and white ball, the sample space is {R, B, Y, W}. What is the sample space for the throw of a die?



Do This

Think of 5 situations with equally likely events and find the sample space.

Let us now try to find the probability of equally likely events that are mutually exclusive.

Example-1. Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.

Solution : In the experiment of tossing a coin once, the number of possible outcomes is two - Head (H) and Tail (T). Let E be the event 'getting a head'. The number of outcomes favourable to E, (i.e., of getting a head) is 1. Therefore,

$$P(E) = P\text{ (head)} = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes}} = \frac{1}{2}$$

Similarly, if F is the event 'getting a tail', then

$$P(F) = P(\text{tail}) = \frac{1}{2} \text{ (Guess why?)}$$

Example-2. A bag contains a red ball, a blue ball and an yellow ball, all the balls being of the same size. Manasa takes out a ball from the bag without looking into it. What is the probability that she takes a (i) yellow ball? (ii) red ball? (iii) blue ball?

Solution : Manasa takes out a ball from the bag without looking into it. So, it is equally likely that she takes out any one of them.

Let Y be the event 'the ball taken out is yellow', B be the event 'the ball taken out is blue', and R be the event 'the ball taken out is red'.

Now, the number of possible outcomes = 3.

(i) The number of outcomes favourable to the event Y = 1.

So, $P(Y) = \frac{1}{3}$. Similarly, $P(R) = \frac{1}{3}$ and $P(B) = \frac{1}{3}$

Remarks

1. An event having only one outcome in an experiment is called an elementary event. In Example 1, both the events E and F are elementary events. Similarly, in Example 2, all the three events, Y, B and R are elementary events.

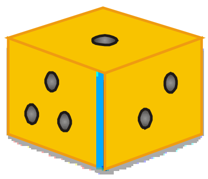
2. In Example 1, we note that : P(E) + P(F) = 1
   In Example 2, we note that : P(Y) + P(R) + P(B) = 1.

   If we find the probability of all the elementary events and add them, we would get the total as 1.

3. In events like a throw of dice, probability of getting less than 3 and of getting a 3 or more than three are not elementary events of the possible outcomes. In tossing two coins {HH}, {HT}, {TH} and {TT} are elementary events.

Example-3. Suppose we throw a die once. (i) What is the probability of getting a number greater than 4? (ii) What is the probability of getting a number less than or equal to 4?

Solution : (i) In rolling an unbaised dice



Sample space	S	= {1, 2, 3, 4, 5, 6}
No. of outcomes	n(S)	= 6
Favourable outcomes for number greater than 4	E	= {5, 6}
No. of favourable outcomes	n(E)	= 2
Probability	P(E)	$= \frac{2}{6} = \frac{1}{3}$

(ii) Let F be the event 'getting a number less than or equal to 4'.

Sample space	S	= {1, 2, 3, 4, 5, 6}
No. of outcomes	n(S)	= 6
Favourable outcomes for number less or equal to 4	F	= {1, 2, 3, 4}
No. of favourable outcomes	n(F)	= 4
Probability	P(F)	$= \frac{4}{6} = \frac{2}{3}$

Note : Are the events E and F in the above example elementary events?

No, they are not elementary events. The event E has 2 outcomes and the event F has 4 outcomes.

13.4.2 Complementary Events and Probability

In the previous section we read about elementary events. Then in example-3, we calculated probability of events which are not elementary. We saw,

$$P(E) + P(F) = \frac{1}{3} + \frac{2}{3} = 1$$

Here F is the same as 'not E' because there are only two events.

We denote the event 'not E' by $\overline{E}$. This is called the **complement** event of event E.

So, P(E) + P(not E) = 1

i.e., $P(E) + P(\overline{E}) = 1$, which gives us $P(\overline{E}) = 1 - P(E)$.

In general, it is true that for an event E, $P(\overline{E}) = 1 - P(E)$

Do This

(i) Is getting a head complementary to getting a tail? Give reasons.

(ii) In case of a die is getting a 1 complementary to events getting 2, 3, 4, 5, 6? Give reasons for your answer.

(iii) Write of five new pair of events that are complementary.

13.4.3 Impossible and Certain Events

Consider the following about the throws of a die with sides marked as 1, 2, 3, 4, 5, 6.

(i) What is the probability of getting a number 7 in a single throw of a die?

We know that there are only six possible outcomes in a single throw of this die. These outcomes are 1, 2, 3, 4, 5 and 6. Since no face of the die is marked 7, there is no outcome favourable to 7, i.e., the number of such outcomes is zero. In other words, getting 7 in a single throw of a die, is impossible.

$$\text{So P(getting 7)} = \frac{0}{6} = 0$$

That is, the probability of an event which is impossible to occur is 0. Such an event is called an **impossible event**.

(ii) What is the probability of getting 6 or a number less than 6 in a single throw of a die?

Since every face of a die is marked with 6 or a number less than 6, it is sure that we will always get one of these when the dice is thrown once. So, the number of favourable outcomes is the same as the number of all possible outcomes, which is 6.

Therefore, P(E) = P(getting 6 or a number less than 6) = $\frac{6}{6} = 1$

So, the probability of an event which is sure (or certain) to occur is 1. Such an event is called a **sure event** or a **certain event**.

Note : From the definition of probability P(E), we see that the numerator (number of outcomes favourable to the event E) is always less than or equal to the denominator (the number of all possible outcomes). Therefore, $\mathbf{0 \le P(E) \le 1}$.

Try This

1. A child has a die whose six faces show the letters A, B, C, D, E and F. The die is thrown once. What is the probability of getting (i) A? (ii) D?
2. Which of the following cannot be the probability of an event?

   (a) 2.3 (b) -1.5 (c) 15% (D) 0.7

Think - Discuss

1. Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of any game?
2. Can $\frac{7}{2}$ be the probability of an event? Explain.
3. Which of the following arguments are correct and which are not correct? Give reasons.

   i) If two coins are tossed simultaneously there are three possible outcomes - two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

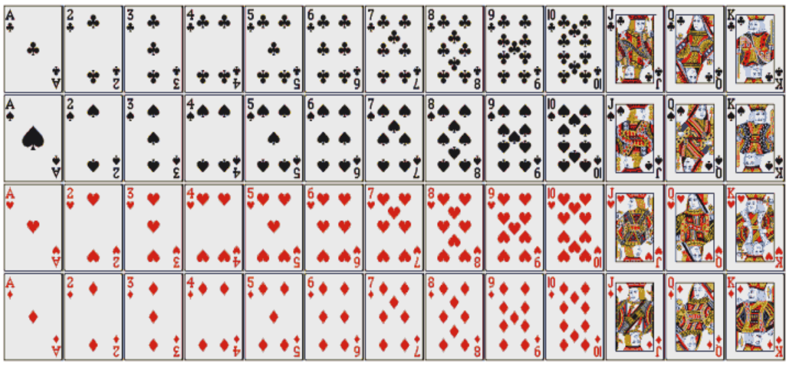
   ii) If a die is thrown, there are two possible outcomes - an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

13.5 Deck of Cards and Probability

Have you seen a deck of playing cards?

A deck of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. They are black spades (♠), red hearts (♥), red diamonds (♦) and black clubs (♣).

The cards in each suit are Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, Queens and Jacks are called face cards. Many games are played with this deck of cards, some games are played with part of the deck and some with two decks even. The study of probability has a lot to do with card and dice games as it helps players to estimate possibilities and predict how the cards could be distributed among players.



Example-4. One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will (i) be an ace, (ii) not be an ace.

Solution : Well-shuffling ensures equally likely outcomes.

(i) There are 4 aces in a deck.

Let E be the event 'the card is an ace'.

The number of outcomes favourable to E = 4

The number of possible outcomes = 52 (Why ?)

Therefore, $P(E) = \frac{4}{52} = \frac{1}{13}$

(ii) Let F be the event 'card drawn is not an ace'.

The number of outcomes favourable to the event F = 52 - 4 = 48 (Why?)

The number of possible outcomes = 52

Therefore, $P(F) = \frac{48}{52} = \frac{12}{13}$

Alternate Method : Note that F is nothing but $\overline{E}$.

Therefore, we can also calculate P(F) as follows:

$P(F) = P(\overline{E}) = 1 - P(E) = 1 - \frac{1}{13} = \frac{12}{13}$

TRY THIS

You have a single deck of well shuffled cards. Then,

1. What is the probability that the card drawn will be a queen?

2. What is the probability that it is a face card?
3. What is the probability it is a spade?
4. What is the probability that is the face card of spades?
5. What is the probability it is not a face card?

13.6 Use of Probability

Let us look at some more occasions where probability may be useful. We know that in sports some countries are strong and others are not so strong. We also know that when two players are playing it is not that they win equal times. The probability of winning of the player or team that wins more often is more than the probability of the other player or team. We also discuss and keep track of birthdays. Sometimes happens it that people we know have the same birthdays. Can we find out whether this is a common event or would it only happen occassionally. Classical probability helps us do this.

Example-5. Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?

Solution : Let S and R denote the events that Sangeeta wins the match and Reshma wins the match, respectively.

The probability of Sangeeta's winning chances = P(S) = 0.62 (given)

The probability of Reshma's winning chances = P(R) = 1 - P(S)

= 1 -0.62 = 0.38 [R and S are complementary]

Example-6. Sarada and Hamida are friends. What is the probability that both will have (i) different birthdays? (ii) the same birthday? (ignoring a leap year).

Solution : Out of the two friends, one girl, say, Sarada's birthday can be any day of the year. Now, Hamida's birthday can also be any day of 365 days in the year. We assume that these 365 outcomes are equally likely.

(i) If Hamida's birthday is different from Sarada's, the number of favourable outcomes for her birthday is 365 - 1 = 364

So, P (Hamida's birthday is different from Sarada's birthday) = $\frac{364}{365}$

(ii) P(Sarada and Hamida have the same birthday) = 1 - P (both have different birthdays)

$$= 1 - \frac{364}{365} \text{ [Using } P(\overline{E}) = 1 - P(E)] = \frac{1}{365}$$

Example-7. There are 40 students in Class X of a school of whom 25 are girls and 15 are boys. The class teacher has to select one student as a class representative. She writes the name of each student on a separate cards, the cards being identical. Then she puts cards in a box and stirs them thoroughly. She then draws one card from the box. What is the probability that the name written on the card is the name of (i) a girl? (ii) a boy?

Solution : There are 40 students, and only one name card has to be chosen.

The number of all possible outcomes is 40

(i) The number of outcomes favourable for a card with the name of a girl = 25 (Why?)

$\therefore$ P (card with name of a girl) = P(Girl) = $\frac{25}{40} = \frac{5}{8}$

(ii) The number of outcomes favourable for a card with the name of a boy = 15 (Why?)

Therefore, P(card with name of a boy) = P(Boy) = $\frac{15}{40} = \frac{3}{8}$

or P(Boy) = 1 - P(not Boy) = 1 - P(Girl) = $1 - \frac{5}{8} = \frac{3}{8}$

Exercise - 13.1

1. Complete the following statements:

   (i) Probability of an event E + Probability of the event 'not E' = ______________

   (ii) The probability of an event that cannot happen is__________.

   Such an event is called __________

   (iii) The probability of an event that is certain to happen is __________.

   Such an event is called______

   (iv) The sum of the probabilities of all the elementary events of an experiment is_________

   (v) The probability of an event is greater than or equal to __________ and less than or equal to _______

2. Which of the following experiments have equally likely outcomes? Explain.

   (i) A driver attempts to start a car. The car starts or does not start.

   (ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.

   (iii) A trial is made to answer a true-false question. The answer is right or wrong.

   (iv) A baby is born. It is a boy or a girl.

3. If P(E) = 0.05, what is the probability of 'not E'?
4. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out

   (i) an orange flavoured candy? (ii) a lemon flavoured candy?
5. Rahim takes out all the hearts from the cards. What is the probability of
   i. Picking out an ace from the remaining pack.
   ii. Picking out a diamonds.
   iii. Picking out a card that is not a heart.
   iv. Picking out the Ace of hearts.
6. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?
7. A die is thrown once. Find the probability of getting

   (i) a prime number; (ii) a number lying between 2 and 6; (iii) an odd number.
8. What is the probability of drawing out a red king from a deck of cards?
9. Make 5 more problem of this kind using dice, cards or birthdays and discuss with friends and teacher about their solutions.

13.7 More Applications of Probability

We have seen some example of use of probability. Think about the contents and ways probability has been used in these. We have seen again that probability of complementary events add to 1. Can you identify in the examples and exercises given above, and those that follow, complementary events and elementary events? Discuss with teachers and friends. Let us see more uses.

Example-8. A box contains 3 blue, 2 white, and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will be

(i) white? (ii) blue? (iii) red?

Solution : Saying that a marble is drawn at random means all the marbles are equally likely to be drawn.

$\therefore$ The number of possible outcomes = 3 +2 + 4 = 9 (Why?)

Let W denote the event 'the marble is white', B denote the event 'the marble is blue' and R denote the event 'marble is red'.

(i) The number of outcomes favourable to the event W = 2

So, $P(W) = \frac{2}{9}$

Similarly, (ii) $P(B) = \frac{3}{9} = \frac{1}{3}$ and (iii) $P(R) = \frac{4}{9}$

Note that $P(W) + P(B) + P(R) = 1$.

Example-9. Harpreet tosses two different coins simultaneously (say, one is of ₹1 and other of ₹2). What is the probability that she gets at least one head?

Solution : We write H for 'head' and T for 'tail'. When two coins are tossed simultaneously, the possible outcomes are (H, H), (H, T), (T, H), (T, T), which are all equally likely. Here (H, H) means heads on the first coin (say on ₹1) and also heads on the second coin (₹2). Similarly (H, T) means heads up on the first coin and tail up on the second coin and so on.

The outcomes favourable to the event E, 'at least one head' are (H, H), (H, T) and (T, H).

So, the number of outcomes favourable to E is 3.

$\therefore P(E) = \frac{3}{4}$ [Since the total possible outcomes = 4]

i.e., the probability that Harpreet gets at least one head is $\frac{3}{4}$

Check This

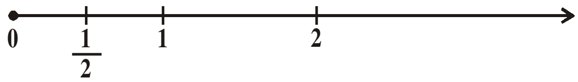
Did you observe that in all the examples discussed so far, the number of possible outcomes in each experiment was finite? If not, check it now.

There are many experiments in which the outcome is number between two given numbers, or in which the outcome is every point within a circle or rectangle, etc. Can you count the number of all possible outcomes in such cases? As you know, this is not possible since there are infinitely many numbers between two given numbers, or there are infinitely many points within a circle. So, the definition of theoretical probability which you have learnt so far cannot be applied in the present form.

What is the way out? To answer this, let us consider the following example:

Example-10. (Not for examination) In a musical chair game, the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the first half-minute after starting?

Solution : Here the possible outcomes are all the numbers between 0 and 2. This is the portion of the number line from 0 to 2



Let E be the event that 'the music is stopped within the first half-minute'.

The outcomes favourable to E are points on the number line from 0 to $\frac{1}{2}$

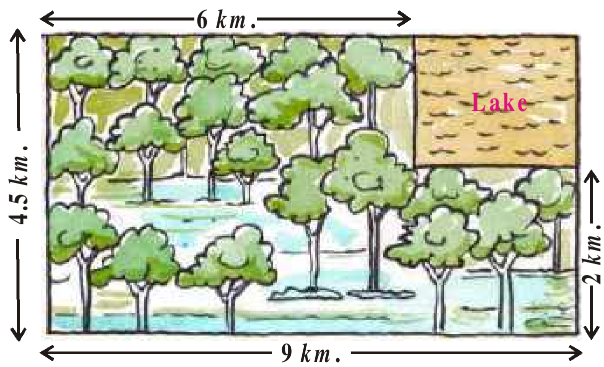
The distance from 0 to 2 is 2, while the distance from 0 to $\frac{1}{2}$ is $\frac{1}{2}$

Since all the outcomes are equally likely, we can argue that, of the total distance is 2 and the distance favourable to the event E is $\frac{1}{2}$

$$\text{So, } P(E) = \frac{\text{Distance favourable to the event E}}{\text{Total distance in which outcomes can lie}} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$

We now try to extend this idea of for finding the probability as the ratio of the favourable area to the total area.

Example-11. A missing helicopter is reported to have crashed somewhere in the rectangular region as shown in the figure. What is the probability that it crashed inside the lake shown in the figure?



Solution : The helicopter is equally likely to crash anywhere in the region. Area of the entire region where the helicopter can crash = $(4.5 \times 9)\ km^2 = 40.5\ km^2$

Area of the lake = $(2 \times 3)\ km^2 = 6\ km^2$

Therefore, P (helicopter crashed in the lake) = $\frac{6}{40.5} = \frac{4}{27}$

Example-12. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jhony, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that

(i) it is acceptable to Jhony? (ii) it is acceptable to Sujatha?

Solution : One shirt is drawn at random from the carton of 100 shirts. Therefore, there are 100 equally likely outcomes.

(i) The number of outcomes favourable (i.e., acceptable) to Jhony = 88 (Why?)

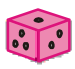
Therefore, P (shirt is acceptable to Jhony) = $\frac{88}{100} = 0.88$

(ii) The number of outcomes favourable to Sujatha = 88 + 8 = 96 (Why?)

So, P (shirt is acceptable to Sujatha) = $\frac{96}{100} = 0.96$

Example-13. Two dice, one red and one white, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is (i) 8 (ii) 13 (iii) less than or equal to 12?

Solution : When the red dice shows '1', the white dice could show any one of the numbers 1, 2, 3, 4, 5, 6. The same is true when the red dice shows '2', '3', '4', '5' or '6'. The possible outcomes of the experiment are shown in the figure; the first number in each ordered pair is the number appearing on the red dice and the second number is that on the white dice.



	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Note that the pair (1, 4) is different from (4, 1). (Why?)

So, the number of possible outcomes n(S) = 6 × 6 = 36.

(i) The outcomes favourable to the event 'the sum of the two numbers is 8' denoted by E, are: (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) (See figure)

i.e., the number of outcomes favourable to E is n(E) = 5.

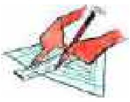
Hence, P(E) = $\frac{n(E)}{n(S)} = \frac{5}{36}$

(ii) As there is no outcome favourable to the event F, 'the sum of two numbers is 13',

So, P(F) = $\frac{0}{36} = 0$

(iii) As all the outcomes are favourable to the event G, 'sum of two numbers is 12',

So, P(G) = $\frac{36}{36} = 1$



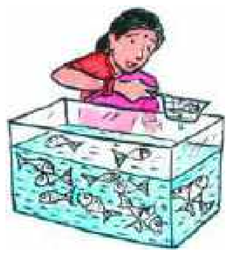
EXERCISE - 13.2

1. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red ? (ii) not red?

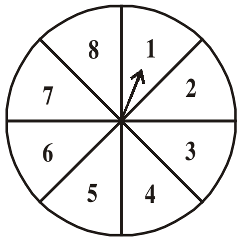
2. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) white ? (iii) not green?

3. A Kiddy bank contains hundred 50p coins, fifty ₹1 coins, twenty ₹2 coins and ten ₹5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin? (ii) will not be a ₹5 coin?

4. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (See figure). What is the probability that the fish taken out is a male fish?



5. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (See figure), and these are equally likely outcomes. What is the probability that it will point at



   (i) 8 ? (ii) an odd number?

   (iii) a number greater than 2? (iv) a number less than 9?

6. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

   (i) a king of red colour (ii) a face card (iii) a red face card

   (iv) the jack of hearts (v) a spade (vi) the queen of diamonds

7. Five cards-the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

   (i) What is the probability that the card is the queen?

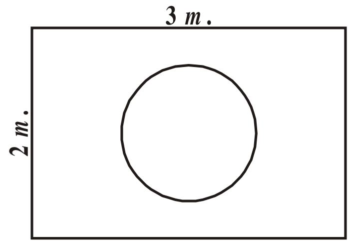
   (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

8. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

9. A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective? Suppose the bulb drawn in previous case is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

10. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

11. Suppose you drop a die at random on the rectangular region shown in figure. What is the probability that it will land inside the circle with diameter 1m?



12. A lot consists of 144 ball pens of which 20 are defective and the others are good. The shopkeeper draws one pen at random and gives it to Sudha. What is the probability that (i) She will buy it? (ii) She will not buy it ?

13. Two dice are rolled simultaneously and counts are added (i) complete the table given below:

Event : 'Sum on 2 dice'	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{12}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.

14. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

15. A die is thrown twice. What is the probability that (i) 5 will not come up either time? (ii) 5 will come up at least once? [**Hint** : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment].

Optional Exercise

[This exercise is not meant for examination]

1. Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on (i) the same day? (ii) consecutive days? (iii) different days?

2. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.

3. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

4. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

What We Have Discussed

In this chapter, you have studied the following points:

1. We have looked at experimental probability and theoretical probability.
2. The theoretical (classical) probability of an event E, written as P(E), is defined as

$$P(E) = \frac{\text{Number of trails in which the event happened}}{\text{Total number of trails}}$$ where we assume that the outcomes of the experiment are equally likely.

3. The probability of a sure event (or certain event) is 1.
4. The probability of an impossible event is 0.
5. The probability of an event E is a number P(E) such that $0 \le P(E) \le 1$
6. An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.
7. For any event E, $P(E) + P(\overline{E}) = 1$, where $\overline{E}$ stands for 'not E'. E and $\overline{E}$ are called complementary events.
8. Some more terms used in the chapter are given below:

Equally likely events : Two or more events are said to be equally likely if each one of them has an equal chance of occurrance.

Mutually Exclusive events : Two or more events are mutually exclusive if the occurrence of each event prevents the every other event.

Complementary events : Consider an event has few outcomes. Event of all other outcomes in the sample survey which are not in the favourable event is called Complementary event.

Exhaustive events : All the events are exhaustive events if their union is the sample space.

Sure events : The sample space of a random experiment is called sure or certain event as any one of its elements will surely occur in any trail of the experiment.

Impossible event : An event which will occur on any account is called an impossible event.