CHAPTER 2

Sets

2.1 INTRODUCTION

Observe the examples given below:

1. Euclid, Pythagoras, Gauss, Leibnitz, Aryabhata, Bhaskar.
2. a,e,i,o,u
3. Happy, sad, angry, anxious, joyful, confused.
4. Cricket, football, kho-kho, kabaddi, basketball.
5. 1, 3, 5, 7, 9.....

What do you observe? Example 1 is a collection of names of some mathematicians, For example 2 is the collection of vowel letters in the English alphabet and example 3 is a collection of feelings. We see that the names/ items/ objects in each example have something in common., i.e. they form a collection. Can you tell what are the collections in examples 4 and 5?

We come across collections in mathematics too. For example, natural numbers, prime numbers, quadrilaterals in a plane etc. All examples seen so far are well defined collection of objects or ideas. A well defined collection of objects or ideas is known as a **set.** Set theory is a comparitively new concept in mathematics. It was developed by Georg Cantor (1845-1918). In this chapter, we will learn about sets and their properties, and what we mean when we say well-defined, elements of a set, types of sets etc.

2.2 WELL DEFINED SETS

What do we mean when we say that a set is a well defined collection of objects. Well defined means that:

1. All the objects in the set should have a common feature or property; and
2. It should be possible to decide whether any given object belongs to the set or not.

Let us understand 'well defined' through some examples. Consider the statement : The collection of all tall students in your class.

What difficulty is caused by this statement? Here, who is tall is not clear. Richa decides that all students taller than her are tall. Her set has five students. Yashodhara also decides that tall means all students taller than her. Her set has ten students. Ganapati decides that tall means every student whose height is more than 5 feet. His set has 3 students. We find that different people get different collections. So, this collection is not well defined.

Now consider the following statement : The collection of all students in your class who are taller than 5 feet 6 inches.

In this case, Richa, Yashodhara and Ganapati, all will get the same collection. So, this collection forms a well defined set.

Do This

1. Write 3 examples of 'sets' from your daily life.
2. Some collections are given below. Tick the ones that form well defined sets.
   (i) Collection of all good students in your class.
   (ii) Red, blue, green, yellow, block.
   (iii) 1,2,3,4,5,6,7,.... (iv) 1, 8, 27, 64, 125, ...

Try This

State which of the following collections are sets.

(i) All even numbers (ii) Stars in the sky

(iii) The collection of odd positive integers. 1, 3, 5, …..

2.3 Naming of Sets and Elements of a Set

We usually denote a set by upper case letters, A, B, C, X, Y, Z etc. A few examples of sets in mathematics are given below.

- The set of all Natural numbers is denoted by N.
- The set of all Integers is denoted by Z.
- The set of all Rational numbers is denoted by Q.
- The set of all Real numbers is denoted by R.

Notice that all the sets given above are well defined collections because given a number we can decide whether it belongs to the set or not. Let us see some more examples of elements.

Suppose we define a set as all days in a week, whose name begins with T. Then we know that Tuesday and Thursday are part of the set but Monday is not. We say that Tuesday and Thursday are **elements** of the set of all days in a week starting with T.

Consider some more examples:

(i) We know that N usually stands for the set of all natural numbers. Then 1,2, 3... are elements of the set. But 0 is not an element of N.

(ii) Let us consider the set B, of quadrilaterals

B = {square, rectangle, rhombus, parallelogram}

Can we put triangle, trapezium or cone in the above set, B? No, a triangle and cone are can not be members of B. But a trapezium can be a member of the set B.

So, we can say that an object belonging to a set is known as a member/ element of the set. We use the symbol $\in$ to denote 'belongs to'. So $1 \in N$ means that 1 belongs to N. Similarly $0 \notin N$ means that 0 does not belong to N.

There are various ways in which we can write sets. For example, we have the set of all vowel letters in the English alphabet. Then, we can write:

(i) V = {a, e, i, o, u}. Here, we list down all the elements of the set between chain/ curly brackets. This is called the **roster** form of writing sets. In roster form, all elements of the set are written, separated by commas, within curly brackets.

(ii) V = {x : x is a vowel letter in the English alphabet}

or V = {x / x is a vowel letter in the English alphabet}

This way of writing a set is known as the set builder form. Here , we use a symbol x (or any other symbol y, z etc.,) for the element of the set. This is is followed by a colon (or a vertical line), after which we write the characteristic property possessed by the elements of the set. The whole is enclosed within curly brackets.

Let C = {2, 3, 5, 7, 11}, a set of prime numbers less than 13. This set can be denoted as:

C = {x / x is a prime number less than 13}or

C = {x: x is a prime number less than 13}.

Example-1. Write the following in roster and set builder forms.

(i) The set of all natural numbers which divide 42.

(ii) The set of natural numbers which are less than 10.

Solution :

(i) Let B be the set of all natural numbers which divide 42. Then, we can write:

$B = \{1, 2, 3, 6, 7, 14, 21, 42\}$ Roster form

$B = \{x : x$ is a natural number which divides $42\}$ Set builder form

(ii) Let A be the set of all natural numbers which are less than 10. Then, we can write:

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ (Roster form)

$B = \{x : x$ is a natural number which is less than $10\}$ (Set builder form)

Note : (i) In roster form, the order in which the elements are listed is **immaterial**. Thus, in example 1, we can also write $\{1, 3, 7, 21, 2, 6, 4, 42\}$.

(ii) While writing the elements of a set in roster form, an element is not repeated. For example, the set of letters forming the word "SCHOOL" is {S, C, H, O, L} and not {S, C, H, O, O, L}

Example-2. Write the set $B = \{x : x$ is a natural number and $x^2 < 40\}$ in the roster form.

Solution : We look at natural numbers and their squares starting from 1. When we reach 7, the sqaure is 49 which is greater than 40. The required numbers are 1, 2, 3, 4, 5, 6.

So, the given set in the roster form is $B = \{1, 2, 3, 4, 5, 6\}$.

DO THIS

1. List the elements of the following sets.
   (i) G = all the factors of 20
   (ii) F = the multiples of 4 between 17 and 61 which are divisible by 7
   (iii) $S = \{x : x$ is a letter in the word 'MADAM'$\}$
   (iv) $P = \{x : x$ is a whole number between 3.5 and $6.7\}$
2. Write the following sets in the roster form.
   (i) B is the set of all months in a year having 30 days.
   (ii) P is the set of all prime numbers less than 10.
   (iii) X is the set of the colours of the rainbow
3. A is the set of factors of 12. Which one of the following is not a member of A.
   (A) 1 (B) 4 (C) 5 (D) 12

TRY THIS

1. Formulate sets of your choice, involving algebraic and geometrical ideas.
2. Match roster forms with the set builder form.

(i)	{P, R, I, N, C, A, L}	(a)	$\{x : x$ is a positive integer and is a divisor of $18\}$
(ii)	{0}	(b)	$\{x : x$ is an integer and $x^2 - 9 = 0\}$
(iii)	{1, 2, 3, 6, 9, 18}	(c)	$\{x : x$ is an integer and $x + 1 = 1\}$
(iv)	{3, -3}	(d)	$\{x : x$ is a letter of the word PRINCIPAL$\}$

EXERCISE - 2.1

1. Which of the following are sets? Justify your answer?
   (i) The collection of all the months of a year begining with the letter "J".
   (ii) The collection of ten most talented writers of India.
   (iii) A team of eleven best cricket batsmen of the world.
   (iv) The collection of all boys in your class.
   (v) The collection of all even integers.
2. If A={0, 2, 4, 6}, B = {3, 5, 7} and C = {p, q, r} then fill the appropriate symbol, $\in$ or $\notin$ in the blanks.

(i) 0 ….. A	(ii) 3 ….. C	(iii) 4 ….. B
(iv) 8 ….. A	(v) p ….. C	(vi) 7 ….. B

3. Express the following statements using symbols.
   (i) The elements 'x' does not belong to 'A'.
   (ii) 'd' is an element of the set 'B'.
   (iii) '1' belongs to the set of Natural numbers N.
   (iv) '8' does not belong to the set of prime numbers P.
4. State whether the following statements are true or false.
   (i) $5 \notin$ {Prime numbers}
   (ii) S = {5, 6, 7} implies $8 \in$ S.
   (iii) $-5 \notin$ W where 'W' is the set of whole numbers
   (iv) $\frac{8}{11} \in$ Z where 'Z' is the set of integers.

5. Write the following sets in roster form.

(i) B = {x : x is a natural number less than 6}

(ii) C = {x : x is a two-digit natural number such that the sum of its digits is 8}.

(iii) D = {x : x is a prime number which is a divisor of 60}.

(iv) E = {the set of all letters in the word BETTER}.

6. Write the following sets in the set-builder form.

(i) {3, 6, 9, 12} (ii) {2, 4, 8, 16, 32}

(iii) {5, 25, 125, 625} (iv) {1, 4, 9, 25, ….. 100}

7. List all the elements of the following sets in roster form.

(i) A = {x : x is a natural number greater than 50 but less than 100}

(ii) B = {x : x is an integer, $x^2 = 4$}

(iii) D = {x : x is a letter in the word "LOYAL"}

8. Match the roster form with set-builder form.

(i)	{1, 2, 3, 6}	(a)	{x : x is prime number and a divisor of 6}
(ii)	{2, 3}	(b)	{x : x is an odd natural number less than 10}
(iii)	{M, A, T, H, E, I, C, S}	(c)	{x : x is a natural number and divisor of 6}
(iv)	{1, 3, 5, 7, 9}	(d)	{x : x is a letter of the word MATHEMATICS}

2.4 Types of Set

Let us consider the following examples of sets:

(i) A = {x : x is natural number smaller than 1}

(ii) D = {x : x is a odd prime number divisible by 2}

How many elements are there in A and D? We find that there is no natural number which is smaller than 1. So set A contains no elements or we say that A is an empty set.

Similarly, there are no prime numbers that are divisible by 2. So, D is also an empty set.

A set which does not contain any element is called an **empty** set, or a **Null** set, or a **void** set. Empty set is denoted by the symbol ϕ or { }.

Here are some more examples of empty sets.

(i) A = $\{x : 1 < x < 2, x$ is a natural number$\}$

(ii) B = $\{x : x^2 - 2 = 0$ and x is a rational number$\}$

(iii) D = $\{x : x^2 = 4, x$ is odd$\}$

Note : ϕ and $\{0\}$ are two different sets. $\{0\}$ is a set containing the single element 0 while $\{\}$ is null set.

Finite & Infinite sets

Now consider the following sets:

(i) A = {the students of your school} (ii) L = {p,q,r,s}

(iii) B = $\{x : x$ is an even number$\}$ (iv) J = $\{x : x$ is a multiple of 7$\}$

Can you list the number of elements in each of the sets given above? In (i), the number of elements will be the number of students in your school. In (ii), the number of elements in set L is 4. We find that it is possible to count the number of elements of sets A and L or that they contain a finite number of elements. Such sets are called **finite sets**.

Now, consider the set B of all even numbers. We cannot count all of them i.e., we see that the number of elements of this set is not finite. Similarly, all the elements of J cannot be listed. We find that the number of elements in B and J is infinite. Such sets are called **infinite sets**.

We can draw many number of straight lines passing though a given point. So this set is infinite. Similarly, it is not possible to find out the last even number or odd number among the collection of all integers. Thus, we can say a set is infinite if it is not finite.

Consider some more examples :

(i) Let 'W' be the set of the days of the week. Then W is finite.

(ii) Let 'S' be the set of solutions of the equation $x^2 - 16 = 0$. Then S is finite.

(iii) Let 'G' be the set of points on a line. Then G is infinite.

Example-3. State which of the following sets are finite or infinite.

(i) $\{x : x \in \mathrm{N}$ and $(x - 1)(x - 2) = 0\}$ (ii) $\{x : x \in \mathrm{N}$ and $x^2 = 4\}$

(iii) $\{x : x \in \mathrm{N}$ and $2x - 2 = 0\}$ (iv) $\{x : x \in \mathrm{N}$ and x is prime$\}$

(v) $\{x : x \in \mathrm{N}$ and x is odd$\}$

Solution :

(i) x can take the values 1 or 2 in the given case. The set is $\{1,2\}$. Hence, it is finite.

(ii) $x^2 = 4$, implies that $x = +2$ or -2. But $x \in$ N or x is a natural number so the set is $\{2\}$. Hence, it is finite.

(iii) In a given set $x = 1$ and $1 \in$ N. Hence, it is finite.

(iv) The given set is the set of all prime numbers. There are infinitely many prime numbers. Hence, set is infinite.

(v) Since there are infinite number of odd numbers, hence the set is infinite.

Now, consider the following finite sets :

$A = \{1, 2, 4\}$; $B = \{6, 7, 8, 9, 10\}$; $C = \{x : x$ is a alphabet in the word "INDIA"$\}$

Here,

Number of elements in set A = 3.

Number of elements in set B = 5.

Number of elements in set C = 4 (In the set C, the element 'I' repeats twice. We know that the elements of a given set should be distinct. So, the number of distinct elements in set C is 4).

The number of elements in a set is called the cardinal number of the set. The cardinal number of the set A is denoted as n(A) = 3.

Similarly, n(B) = 5 and n(C) = 4.

Note **:** There are no elements in a null set. The cardinal number of that set is 0. $\therefore$ n(ϕ) = 0

Example-4. If $A = \{1, 2, 3\}$; $B = \{a, b, c\}$ then find n(A) and n(B).

Solution : The set A contains three distinct elements $\therefore$ n(A) = 3

and the set B contains three distinct elements $\therefore$ n(B) = 3

Do These

1. Which of the following are empty sets? Justify your answer.

(i) Set of integers which lie between 2 and 3.

(ii) Set of natural numbers that are less than 1.

(iii) Set of odd numbers that have remainder zero, when divided by 2.

2. State which of the following sets are finite and which are infinite. Give reasons for your answer.

(i) $A = \{x : x \in N \text{ and } x < 100\}$ (ii) $B = \{x : x \in N \text{ and } x \leq 5\}$

(iii) $C = \{1^2, 2^2, 3^2, ...\}$ (iv) $D = \{1, 2, 3, 4\}$

(v) $\{x : x \text{ is a day of the week}\}$.

3. Tick the set which is infinite

(A) The set of whole numbers < 10 (B) The set of prime number < 10

(C) The set of integers < 10 (D) The set of factors of 10

Try This

1. Which of the following sets are empty sets? Justify your answer.

(i) $A = \{x : x^2 = 4 \text{ and } 3x = 9\}$.

(ii) The set of all triangles in a plane having the sum of their three angles less than 180.

2. $B = \{x : x + 5 = 5\}$ is not an empty set. Why?

Think - Discuss

An empty set is a finite set. Is this statement true or false? Why?

Exercise - 2.2

1. State which of the following sets are empty and which are not?

(i) The set of straight lines passing through a point.

(ii) Set of odd natural numbers divisible by 2.

(iii) $\{x : x \text{ is a natural number}, x < 5 \text{ and } x > 7\}$

(iv) $\{x : x \text{ is a common point to any two parallel lines}\}$

(v) Set of even prime numbers.

2. Which of the following sets are finite or infinite.

(i) The set of months in a year. (ii) $\{1, 2, 3, \ldots, 99, 100\}$

(iii) The set of prime numbers less than 99.

3. State whether each of the following set is finite or infinite.

   (i) The set of letters in the English alphabet.

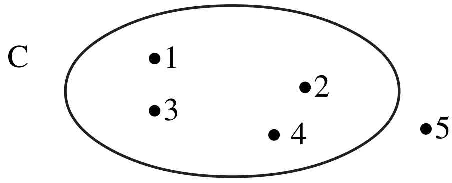
   (ii) The set of lines which are parallel to the X-Axis.

   (iii) The set of numbers which are multiplies of 5.

   (iv) The set of circles passing through the origin (0, 0).

2.5 Using Diagrams to Represent Sets

If S is a **set and *x* is an object then either** $x \in S$ or $x \notin S$. Every set can be represented by a drawing a closed curve C where elements of C are represented by points within C and elements not in the set by points outside C. For example, the set C = {1, 2, 3, 4} can be represented as shown below:



2.6 Universal Set and Subsets

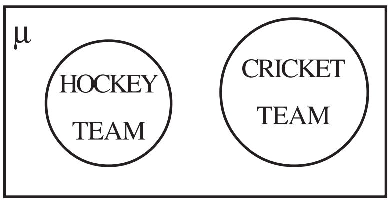
Let us consider that a cricket team is to be selected from your school. What is the set from which the team can be selected? It is the set of all students in your school. Now, we want to select the hockey team. Again, the set from which the team will be selected is the set of all students in your school. So, for selection of any school team, the students of your school are considered as the universal set.

Let us see some more examples of universal sets:

(i) If we want to study the various groups of people of our state, universal set is the set of all people in Andhra Pradesh.

(ii) If we want to study the various groups of people in our country, universal set is the set of all people in India.

The universal set is denoted by 'μ'. The Universal set is usually represented by rectangles.



If the set of real numbers R; is the universal set then what about rational and irrational numbers?

Let us consider the set of rational numbers,

$$Q = \{x : x = \frac{p}{q}, \text{p, q} \in \text{z and q} \neq 0\}$$

which is read as 'Q' is the set of all numbers such that x equals $\frac{p}{q}$, where p and q are integers and q is not zero. Then we know that every element of Q is also an element of R. So, we can say that Q is a subset of R. If Q is a subset of R, then we write is as $Q \subset R$.

Note : It is often convenient to use the symbol '$\Rightarrow$' which means implies.

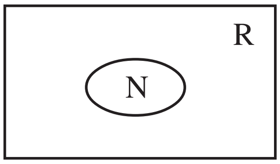
Using this symbol, we can write the definition of subset as follows:

$A \subset B$ if $a \in A \Rightarrow a \in B$, where A, B are two sets.

We read the above statement as "A is a subset of B if 'a' is an element of A implies that 'a' is also an element of B".

Real numbers R; has many subsets. For example,

The set of natural numbers $N = \{1, 2, 3, 4, 5, ...\}$



The set of whole number $W = \{0, 1, 2, 3, ...\}$

The set of integers $Z = \{...., -3, -2, -1, 0, 1, 2, 3, ...\}$

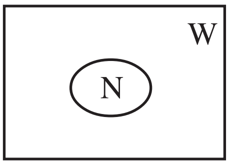
The set of irrational numbers Q', is composed of all real numbers that are not rational.

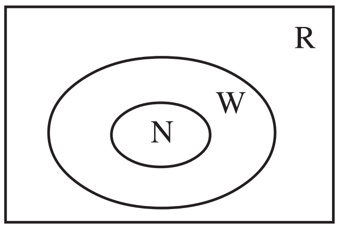
Thus, $Q' = \{x : x \in R \text{ and } x \notin Q\}$ i.e., all real numbers that are not rational. e.g. $\sqrt{2}$, $\sqrt{5}$ and π.

Similarly, the set of natural numbers, N is a subset of the set of whole numbers W and we can write $N \subset W$. Also W is a subet of R.

That is $N \subset W$ and $W \subset R$

$\Rightarrow N \subset W \subset R$





Some of the obvious relations among these subsets are $N \subset Z \subset Q$, $Q \subset R$, $Q' \subset R$, and $N \not\subset Q'$.

Example-5. Consider a set of vowels letters, $V = \{a, e, i, o, u\}$. Also consider the set A, of all letters in the English alphabet. $A = \{a, b, c, d, ..., z\}$. Identify the universal set and the subset in the given example.

Solution : We can see that every element of set V is also an element A. But every element of A is not a part of V. In this case, V is the subset of A.

In other words $V \subset A$ since whenever $a \in V$, then $a \in A$.

Note **:** Since the empty set ϕ has no elements, we consider that ϕ is a subset of every set.

If A is not a subset of B ($A \not\subset B$), that means there is at least one element in A that is not a member of B.

Let us consider some more examples of subsets.

- The set C = {1, 3, 5} is a subset of D = {5, 4, 3, 2, 1}, since each number 1, 3, and 5 belonging to C also belongs to D.
- Let A = {a, e, i, o, u} and B = {a, b, c, d} then A is not a subset of B. Also B is not a subset of A.

2.6.1 Equal Sets

Consider the following sets.

A = {Sachin, Dravid, Kohli}

B = {Dravid, Sachin, Dhoni}

C = {Kohli, Dravid, Sachin}

What do you observe in the above three sets A, B and C? All the players that are in A are in C but not in B. Thus, A and C have same elements but some elements of A and B are different. So, the sets A and C are equal sets but sets A and B are not equal.

Two sets A and C are said to be **equal** if every element in A belongs to C and every element in C belongs to A. If A and C are equal sets, than we write A = C.

Example-6. Consider the following sets:

$A = \{p, q, r\}$ $\qquad$ $B = \{q, p, r\}$

In the above sets, every element of A is also an element of B. $\therefore A \subset B$.

Similarly every element of B is also in A. $\qquad \therefore B \subset A$.

Thus, we can also write that if $B \subset A$ and $A \subset B \Leftrightarrow A = B$. Here $\Leftrightarrow$ is the symbol for two way implication and is usually read as, **if and only if** (briefly written as "iff").

Examples-7. If A = {1, 2, 3, ….} and N is a set of natural numbers, check whether A and N are equal?

Solution : The elements are same in both the sets. Therefore, both A and N are the set of Natural numbers. Therefore the sets A and N are equal sets or A = N.

Example-8. Consider the sets $A = \{p, q, r, s\}$ and $B = \{1, 2, 3, 4\}$. Are they equal?

Solution : A and B do not contain the same elements. So, $A \neq B$.

Example-9. Let A be the set of prime numbers less than 6 and P the set of prime factors of 30. Check if A and P are equal.

Solution : The set of prime numbers less than 6, $A = \{2,3,5\}$

The prime factors of 30 are 2, 3 and 5. So, $P = \{2,3,5\}$

Since the elements of A are the same as the elements of P, therefore, A and P are equal.

Example-10. Show that the sets A and B are equal, where

$A = \{x : x$ is a letter in the word 'ASSASSINATION'$\}$

$B = \{x : x$ is a letter in the word STATION$\}$

Solution : Given, $A = \{x : x$ is a letter in the word 'ASSASSINATION'$\}$

This set A can also be written as $A = \{A,S,I,N,T,O\}$ since generally elements in a set are not repeated.

Also given that $B = \{x : x$ is a letter in the word STATION$\}$

'B' can also be written as $B = \{A,S,I,N,T,O\}$

So, the elements of A nd B are same and $A = B$

Exercise - 2.3

1. Which of the following sets are equal?

   (i) $A = \{x : x$ is a letter in the word FOLLOW$\}$

   (ii) $B = \{x : x$ is a letter in the word FLOW$\}$

   (iii) $C = \{x : x$ is a letter in the word WOLF$\}$

2. Consider the following sets and fill up the blank in the statement given below with = or $\neq$ so as to make the statement true.

   $A = \{1, 2, 3\}$; $B = \{$The first three natural numbers$\}$

   $C = \{a, b, c, d\}$; $D = \{d, c, a, b\}$

   $E = \{a, e, i, o, u\}$; $F = \{$set of vowels in English Alphabet$\}$

(i) A ···· B (ii) A ···· E (iii) C ···· D

(iv) D ···· F (v) F ···· A (vi) D ···· E

(vii) F ···· B

3. In each of the following, state whether A = B or not.

(i) $A = \{a, b, c, d\}$ $B = \{d, c, a, b\}$

(ii) $A = \{4, 8, 12, 16\}$ $B = \{8, 4, 16, 18\}$

(iii) $A = \{2, 4, 6, 8, 10\}$ $B = \{x : x \text{ is a positive even integer and } x < 10\}$

(iv) $A = \{x : x \text{ is a multiple of } 10\}$ $B = \{10, 15, 20, 25, 30, \ldots\}$

Consider the set $E = \{2, 4, 6\}$ and $F = \{6, 2, 4\}$. Note that E = F. Now, since each element of E also belongs to F, therefore E is a subset of F. But each element of F is also an element of E. So F is a subset of E. In this manner it can be shown that every set is a subset of itself.

If A and B contain the same elements, they are equal i.e. A = B. By this observation we can say that "Every set is subset of itself".

Example-11. Consider the sets ϕ, $A = \{1, 3\}$, $B = \{1, 5, 9\}$, $C = \{1, 3, 5, 7, 9\}$. Insert the symbol $\subset$ or $\not\subset$ between each of the following pair of sets.

(i) ϕ ….. B (ii) A ….. B (iii) A ….. C (iv) B ….. C

Solution : (i) $\phi \subset B$, as ϕ is a subset of every set.

(ii) $A \not\subset B$, for $3 \in A$ but $3 \notin B$.

(iii) $A \subset C$ as $1, 3 \in A$ also belong to C.

(iv) $B \subset C$ as each element of B is also an element of C.

Do This

1. $A = \{1, 2, 3, 4\}$, $B = \{2, 4\}$, $C = \{1, 2, 3, 4, 7\}$, $F = \{\ \}$.

Fill in the blanks with $\subset$ or $\not\subset$.

(i) A ….. B (ii) C ….. A (iii) B ….. A

(iv) A ….. C (v) B ….. C (vi) ϕ ….. B

2. State which of the following statement are true.

(i) $\{\ \} = \phi$ (ii) $\phi = 0$ (iii) $0 = \{0\}$

Try This

1. A = {quadrilaterals}, B = {square, rectangle, trapezium, rhombus}. State whether $A \subset B$ or $B \subset A$. Justify your answer.

2. If A = {a, b, c, d}. How many subsets does the set A have?
   (A) 5 (B) 6 (C) 16 (D) 65

3. P is the set of factors of 5, Q is the set of factors of 25 and R is the set of factors of 125. Which one of the following is false?
   (A) $P \subset Q$ (B) $Q \subset R$ (C) $R \subset P$ (D) $P \subset R$

4. A is the set of prime numbers less than 10, B is the set of odd numbers < 10 and C is the set of even numbers < 10. How many of the following statements are true?
   (i) $A \subset B$ (ii) $B \subset A$ (iii) $A \subset C$
   (iv) $C \subset A$ (v) $B \subset C$ (vi) $X \subset A$

Consider the following sets:

A = {1, 2, 3}, B = {1, 2, 3, 4}, C = {1, 2, 3, 4, 5}

All the elements of A are in B $\therefore A \subset B$.

All the elements of B are in C $\therefore B \subset C$.

All the elements of A are in C $\therefore A \subset C$.

That is, $A \subset B, B \subset C \Rightarrow A \subset C$.

Exercise - 2.4

1. State which of the following statements are true given that. A = {1, 2, 3, 4}
   (i) $2 \in A$ (ii) $2 \in \{1, 2, 3, 4\}$
   (iii) $A \subset \{1, 2, 3, 4\}$ (iv) $\{2, 3, 4\} \subset \{1, 2, 3, 4\}$

2. State the reasons for the following :
   (i) $\{1, 2, 3, \ldots, 10\} \neq \{x : x \in N \text{ and } 1 < x < 10\}$
   (ii) $\{2, 4, 6, 8, 10\} \neq \{x : x = 2n+1 \text{ and } x \in N\}$

(iii) {5, 15, 30, 45} $\neq$ {x : x is a multiple of 15}

(iv) {2, 3, 5, 7, 9} $\neq$ {x : x is a prime number}

3. List all the subsets of the following sets.

(i) B = {p, q} (ii) C = {x, y, z} (iii) D = {a, b, c, d}

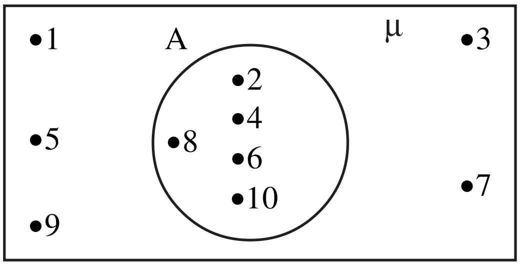
(iv) E = {1, 4, 9, 16} (v) F = {10, 100, 1000}

2.7 Venn Diagrams

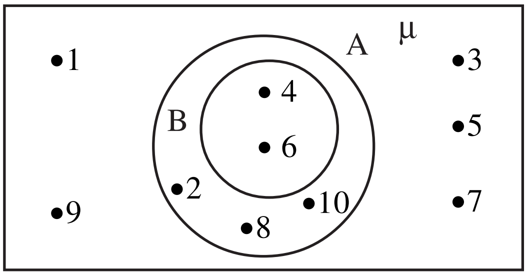
We have already seen some ways of representing sets using diagrams. Let us study it in more detail now. Venn-Euler diagram or simply Venn-diagram is a way of representing the relationships between sets. These diagrams consist of rectangles and closed curves usually circles.

As mentioned earlier in the chapter, the universal set is usually represented by a rectangle.

(i) Consider that μ = {1, 2, 3, …., 10} is the universal set of which, A = {2, 4, 6, 8, 10} is a subset. Then the venn-diagrams is as:

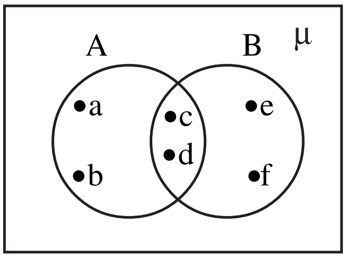


(ii) μ = {1, 2, 3, …., 10} is the universal set of which, A = {2, 4, 6, 8, 10} and B = {4, 6} are subsets and also $B \subset A$. Then we have the following figure:



(iii) Let A = {a, b, c, d} and B = {c, d, e, f}.

Then we illustrate these sets with a Venn diagram as



2.8 Basic Operations on Sets

We know that arithmetic has operations of additions, subtraction and multiplication of numbers. Similarly in sets, we define the operation of union, intersection and difference of sets.

2.8.1 Union of Sets

Example-12. Suppose A is the set of students in your class who were absent on Tuesday and B the set of students who were absent on Wednesday. Then,

A = {Roja, Ramu, Ravi} and

B = {Ramu, Preethi, Haneef}

Now, we want to find K, the set of students who were absent on either Tuesday or Wednesday. Then, does Roja $\in$ K? Ramu $\in$ K? Ravi $\in$ K? Haneef $\in$ K? Preeti $\in$ K? Akhila $\in$ K?

Roja, Ramu, Ravi, Haneef and Preeti all belong to K but Ganpati does not.

Hence, K = {Roja, Ramu, Raheem, Prudhvi, Preethi}

Here K is the called the union of sets A and B. The union of A and B is the set which consists of all the elements of A and B and the common elements being taken only once. The symbol 'μ' is used to denote the union. Symbolically, we write $A \cup B$ and usually read as 'A union B'.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Example-13. Let A = {2, 5, 6, 8} and B = {5, 7, 9, 1}. Find $A \cup B$.

Solution : We have $A \cup B$ = {1, 2, 5, 6, 7, 8, 9}.

Note that the common element 5 was taken only once while writing $A \cup B$.

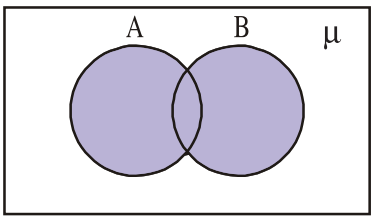
Example-14. Let A = {a, e, i, o, u} and B = {a, i, u}. Show that $A \cup B = A$.

Solution : We have $A \cup B$ = {a, e, i, o, u} = A.

This example illustrates that union of sets A and its subset B is the Set A itself.

i.e, if $B \subset A$, then $A \cup B = A$.

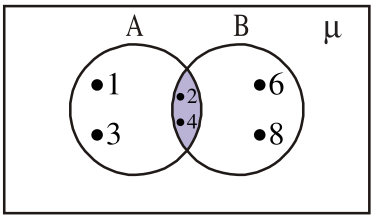
The union of the sets can be represented by a Venn-diagram as shown (shaded portion)



Example-15. Illustrate $A \cup B$ in Venn-diagrams where.

A = {1, 2, 3, 4} and B = {2, 4, 6, 8}

Solution :



$A \cup B = \{1, 2, 3, 4, 6, 8\}$

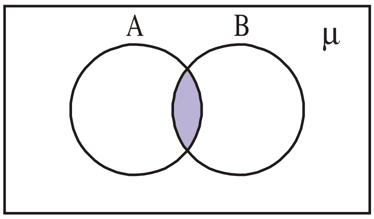
2.8.2 Intersection of Sets

Let us again consider the example of absent students. This time we want to find the set L of students who were absent on both Tuesday and Wednesday. We find that L = {Ramu}. Here, L is called the intersection of sets A and B.

In general, the intersection of sets A and B is the set of all elements which are common to A and B. i.e., those elements which belong to A and also belong to B. We denote intersection by $A \cap B$. (read as "A intersection B"). Symbolically, we write

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

The intersection of A and B can be illustrated in the Venn-diagram as shown in the shaded portion in the adjacent figure.



$A \cap B$

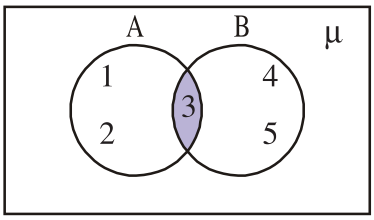
Example-16. Find $A \cap B$ when $A = \{5, 6, 7, 8\}$ and $B = \{7, 8, 9, 10\}$.

Solution : The common elements in both A and B are 7, 8.

$\therefore\ A \cap B = \{7, 8\}$.

Example-17. Illustrate $A \cap B$ in Venn-diagrams where $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$

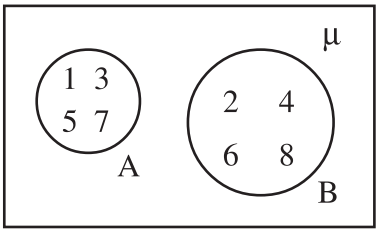
Solution : The intersection of A and B can be illustrated in the Venn-diagram as follows:



$A \cap B = \{3\}$

2.8.3 Disjoint Set

Suppose A {1, 3, 5, 7} and B = {2, 4, 6, 8}. We see that there are no common elements in A and B. Such sets are known as disjoint sets. The disjoint sets can be represented by means of the Venn-diagram as follows:



$A \cap B = \phi$

Do This

1. Let A = {1, 3, 7, 8} and B = {2, 4, 7, 9}. Find $A \cap B$.
2. If A = {6, 9, 11}; B = { }, find $A \cup \phi$.
3. A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}; B = {2, 3, 5, 7}. Find $A \cap B$ and show that $A \cap B = B$.
4. If A = {4, 5, 6}; B = {7, 8} then show that $A \cup B = B \cup A$.

Try This

1. List out some sets A and B and choose their elements such that A and B are disjoint
2. If A = {2, 3, 5}, find $A \cup \phi$ and $\phi \cup A$ and compare.
3. If A = {1, 2, 3, 4}; B = {1, 2, 3, 4, 5, 6, 7, 8} then find $A \cup B$, $A \cap B$. What do you notice about the result?
4. A = {1, 2, 3, 4, 5, 6}; B = {2, 4, 6, 8, 10}. Find the intersection of A and B.

Think - Discuss

The intersection of any two disjoint sets is a null set. Justify your answer.

2.8.4 Difference of Sets

The difference of sets A and B is the set of elements which belong to A but do not belong to B. We denote the difference of A and B by A – B or simply "A Minus B".

$$A - B = \{x : x \in A \text{ and } x \notin B\}.$$

Example-18. Let A = {1, 2, 3, 4, 5}; B = {4, 5, 6, 7}. Find A – B.

Solution : Given A = {1, 2, 3, 4, 5} and B = {4, 5, 6, 7}. Only the elements which are in A but not in B should be taken.

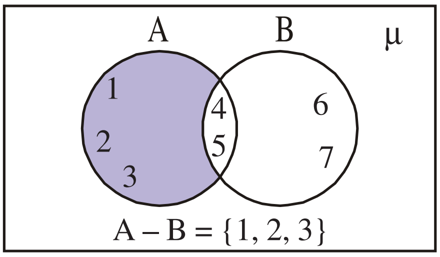
$\therefore$ A – B = {1, 2, 3}. Since 4, 5 are the elements in B they are not taken.

Similarly for B – A, the elements which are only in B are taken.

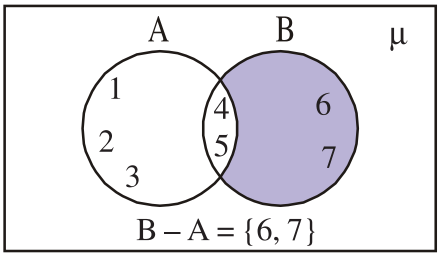
$\therefore$ B – A = {6, 7} (4, 5 are the elements in A).

Note that A – B $\neq$ B – A

The Venn diagram of A-B is as shown.



A – B = {1, 2, 3}



B – A = {6, 7}

Example-19. Observe the following

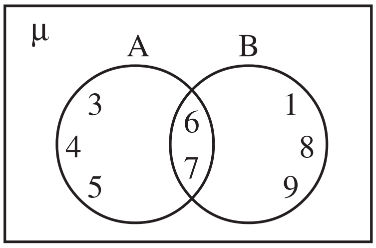
A = {3, 4, 5, 6, 7} $\therefore$ n(A) = 5

B = {1, 6, 7, 8, 9} $\therefore$ n(B) = 5

A $\cup$ B = {1, 3, 4, 5, 6, 7, 8, 9} $\therefore$ n(A $\cup$ B) = 8

A $\cap$ B = {6, 7} $\therefore$ n(A $\cap$ B) = 2

$\therefore$ n(A $\cup$ B) = 5 + 5 – 2 = 8



We observe that **n(A $\cup$ B) = n(A) + n(B) – n(A $\cap$ B)**

Do This

1. If A = {1, 2, 3, 4 ,5}; B = {4, 5, 6, 7}then find A – B and B – A. Are they equal?
2. If V = {a, e, i, o, u} and B = {a, i,k, u}, find V – B and B – V.

Think - Discuss

The sets A – B, B – A and A $\cap$ B are mutually disjoint sets. Use examples to observe if this is true.

Exercise - 2.5

1. If $A = \{1, 2, 3, 4\}$; $B = \{1, 2, 3, 5, 6\}$ then find $A \cap B$ and $B \cap A$. Are they equal?
2. $A = \{0, 2, 4\}$, find $A \cap \phi$ and $A \cap A$. Comment.
3. If $A = \{2, 4, 6, 8, 10\}$ and $B = \{3, 6, 9, 12, 15\}$, find $A - B$ and $B - A$.
4. If A and B are two sets such that $A \subset B$ then what is $A \cup B$?

5. If $A = \{x : x \text{ is a natural number}\}$

   $B = \{x : x \text{ is an even natural number}\}$

   $C = \{x : x \text{ is an odd natural number}\}$

   $D = \{x : x \text{ is a prime number}\}$

   Find $A \cap B$, $A \cap C$, $A \cap D$, $B \cap C$, $B \cap D$, $C \cap D$.
6. If $A = \{3, 6, 9, 12, 15, 18, 21\}$; $B = \{4, 8, 12, 16, 20\}$

   $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$; $D = \{5, 10, 15, 20\}$ find

   (i) $A - B$ (ii) $A - C$ (iii) $A - D$ (iv) $B - A$ (v) $C - A$

   (vi) $D - A$ (vii) $B - C$ (viii) $B - D$ (ix) $C - B$ (x) $D - B$
7. State whether each of the following statement is true or false. Justify you answers.

   (i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.

   (ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.

   (iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets.

   (iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets.

What We Have Discussed

1. A set is a well defined collection of objects where well defined means that:

   (i) All the objects in the set have a common feature or property; and

   (ii) It is possible to decide whether any given object belongs to the set or not.
2. An object belonging to a set is known as an element of the set. We use the symbol '$\in$' to denote 'belongs to'.

3. Sets can be written in the roster form where all elements of the set are written, separated by commas, within { } curly brackets.
4. Sets can also be written in the set-builder form.
5. A set which does not contain any element is called an **empty** set, or a **Null** set, or a **void** set.
6. A set is called a finite set if it is possible to count the number of elements of that set.
7. We can say that a set is infinite if it is not finite.
8. The number of elements in a set is called the cardinal number of the set.
9. The universal set is denoted by 'μ'. The Universal set is usually represented by rectangles.
10. A is a subset of B if 'a' is an element of A implies that 'a' is also an element of B. This is written as $A \subset B$ if $a \in A \Rightarrow a \in B$, where A, B are two sets.
11. Two sets, A and B are said to be **equal** if every element in A belongs to B and every element in B belongs to A.
12. A union B is written as $A \cup B = \{x : x \in A \text{ or } x \in B\}$.
13. A intersection B is written as $A \cap B = \{x : x \in A \text{ and } x \in B\}$
14. The difference of two sets A, B is denoted as $A - B$ or $B - A$
15. Venn diagrams are a convenient way of showing operations between sets.