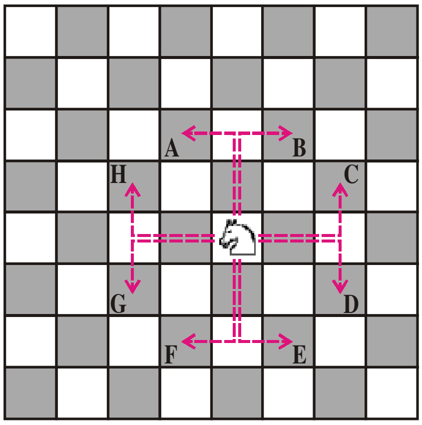
CHAPTER 7

Coordinate Geometry

7.1 Introduction

You know that in chess, the Knight moves in 'L' shape or two and a half steps (see figure). It can jump over other pieces too. A Bishop moves diagonally, as many steps as are free in front of it.



Find out how other pieces move. Also locate Knight, Bishop and other pieces on the board and see how they move.

Consider that the Knight is at the origin (0, 0). It can move in 4 directions as shown by dotted lines in the figure. Find the coordinates of its position after the various moves shown in the figure.

Do This

i. From the figure write coordinates of the points A, B, C, D, E, F, G, H.

ii. Find the distance covered by the Knight in each of its 8 moves i.e. find the distance of A, B, C, D, E, F, G and H from the origin.

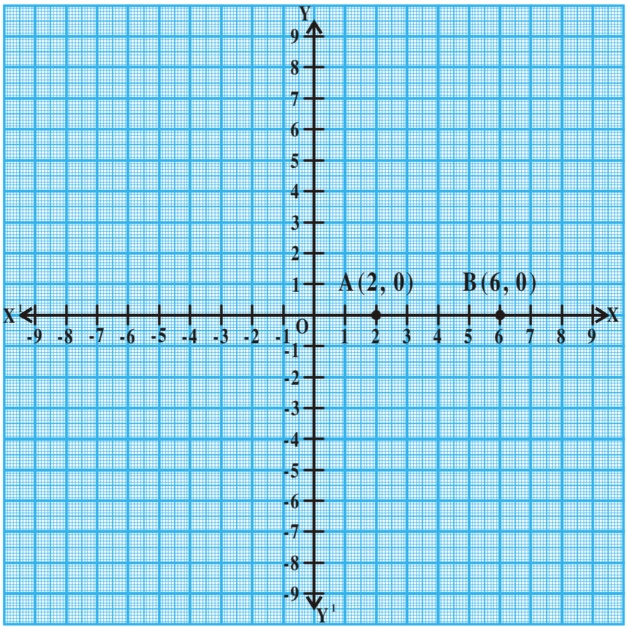
iii. What is the distance between two points H and C? and also find the distance between two points A and B

7.2 Distance Between Two Points

The two points (2, 0) and (6, 0) lie on the X-axis as shown in figure.

It is easy to see that the distance between two points A and B as 4 units.

We can say the distance between points lying on X-aixs is the difference between the *x*-coordinates.



What is the distance between (–2, 0) and (–6, 0)?

The difference in the value of x-coordinates is

$(-6) - (-2) = -4$ (Negative)

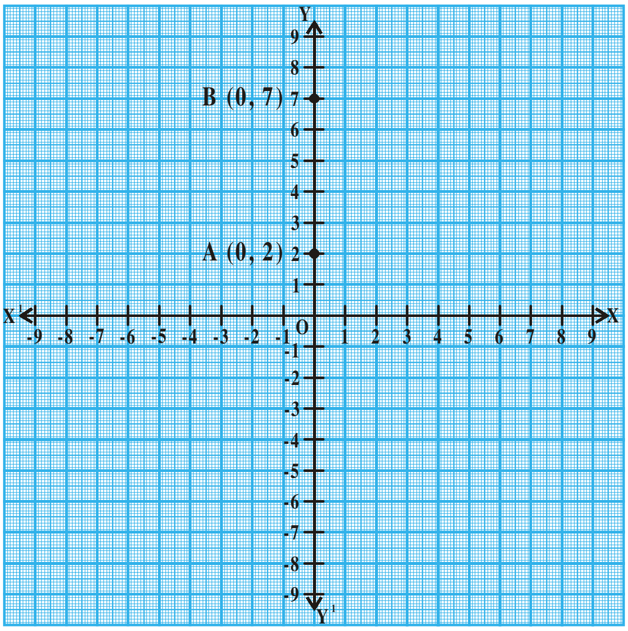
We never say the distance in negative values.

So, we calculate that absolute value of the distance.

Therefore, the distance

$= |(-6) - (-2)| = |-4| = 4$ units.

So, in general for the points $A(x_1, 0)$, $B(x_2, 0)$ on the X-axis, the distance between A and B is $|x_2 - x_1|$



Similarly, if two points lie on Y-axis, then the distance between the points A and B would be the difference between their y coordinates of the points.

The distance between two points $(0, y_1)$ $(0, y_2)$ would be $|y_2 - y_1|$.

For example, Let the points be A(0, 2) and B(0, 7)

Then, the distance between A and B is $|7 - 2| = 5$ units.

Do this

1. Where do these following points lie (–4, 0), (2, 0), (6, 0), (–8, 0).
2. What is the distance between points (–4, 0) and (6, 0)?

Try this

1. Where do these following points lie (0, –3), (0, –8), (0, 6), (0, 4)
2. What is the distance between (0, –3), (0, –8) and justify that the distance between two points on Y-axis is $|y_2 - y_1|$.

Think - Discuss

How will you find the distance between two points in which x or y coordinates are same but not zero?

7.3 Distance Between Two Points on a Line Parallel to the Coordinate Axes.

Consider the points $A(x_1, y_1)$ and $B(x_2, y_1)$. Since the y-coordinates are equal, points lie on a line, parallel to X-axis.

AP and BQ are drawn perpendicular to X-axis.

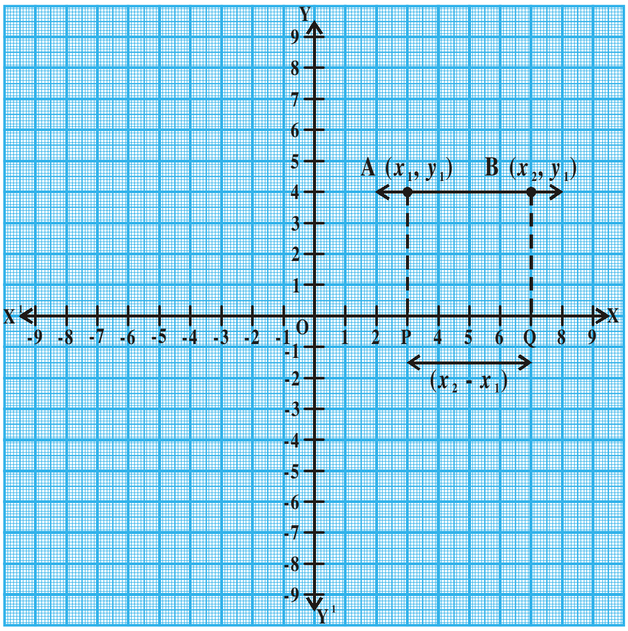
Observe the figure. The distance between two points A and B is equal to the distance between P and Q.

Therefore,

Distance AB = Distance

PQ = $|x_2 - x_1|$ (i.e., The difference between x coordinates)

Similarly, line joining two points $A(x_1, y_1)$ and $B(x_1, y_2)$ parallel to Y-axis, then the distance between these two points is $|y_2 - y_1|$ (i.e. the difference between y coordinates)



Example-1. What is the distance between A (4,0) and B (8, 0).

Solution : The difference in the x coordinates is $|x_2 - x_1| = |8 - 4| = 4$ units.

Example-2. A and B are two points given by (8, 3), (–4, 3). Find the distance between A and B.

Solution : Here x_1 and x_2 are lying in two different quadrants and y-coordinate are equal.

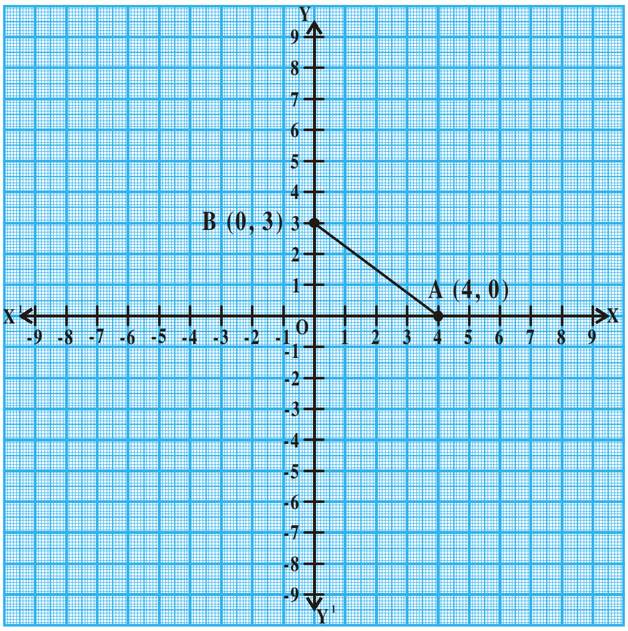
Distance AB = $|x_2 - x_1| = |-4 - 8| = |-12| = 12$ units

i. (3, 8), (6, 8)

ii. (–4,–3), (–8,–3)

iii. (3, 4), (3, 8)

(iv) (–5, –8), (–5, –12)



Let A and B denote the points (4, 0) and (0, 3) and 'O' be the origin.

The ΔAOB is a right angle triangle.

From the figure

OA = 4 units (x-coordinate)

OB = 3 units (y-coordinate)

Then distance AB = ?

Hence, by using pythagoran theorem

$$AB^2 = AO^2 + OB^2$$

$$AB^2 = 4^2 + 3^2$$

$AB = \sqrt{16+9} = \sqrt{25} = 5$ units $\Rightarrow$ is the distance between A and B.

Do This

Find the distance between the following points (i) A = (2, 0) and B(0, 4) (ii) P(0, 5) and Q(12, 0)

Try This

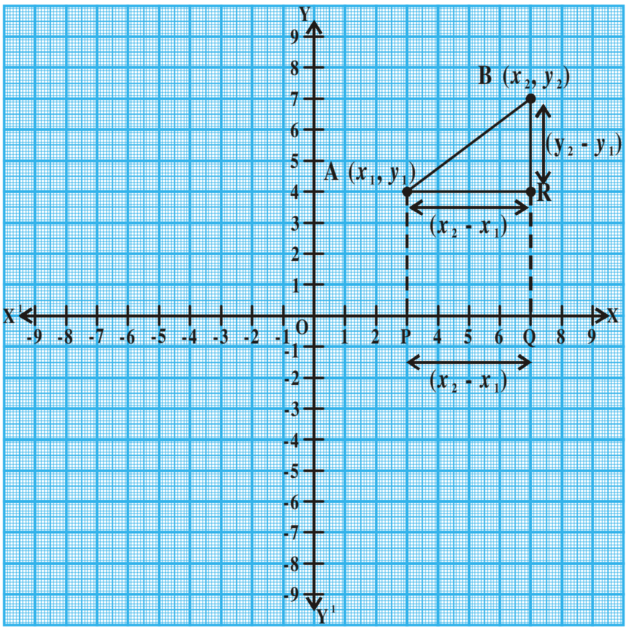
Find the distance between points 'O' (origin) and 'A' (7, 4).

Think - Discuss

1. Ramu says the distance of a point $P(x_1, y_1)$ from the origin O(0, 0) is $\sqrt{x^2 + y^2}$. Do you agree with Ramu or not? Why?

2. Ramu also writes the distance formulas as $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (why?)

7.4 Distance Between Any Two Points on a line in the X-Y plane



Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two points (on a line) in a plane as shown in figure.

Draw AP and BQ perpendiculars to X-axis

Draw a perpendicular ARfrom the point A on BQ to meet at the point R.

Then $OP = x_1$, $OQ = x_2$

So $PQ = OQ - OP = x_2 - x_1$

Observe the shape of APQR. It is a rectangle.

So $PQ = AR = x_2 - x_1$.

Also $QB = y_2$, $QR = y_1$,

So $BR = QB - QR = y_2 - y_1$

from ΔARB (right angle triangle)

$$AB^2 = AR^2 + RB^2 \qquad \text{(By Pythagoras theorem)}$$

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

i.e., $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Hence, the distance between the points A and B is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

this is called the distance formula.

Example-3. Let's find the distance between two points A(4, 2) and B(8, 6)

Solution : Compare these points with $(x_1, y_1), (x_2, y_2)$

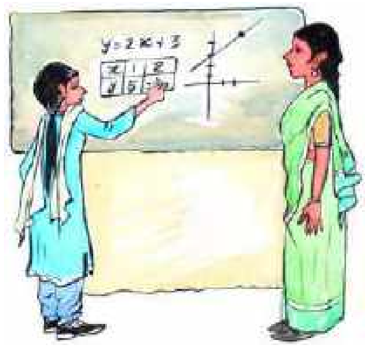
$x_1 = 4,\ x_2 = 8,\ y_1 = 3,\ y_2 = 6$

Using distance formula

$$\text{distance AB} = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8-4)^2 + (6-3)^2} = \sqrt{4^2 + 3^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units.}$$



Do This

Find the distance between the following pairs of points

(i) (7, 8) and (–2, 3) (ii) (–8, 6) and (2, 0)

Try This

Find the distance between A(1, –3) and B(–4, 4) and rounded to are decimal

Think - Discuss

Sridhar calculated the distance between T(5, 2) and R(–4, –1) to the nearest tenth is 9.5 units.

Now you find the distance between P (4, 1) and Q (-5, -2). Do you get the same answer that sridhar got? Why?

Let us see some examples

Example-4. Show that the points A (4, 2), B (7, 5) and C (9, 7) are three points lie on a same line.

Solution : Now, we find the distances AB, BC, AC

By using distance formula $= d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

So, $d = AB = \sqrt{(7-4)^2+(5-2)^2} = \sqrt{3^2+3^2} = \sqrt{9+9} = \sqrt{18}$

$= \sqrt{9\times 2} = 3\sqrt{2}$ units.

$BC = \sqrt{(9-7)^2+(7-5)^2} = \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

$AC = \sqrt{(9-4)^2+(7-2)^2} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$= \sqrt{25\times 2} = 5\sqrt{2}$ units.

Now AB + BC = $3\sqrt{2}+2\sqrt{2} = 5\sqrt{2} = AC$. Therefore, that the three points (4, 2), (7, 5) and (9, 7) lie on a straight line. (Points that lie on the same line are called collinear points).

Example-5. Are the points (3, 2), (–2, –3) and (2, 3) form a triangle?

Solution : Let us apply the distance formula to find the distances PQ, QR and PR, where P(3, 2), Q(–2, –3) and R(2, 3) are the given points. We have

$PQ = \sqrt{(-2-3)^2+(-3-2)^2} = \sqrt{(-5)^2+(-5)^2} = \sqrt{25+25} = \sqrt{50} = 7.07$ units (approx)

$QR = \sqrt{(2-(-2))^2+(3-(-3))^2} = \sqrt{(4)^2+(6)^2} = \sqrt{52} = 7.21$ units (approx)

$PR = \sqrt{(2-3)^2+(3-2)^2} = \sqrt{(-1)^2+1^2} = \sqrt{2} = 1.41$ units (approx)

Since the sum of any two of these distances is greater than the third distance, therefore, the points P, Q and R form a triangle and all the sides of triangle is unequal.

Example-6. Show that the points (1, 7), (4, 2), (–1, –1) and (–4, 4) are the vertices of a square.

Solution : Let A(1, 7), B(4, 2), C(–1, –1)and D(–4, 4) be the given points.

One way of showing that ABCD is a square is to use the property that all its sides should be equal and both its digonals should also be equal. Now

So sides are $AB = d = \sqrt{(1-4)^2+(7-2)^2} = \sqrt{9+25} = \sqrt{34}$ units

$BC = \sqrt{(4+1)^2+(2+1)^2} = \sqrt{25+9} = \sqrt{34}$ units

$CD = \sqrt{(-1+4)^2+(-1-4)^2} = \sqrt{9+25} = \sqrt{34}$ units

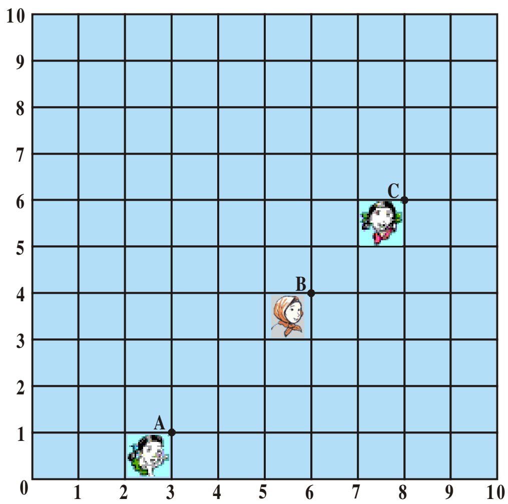
$$DA = \sqrt{(-4-1)^2 + (4-7)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

and digonal are $AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$ units

$$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68} \text{ units}$$

Since AB = BC = CD = DA and AC = BD. So all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is square.

Example-7. Figure shows the arrangement of desks in a class room.



Madhuri, Meena, Pallavi are seated at A(3, 1), B(6, 4) and C(8, 6) respectively.

Do you think they are seated in a line ?

Give reasons for your answer.

Solution : Using the distance formula, we have

$AB = \sqrt{(6-3)^2 + (4-1)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$BC = \sqrt{(18-6)^2 + (6-4)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

$$AC = \sqrt{(8-3)^2 + (6-1)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \text{ units}$$

Since, $AB + BC = 3\sqrt{2} + 2\sqrt{2} + 5\sqrt{2} = AC$, we can say that the points A, B and C are collinear. Therefore, they are seated in a line.

Example-8. Find a relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5).

Solution : Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5).

Given that AP = BP. So, $AP^2 = BP^2$

i.e., $(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$

i.e., $(x^2 - 14x + 49) + (y^2 - 2y + 1) = (x^2 - 6x + 9) + (y^2 - 10y + 25)$

$(x_2 + y^2 - 14x - 2y + 50) - (x^2 + y^2 - 6x - 10y + 34) = 0$

$-8x + 8y = -16$

i.e., $x - y = 2$ which is the required relation.

Example-9. Find a point on the y-axis which is equidistant from both the points A(6, 5) and B(– 4, 3).

Solution : We know that a point on the Y-axis is of the form (0, y). So, let the point P(0, y) be equidistant from A and B. Then

$$PA = \sqrt{(6-0)^2 + (5-y)^2}$$

$$PB = \sqrt{(-4-0)^2 + (3-4)^2}$$

$$PA^2 = PB^2$$

So, $(6-0)^2 + (5-y)^2 = (-4-0)^2 + (3-y)^2$

i.e., $36 + 25 + y^2 - 10y = 16 + 9 + y^2 - 6y$

i.e., $4y = 36$

i.e., $y = 9$

So, the required point is (0, 9).

Let us check our solution : $AP = \sqrt{(6-0)^2 + (5-9)^2} = \sqrt{36+16} = \sqrt{52}$

$BP = \sqrt{(-4-0)^2 + (3-9)^2} = \sqrt{16+36} = \sqrt{52}$

So (0, 9) is equidistant from (6, 5) and (4, 3).

Exercise 7.1

1. Find the distance between the following pairs of points

   (i) (2, 3) and (4, 1) (ii) (–5, 7) and (–1, 3)

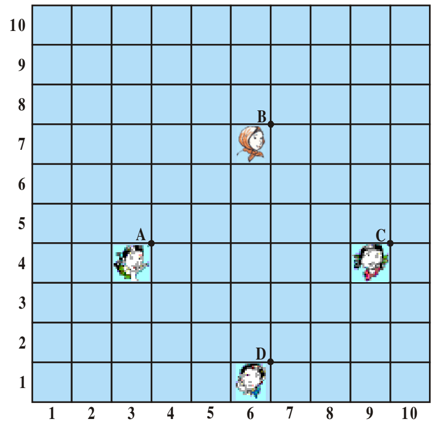
   (iii) (–2, –3) and (3, 2) (iv) (a, b) and (–a, –b)

2. Find the distance between the points (0, 0) and (36, 15).

3. Verify that the points (1, 5), (2, 3) and (–2, –1) are collinear or not.

4. Check whether (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle.

5. In a class room, 4 friends are seated at the points A, B, C and D as shown in Figure. Jarina and Phani walk into the class and after observing for a few minutes Jarina asks phani "Don't you think ABCD is a square?" Phani disagrees.
Using distance formula, find which of them is correct. Why ?



6. Show that the following points form a equilateral triangle A(a, 0), B($-a$, 0), C(0, $a\sqrt{3}$)

7. Prove that the points (–7, –3), (5, 10), (15, 8) and (3, –5) taken in order are the corners of a parallelogram. And find its area.

(Hint : Area of rhombus = $\frac{1}{2}\times$ product of its diagonals)

8. Show that the points (–4, –7), (–1, 2), (8, 5) and (5, –4) taken in order are the vertices of a rhombus.

9. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

(i) (–1, –2), (1, 0), (–1, 2), (–3, 0) (ii) (–3, 5), (3, 1), (0, 3), (–1, –4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

10. Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).

11. If the distance between two points (x, 7) and (1, 15) is 10, find the value of x.

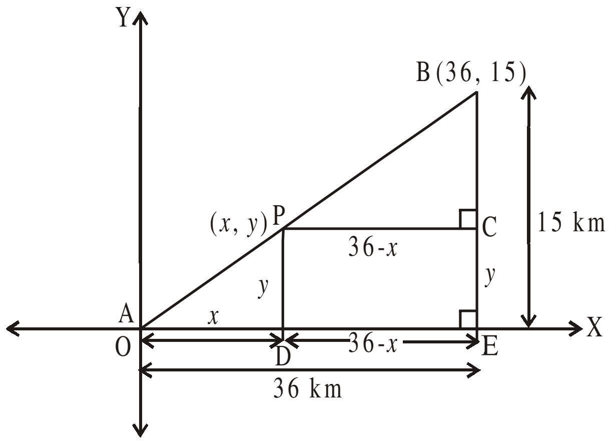
12. Find the values of y for which the distance between the points P(2, –3) and Q(10, y) is 10 units.

13. Find the radius of the circle whose centre is (3, 2) and passes through (–5, 6).

14. Can you draw a triangle with vertices (1, 5), (5, 8) and (13, 14) ? Give reason.

15. Find a relation between x and y such that the point (x, y) is equidistant from the points (–2, 8) and (–3, –5)

7.5 Section Formula



Suppose a telephone company wants to position a relay tower at P between A and B in such a way that the distance of the tower from B is twice its distance from A. If P lies on AB, it will divide AB in the ratio 1 : 2 (See figure). If we take A as the origin O, and 1 km as one unit on both the axis, the coordinates of B will be (36, 15). In order to know the position of the tower, we must know the coordinates of P. How do we find these coordinates?

Let the coordinates of P be (x, y). Draw perpendiculars from P and B to the x-axis, meeting it in D and E, respectively. Draw PC perpendicular to BE. Then, by the AA similarity criterion, studied earlier, ΔPOD and ΔBPC are similar.

Therefore, $\dfrac{OD}{PC} = \dfrac{OP}{PB} = \dfrac{1}{2}$ and $\dfrac{PD}{BC} = \dfrac{OP}{PB} = \dfrac{1}{2}$

So, $\dfrac{x}{36 - x} = \dfrac{1}{2}$ $\qquad$ $\dfrac{y}{15 - y} = \dfrac{1}{2}$.

$2x = (36 - x)$ $\qquad$ $2y = 15 - y$

$3x = 36$ $\qquad$ $3y = 15$

$x = 12$ $\qquad$ $y = 5$

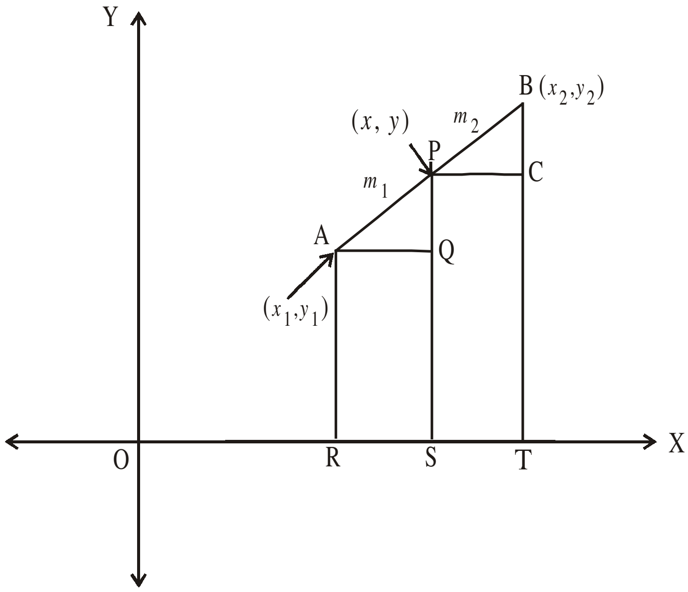
These equations give $x = 12$ and $y = 5$.

You can check that P(12, 5) meets the condition that OP : PB = 1 : 2.

Consider any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ and assume that $P(x, y)$ divides AB internally in the ratio $m_1 : m_2$,

i.e., $\dfrac{AP}{PB} = \dfrac{m_1}{m_2}$ ..... (1)

(See figure).

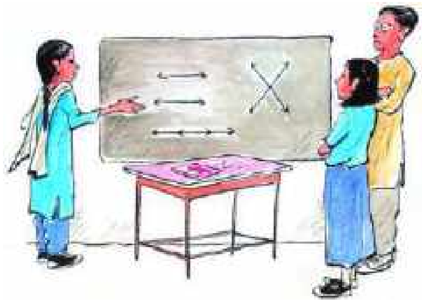


Draw AR, PS and BT perpendicular to the x-axis. Draw

AQ and PC parallel to the X-axis. Then, by the AA similarity criterion,

$$\Delta PAQ \sim \Delta BPC$$

Therefore, $\dfrac{AP}{PB} = \dfrac{AQ}{PC} = \dfrac{PQ}{BC}$ .....(2)



Now, $AQ = RS = OS - OR = x - x_1$

$PC = ST = OT - OS = x_2 - x$

$PQ = PS - QS = PS - AR = y - y_1$

$BC = BT - CT = BT - PS = y_2 - y$

Substituting these values in (1), we get

$$\frac{m_1}{m_2} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y} \quad \left[\because \frac{AP}{PB} = \frac{m_1}{m_2} \text{ from(1)}\right]$$

Taking $\dfrac{m_1}{m_2} = \dfrac{x - x_1}{x_2 - x}$, we get $x = \dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$

Similarly, taking $\dfrac{m_1}{m_2} = \dfrac{y - y_1}{y_2 - y}$, we get $y = \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

So, the coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2), **internally** in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right) \quad .....(3)$$

This is known as the **section formula**.

This can also be derived by drawing perpendiculars from A, P and B on the Y-axis and proceeding as above.

If the ratio in which P divides AB is $k : 1$, then the coordinates of the point P are

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right).$$

Special Case : The mid-point of a line segment divides the line segment in the ratio 1 : 1. Therefore, the coordinates of the mid-point P of the join of the points A(x_1, y_1) and B(x_2, y_2) are

$$\left(\frac{1.x_1+1.x_2}{1+1}, \frac{1.y_1+1.y_2}{1+1}\right)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

Let us solve few examples based on the section formula.

Example-10. Find the coordinates of the point which divides the line segement joining the points (4, –3) and (8, 5) in the ratio 3 : 1 internally.

Solution : Let P(x, y) be the required point. Using the section formuls

$$P(x, y)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right), \text{ we get}$$

$$x=\frac{3(8)+1(4)}{3+1}=\frac{24+4}{4}=\frac{28}{4}=7,$$

$$y=\frac{3(5)+1(-3)}{3+1}=\frac{15-3}{4}=\frac{12}{4}=3$$

P(x, y) = (7, 3) is the required point.

Example-11. Find the mid point of the line segment joining the points (3, 0) and (-1, 4)

Solution : The mid point M(x, y) of the line segment joining the points (x_1, y_1) and (x_2, y_2).

$$M(x, y)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$\therefore$ The mid point of the line segment joining the points (3, 0) and (–1, 4) is

$$M(x, y)=\left(\frac{3+(-1)}{2}, \frac{0+4}{2}\right)=\left(\frac{2}{2}, \frac{4}{2}\right)=(1, 2).$$

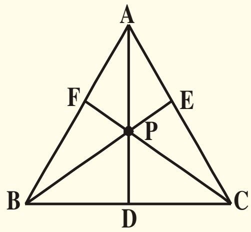
Do This

1 Find the point which divides the line segment joining the points (3, 5) and (8, 10) internally in the ratio 2 : 3

2. Find the midpoint of the line segement joining the points (2, 7) and (12, –7).

Try This

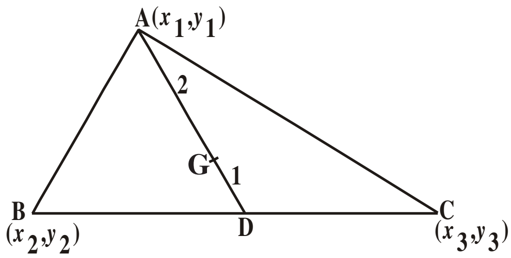
Let A(4, 2), B(6, 5) and C(1, 4) be the vertices of ΔABC



1. The median from A meets BC at D. Find the coordinates of the point D.
2. Find the coordinates of the point P on AD such that AP : PD = 2 : 1.
3. Find the coordinates of points Q and R on medians BE and CF.
4. Find the points which divide the line segment BE in the ratio 2 : 1 and also that divide the line segment CF in the ratio 2 : 1.
5. What do you observe ?

Justify the point that divides each median in the ratio 2 : 1 is the centriod of a triangle.

7.6 Centroid of a Triangle



The centroid of a triangle is the point of intersection of its medians.

Let A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the vertices of the triangle ABC.

Let AD be the median bisecting its base. Then,

$$D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

Now the point G on AD which divides it internally in the ratio 2 : 1, is the centroid. If (x, y) are the coordinates of G, then

$$G(x, y) = \left[\frac{2\left(\frac{x_2 + x_3}{2}\right) + 1(x_1)}{2+1}, \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1(y_1)}{2+1}\right]$$

$$= \left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$$

Hence, the coordinates of the centroid are given by

$$\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right].$$

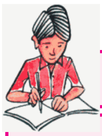
Example-12. Find the centroid of the triangle whose vertices are (3, –5), (–7, 4), (10, –2) respectively.

Solution : The coordinates of the centroid are

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$\left(\frac{3+(-7)+10}{3}, \frac{(-5)+4+(-2)}{3}\right) = (2, -1)$$

$\therefore$ the centroid is (2, –1).



Do This

Find the centroid of the triangle whose vertices are (–4, 6), (2, –2) and (2, 5) respectively.

Try This

The points (2, 3), (x, y), (3, –2) are vertices of a triangle. If the centroid of this triangle is again find (x, y).

Example-13. In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

Solution : Let (– 4, 6) divide AB internally in the ratio $m_1 : m_2$. Using the section formula, we get

$$(-4, 6) = \left(\frac{3m_1 - 6m_2}{m_1+m_2}, \frac{-8m_1+10m_2}{m_1+m_2}\right) \qquad .....(1)$$

We know that if $(x, y) = (a, b)$ then $x = a$ and $y = b$.

So, $\quad -4 = \dfrac{3m_1 - 6m_2}{m_1+m_2}$ and $6 = \dfrac{-8m_1+10m_2}{m_1+m_2}$

Now, $-4 = \dfrac{3m_1 - 6m_2}{m_1 + m_2}$ gives us

$$-4m_1 - 4m_2 = 3m_1 - 6m_2$$

i.e., $7m_1 = 2m_2$

$$\frac{m_1}{m_2} = \frac{2}{7}$$

i.e., $m_1 : m_2 = 2 : 7$

We should verify that the ratio satisfies the y-coordinate also.

Now, $\dfrac{-8m_1 + 10m_2}{m_1 + m_2} = \dfrac{-8\dfrac{m_1}{m_2} + 10}{\dfrac{m_1}{m_2} + 1}$ (Dividing throughout by m_2)

$$= \frac{-8 \times \frac{2}{7} + 10}{\frac{2}{7} + 1} = \frac{\frac{-16}{7} + 10}{\frac{9}{7}} = \frac{-16 + 70}{9} = \frac{54}{9} = 6$$

Therefore, the point (–4, 6) divides the line segment joining the points A(–6, 10) and B (3, –8) in the ratio 2 : 7.

Think - Discuss

The line joining points A(6, 9) and B(–6, –4) are given

(a) In which ratio does origin divide $\overline{AB}$? And what it is called for $\overline{AB}$?

(b) In which ratio does the point P(2, 3) divide $\overline{AB}$?

(c) In which ratio does the point Q(–2, –3) divide $\overline{AB}$?

(d) In how many equal parts is $\overline{AB}$ divided by P and Q?

(e) What do we call P and Q for $\overline{AB}$?

7.7 Trisectional Points of a Line

Example-14. Find the coordinates of the points of trisection (*The points which divide a line segment into 3 equal parts are said to be the sectional points*) of the line segment joining the points A(2,–2) and B(–7, 4).

Solution : Let P and Q be the points of trisection of AB i.e., AP=PQ=QB (see figure below).

Therefore, P divides AB internally in the ratio 1 : 2.

Therefore, the coordinates of P are (by applying the section formula)

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$\left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right)$$

$$\text{i.e., } \left(\frac{-7+4}{3}, \frac{4-4}{3}\right) = \left(\frac{-3}{3}, \frac{0}{3}\right) = (-1, 0)$$

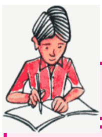
Now, Q also divides AB internally in the ratio 2:1.

So,the coordinates of Q are

$$= \left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right)$$

$$\text{i.e., } \left(\frac{-14+2}{3}, \frac{8-2}{3}\right) = \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$$

Therefore, the coordinates of the points of trisection of the line segment are P(-1, 0) and Q(-4, 2)



Do This

1. Find the trisectional points of line joining (2, 6) and (–4, 8).
2. Find the trisectional points of line joining (–3, –5) and (–6, –8).

Example-15. Find the ratio in which the y-axis divides the line segment joining the points (5, –6) and (1, –4). Also find the point of intersection.

Solution : Let the ratio be K : 1. Then by the section formula, the coordinates of the point which divides AB in the ratio K : 1 are

$$\left(\frac{K(-1)+1(5)}{K+1}, \frac{K(-4)+1(-6)}{K+1}\right)$$

$$\text{i.e., } \left(\frac{-K+5}{K+1}, \frac{-4K-6}{K+1}\right)$$

This point lies on the y-axis, and we know that on the y-axis the abscissa is 0.

$$\text{Therefore, } \frac{-K+5}{K+1} = 0$$

$$-K + 5 = 0 \Rightarrow K = 5.$$

So, the ratio is K : 1 = 5 : 1

Putting the value of K = 5, we get the point of intersection as

$$= \left(\frac{-5+5}{5+1}, \frac{-4(5)-6}{5+1}\right) = \left(0, \frac{-20-6}{6}\right) = \left(0, \frac{-26}{6}\right) = \left(0, \frac{-13}{3}\right)$$

Example-16. Show that the points A(7, 3), B(6, 1), C(8, 2) and D(9, 4) taken in that order are vertices of a parallelogram.

Solution : Let the points A(7, 3), B(6, 1), C(8, 2) and D(9, 4) are vertices of a parallelogram.

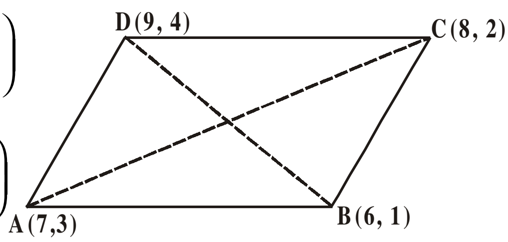
We know that the diagonals of a parallelogram bisect each other.

$\therefore$ So the midpoints of the diagonals AC and DB should be equal.

Now, we find the mid points of AC and DB by using $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ formula.

$$\text{midpoint of } AC = \left(\frac{7+8}{2}, \frac{3+2}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$$

$$\text{midpoint of } DB = \left(\frac{9+6}{2}, \frac{4+1}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$$



Hence, midpoint of AC = midpoint of DB.

Therefore, the points A, B, C, D are vertices of a parallelogram.

Example-17. If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken inorder, find the value of P.

Solution : We know that diagonals of parallelogram bisect each other.

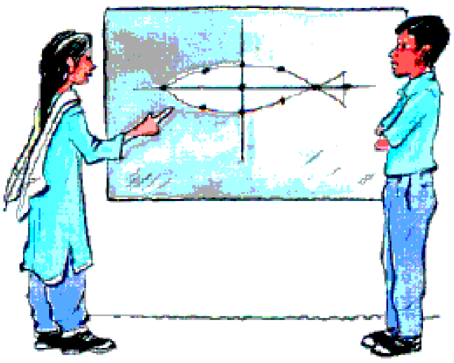
So, the coordinates of the midpoint of AC = Coordinates of the midpoint of BD.

i.e., $$\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$$

$$\left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$$

$$\frac{15}{2} = \frac{8+p}{2}$$

$$15 = 8 + p \Rightarrow p = 7.$$



EXERCISE - 7.2

1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.

2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

3. Find the ratio in which the line segment joining the points (–3, 10) and (6, –8)is divided by (–1, 6).

4. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

5. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).

6. If A and B are (–2, –2) and (2, –4) respectively. Find the coordinates of P such that $AP = \frac{3}{7}$ AB and P lies on the segment AB.

7. Find the coordinates of points which divide the line segment joining A(–4, 0) and B(0, 6) into four equal parts.

8. Find the coordinates of the points which divides the line segment joining A(–2, 2) and B(2, 8) into four equal parts.

9. Find the coordinates of the point which divides the line segment joining the points $(a+b, a-b)$ and $(a-b, a+b)$ in the ratio 3 : 2 internally.

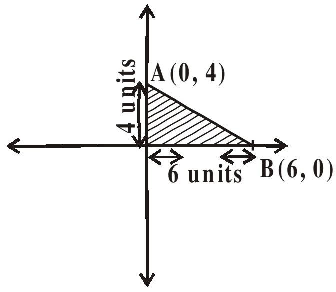
10. Find the coordinates of centroid of the following:

    i. (-1, 3), (6, -3) and (-3, 6) ii. (6, 2), (0, 0) and (4, -7)

    iii. (1, -1), (0, 6) and (-3, 0)

7.8 Area of the Triangle

Consider the points A(0, 4) and B(6, 0) which form a triangle with origin O on a plane as shown in figure.



What is the area of the ΔAOB?

ΔAOB is right angle triangle and the base is 6 units (i.e., x coordinate) and height is 4 units (i.e., y coordinate).

$$\therefore \text{Area of } \Delta AOB = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 6 \times 4 = 12 \text{ square units.}$$

Try This

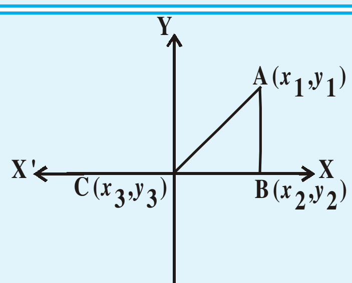
Take a point A on X-axis and B on Y-axis and find area of the triangle AOB. Discuss with your friends what did they do?

Think - Discuss

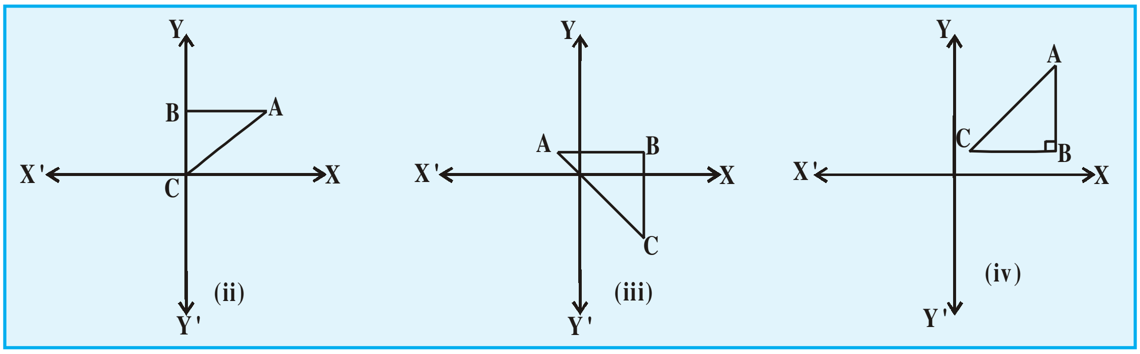
Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$.

Then find the area of the following triangles in a plane.

And discuss with your friends in groups about the area of that triangle.



(i)



Area of the triangle

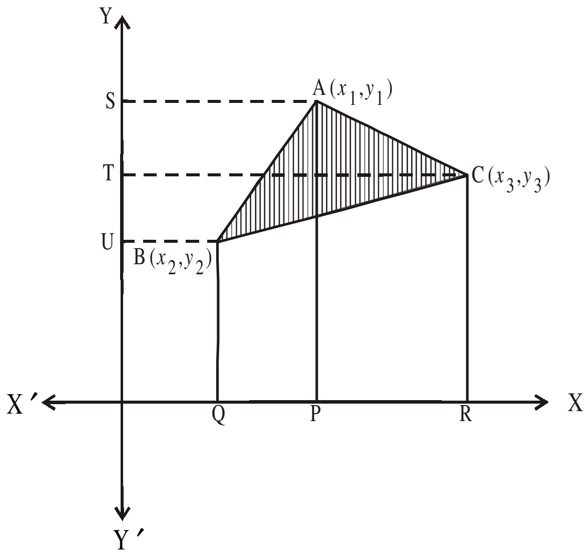
Let ABC be any triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Draw AP, BQ and CR perpendiculars from A, B and C respectively to the x-axis.

Clearly ABQP, APRC and BQRC are all trapezia as shown in figure.

Now from figure, it is clear that

Area of ΔABC = area of trapezium ABQP + area of trapezium APRC – area of trapezium BQRC



$\because$ Area of trapezium $= \frac{1}{2}$ (sum of the parallel sides) (distance between them)

$$\text{Area of } \Delta ABC = \frac{1}{2}(BQ + AP)QP + \frac{1}{2}(AP + CR)PR - \frac{1}{2}(BQ + CR)QR \quad \text{.... (1)}$$

Here from the figure

$BQ = y_2$, $AP = y_1$, $QP = OP - OQ = x_1 - x_2$

$CR = y_3$, $PR = OR - OP = x_3 - x_1$

$QR = OR - OQ = x_3 - x_2$

Therefore, Area of ΔABC [from (1)]

$$= \frac{1}{2}\left|(y_2 + y_1)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_3 + y_3)(x_3 - x_2)\right|$$

$$= \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

Thus, the area of ΔABC is the numerical value of the expression

$$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

Let us try some examples.

Example-18. Find the area of a triangle whose vertices are (1, –1), (–4, 6) and (–3, –5).

Solution : The area of the triangle formed by the vertices A(1, –1), B(–4, 6) and C(–3, –5), by using the formula above

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

is given by

$$= \frac{1}{2}\left|1(6+5) + (-4)(-5+1) + (-3)(-1-6)\right|$$

$$= \frac{1}{2}\left|11+16+21\right| = 24$$

So the area of the triangle is 24 square units.

Example-19. Find the area of a triangle formed by the points A(5, 2), B(4, 7) and C(7, –4).

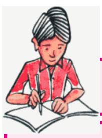
Solution : The area of the triangle formed by the vertices A(5, 2), B(4, 7) and C(7, –4) is given by

$$\frac{1}{2}\left|5(7+4) + 4(-4-2) + 7(2-7)\right|$$

$$= \frac{1}{2}\left|55-24-35\right| = \left|\frac{-4}{2}\right| = \left|-2\right|$$

Since area is a measure, which cannot be negative, we will take the numberical value of -2 or absolute value i.e., $|-2| = 2$.

Therefore, the area of the triangle = 2 square units.



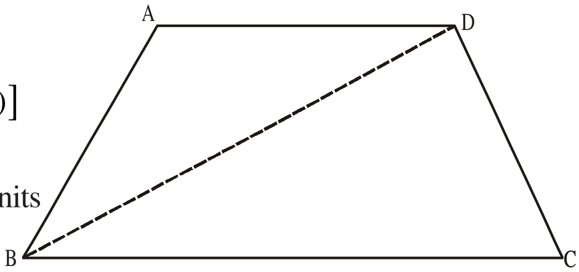
Do This

1. Find the area of the triangle whose vertices are
2. (5, 2) (3, –5) and (–5, –1)
3. (6, –6), (3, –7) and (3, 3)

Example-20. If A(–5, 7), B(–4,–5), C(–1, –6) and D(4,5) are the vertices of a quadrilateral. Then, find the area of the quadrilateral ABCD.

Solution : By joining B to D, you willl get two triangles ABD, and BCD.

The area of ΔABD



$$= \frac{1}{2}[-5(-5-5)+(-4)(5-7)+4(7+5)]$$

$$= \frac{1}{2}(50+8+48) = \frac{106}{2} = 53 \text{ square units}$$

Also, The area of ΔBCD $= \frac{1}{2}[-4(-6-5)-1(5+5)+4(-5+6)]$

$$= \frac{1}{2}[44-10+4] = 19 \text{ Square units}$$

Area of ΔABD + area of ΔBCD

So, the area of quadrilateral ABCD = 53+19 = 72 square units.

Try This

Find the area of the square formed by (0, –1), (2, 1) (0, 3) and (–2, 1) taken inorder are as vertices.

Think - discuss

Find the area of the triangle formed by the following points

(i) (2, 0), (1, 2), (1, 6)

(ii) (3, 1), (5, 0), (1, 2)

(iii) (–1.5, 3), (6, 2), (–3, 4)

What do you observe?

Plot these points three different graphs. What do you observe?

Can we have a triangle is 0 square units ?

What does it mean?

7.8.1. Collinearity

Suppose the points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are lying on a line. Then, they can not form a triangle. i.e. area of ΔABC is zero.

When the area of a triangle is zero then the three points said to be collinear points.

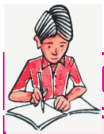
Example-21. The points (3, –2) (–2, 8) and (0, 4) are three points in a plane. Show that these points are collinear.

Solution : By using area of the triangle formula

$$\Delta = \frac{1}{2}|3(8-4)+(-2)(4-(-2))+0((-2)-8)|$$

$$= \frac{1}{2}|12-12| = 0$$

The area of the triangle is 0. Hence the three points are collinear or the lie on the same line.



Do This

Verify wheather the following points are

(i) (1, –1), (4, 1), (–2, –3)

(ii) (1, –1), (2, 3), (2, 0)

(iii) (1, –6), (3, –4), (4, –3)

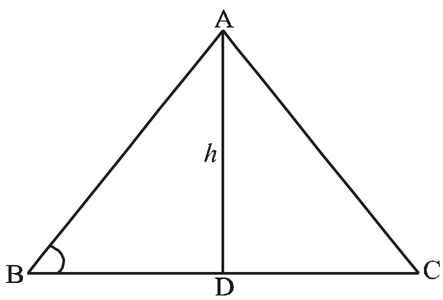
7.8.2. Area of a Triangle- 'Heron's Formula'

We know the formula for area of the triangle is $\frac{1}{2}\times \text{base} \times \text{height}$.

Any given triangle is may be a right angle triangle, equillateral triangle and isosceles triangle. Can we calculate the area of the triangle ?

If we know the base and height directly we apply the above formula to find the area of a triangle.

The height (h) is not known, how can we find its area?

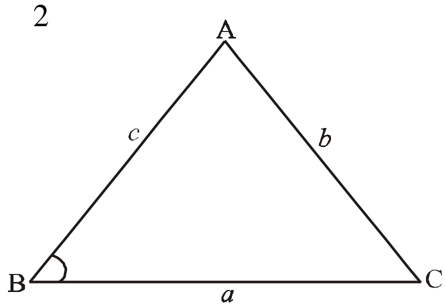


For this Heron, a Ancient Greek mathematician, derived a formula for a triangle whose lengths of sides are a, b and c.

$$A = \sqrt{S(S-a)(S-b)(S-c)}, \quad \text{where} \quad S = \frac{a+b+c}{2}$$

For example, we find the area of the triangle whose lengths of sides are 12m, 9m, 15m by using Heron's formula we get

$$A = \sqrt{S(S-a)(S-b)(S-c)}, \quad \text{where} \quad S = \frac{a+b+c}{2}$$



$$S = \frac{12+9+15}{2} = \frac{36}{2} = 18m$$

Then $S - a = 18 - 12 = 6\text{m}$

$S - b = 18 - 9 = 9\text{m}$

$S - c = 18 - 15 = 3\text{m}$

$A = \sqrt{18(6)(9)(3)} = \sqrt{2916} = 54$ square meter.

Do This

(i) Find the area of the triangle whose lengths of sides are 15m, 17m, 21m (use Heron's Formula) and verify your answer by using the formula $A = \frac{1}{2}bh$.

(ii) Find the area of the triangle formed by the points (0, 0), (4, 0), (4, 3) by using Heron's formula.

Example-22. Find the value of 'b' for which the points are collinear.

Solution : Let given points A(1, 2), B(–1, b), C(–3, –4)

Then $x_1 = 1, y_1 = 2;$ $\quad x_2 = -1, y_2 = b;$ $\quad x_3 = -3, \; y_3 = -4$

We know, area of $\Delta ABC = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$\therefore \; \frac{1}{2}\left|1(b+4) + (-1)(-4,-2) + (-3)(2-b)\right| = 0$ $\quad$ ($\because$ The given points are collinear)

$|b + 4 + 6 - 6 + 36| = 0$

$|4b + 4| = 0$

$4b + 4 = 0$

$\therefore \; b = -1$

Exercise - 7.3

1. Find the area of the triangle whose vertices are

   (i) (2, 3) (–1, 0), (2, –4) (ii) (–5, –1), (3, –5), (5, 2)

   (iii) (0, 0), (3, 0) and (0, 2)

2. Find the value of 'K' for which the points are collinear.

   (i) (7, –2) (5, 1) (3, K) (ii) (8, 1), (K, –4), (2, –5)

   (iii) (K, K) (2, 3) and (4, –1).

3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

4. Find the area of the quadrilateral whose vertices, taken inorder, are (–4, –2), (–3, –5), (3, –2) and (2, 3).

5. Find the area of the triangle formed by the points (8, –5), (–2, –7) and (5, 1) by using Herones formula.

7.9 Straight Lines

Bharadwaj and Meena are discussing to find solutions for a linear equation in two variable.

Bharadwaj : Can you find solutions for $2x + 3y = 12$

Meena : Yes, I have done this, see

x	0	3	6	–3
y	4	2	0	6

$$2x+3y = 12$$

$$3y = 12 - 2x$$

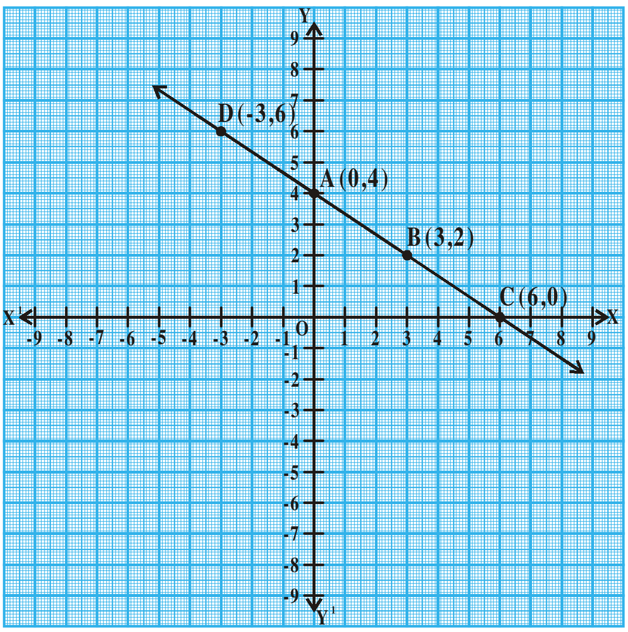
$$y = \frac{12-2x}{3}$$

Meena : Can you write these solutions in order pairs

Bharadwaj : Yes, (0, 4), (3, 2), (6, 0), (–3, 6)

Meena, can you plot these points on the coordinate plane.

Meena : I have done case like this case



Bharadwaj : What do you observe?

What does this line represent?

Meena : It is a straight line.

Bharadwaj : Can you identify some more points on this line?

Can you help Meena to find some more points on this line?

.................., ..., ..., ...

And In this line, what is $\overline{AB}$ called ?

$\overline{AB}$ is a line segment.

Do This

Plot these points on the coordinates axis and join Them:

1. A(1, 2), B(–3, 4), C(7, –1)
2. P(3, –5) Q(5, –1), R(2, 1), S(1, 2)

Which gives a straight line? Which is not? why?

Think - Discuss

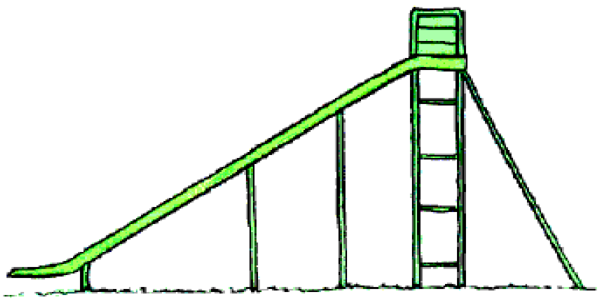
Is $y = x + 7$ represent a straight line? draw the line on the coordinate plane.

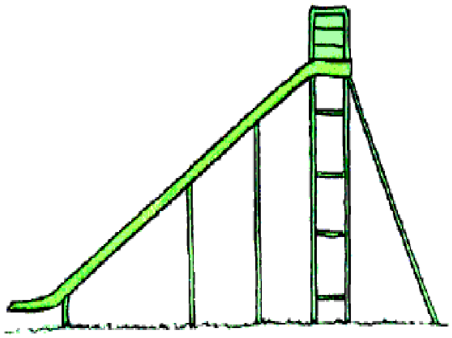
At which point does this line intersect Y-axis?

How much angle does it make with X-axis? Discuss with your friends

7.9.1 Slope of the straight line

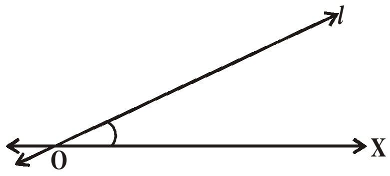
You might have seen a slider in a park. Two sliders have been given here. On which slider you can slide faster?

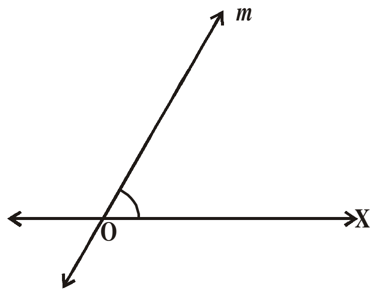




Obviousely your answer will be second "Why"?

Observe these lines.





Which line makes more angle with OX ?

Since the line "*m*" makes a greater angle with OX than line '*l*'.

line '*m*' has a greater "slope" than line '*l*'. We may also term the "Steepness" of a line as its slope.

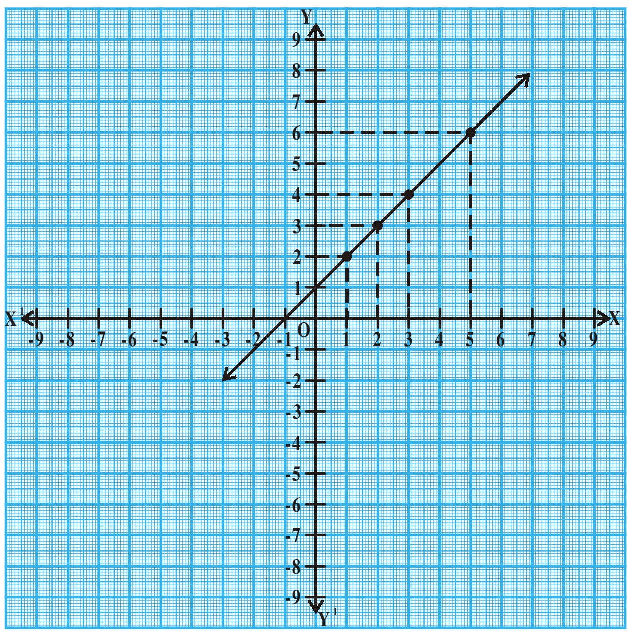
How we find the slope of a line?

Activity

Consider the line given in the figure indentify the points on the line and fill the table below.

x coordinate	1	-	-	4	-
y coordinate	2	3	4	-	6

We can observe that y coordinates change when x coordinates change.



When y coordinate increases from $y_1 = 2$ to $y_2 = 3$,

So the change in y is = ...

Then corresponding change in 'x' is = ...

$$\therefore \frac{\text{change in } y}{\text{change in } x} = \text{..................}$$

When y coordinate increases from $y_1 = 2$, $y_3 = 4$

So, the change in y is = ...

The corresponding change in x is ....

$$\text{So, } \frac{\text{change in } y}{\text{change in } x} = \text{...............}$$

Then can you try other points on the line choose any two points and fill in the table.

y value		Change in y	x	Change in x		$\dfrac{\text{change in } y}{\text{change in } x}$
2	4	-	1	2	1	-
-	-	-	-	-	-	-
-	-	-	-	-	-	-

What can you conclude from above activity?

Therefore, there is a relation between the ratio of change in y to change in x on a line has relation with angle made by it with X-axis.

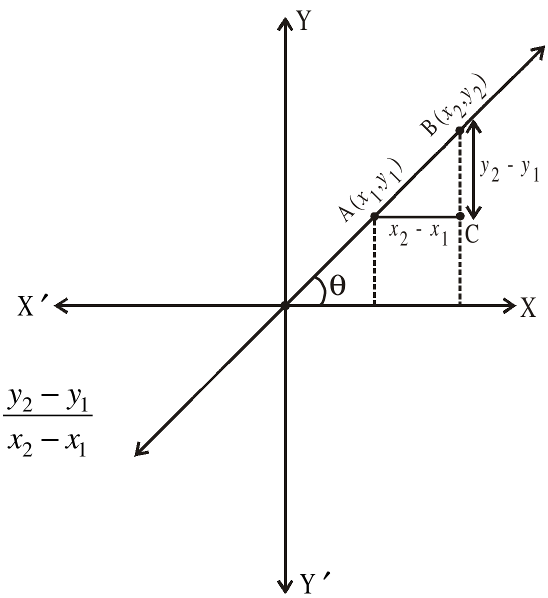
You will learn the concept of tan θ from trigonometry

i.e., $\tan\theta = \dfrac{\text{Opposite side of angle }\theta}{\text{adjecent side of angle }\theta} = \dfrac{\text{Change in } y}{\text{Change in } x}$

7.9.2 Slope of a line joining Two Points

Let A(x_1, y_1) and B(x_2, y_2) be two points on a line 'l' as shown in figure

The slope of a line $= \dfrac{\text{change in } y}{\text{change in } x}$



Slope of $\overline{AB} = m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Slope will be denoted by 'm' and the line 'l' makes the angle θ with X-axis.

So AB line segment makes the same angle θ with AC also.

$\therefore \tan\theta = \dfrac{\text{Opposite side of angle }\theta}{\text{adjecent wide of angle }\theta} = \dfrac{BC}{AC} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\therefore \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1} = m$

Hence $\therefore m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$

It is the formula to find slope of line segment $\overline{AB}$ which is having end points are (x_1, y_1), (x_2, y_2).

If θ is angle made by the line with X-axis, then $m = \tan\theta$.

Example-22. The end points of a line are (2, 3), (4, 5). Find the slope of the line.

Solution : Points of a line are (2, 3), (4, 5) then slope of the line

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{4-2} = \frac{2}{2} = 1$$

Slope of the given line is 1.

Do This

Find the slope of $\overleftrightarrow{AB}$ with the given end points.

1. A(4, –6) B(7, 2)
2. A(8, –4), B(–4, 8)
3. A(–2, –5), B(1, –7)

Try This

Find the slope of $\overleftrightarrow{AB}$ with the points lying on

1. A(2, 1), B(2, 6)
2. A(–4, 2), B(–4, –2)
3. A(–2, 8), B(–2, –2)
4. Justify that the line $\overleftrightarrow{AB}$ line segment formed by given points is parallel to Y-axis. What can you say about their slope? Why?

Think - Discuss

Find the slope $\overleftrightarrow{AB}$ with the points lying on A(3, 2), (–8, 2)

When the line $\overleftrightarrow{AB}$ parallel to X-axis ? Why ?

Think and discuss with your friends in groups.

Example-23. Determine x so that 2 is the slope of the line through P(2, 5) and Q(x, 3).

Solution : Slope of the line passing through P(2, 5) and Q(x, 3) is 2.

Here, $x_1 = 5$, $y_1 = 5$, $x_2 = x$, $y_2 = 3$

$$\text{Slope of a } \overline{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{x-2} = \frac{-2}{x-2} \Rightarrow \frac{-2}{x-2} = 2$$

$$\Rightarrow -2 = 2x - 4 \qquad \Rightarrow 2x = 2 \qquad \Rightarrow x = 1$$

Exercise - 7.4

1. Find the slope of the line joining the two given points

(i) (4, –8) and (5, –2)

(ii) (0, 0) and $(\sqrt{3}, 3)$

(iii) (2a, 3b) and (a, –b)

(iv) (a, 0) and (0, b)

(v) A(–1.4, –3.7), B(–2.4, 1.3)

(vi) A(3, –2), B(–6, –2)

(vii) $A\left(-3\frac{1}{2}, 3\right)$, $B\left(-7, 2\frac{1}{2}\right)$

(viii) A(0, 4), B(4, 0)

Optional Exercise

[This exercise is not meant for examination]

1. Centre of the circle Q is on the Y-axis. And the circle passes through the points (0, 7) and (0, –1). Circle intersects the positive X-axis at (P, 0). What is the value of 'P'.

2. A triangle ΔABC is formed by the points A(2, 3), B(–2, –3), C(4, –3). What is the point of intersection of side BC and angular bisector of A.

3. The side of BC of an equilateral triangle ΔABC is parallel to X-axis. Find the slopes of line along sides BC, CA and AB.

4. A right triangle has sides 'a' and 'b' where $a > b$. If the right angle is bisected then find the distance between orthocentres of the smaller triangles using coordinate geometry.

5. Find the centroid of the triangle formed by the line $2x + 3y - 6 = 0$. With the coordinate axes.

What We Have Discussed

1. The distance between two points P(x_1, y_1) and Q(x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

2. The distance of a point P(x, y) from the origin is $\sqrt{x^2 + y^2}$.

3. The distance between two points (x_1, y_1) and (x_1, y_2) on a line parallel to Y-axis is $|y_2 - y_1|$.

4. The distance between two point (x_1, y_1) and (x_2, y_2) on a line parallel to X-axis is $|x_2 - x_1|$.

5. The coordinates of the point P(x, y) which divides the line segment joining the points A (x_1, y_1) and B(x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left[\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right]$.

6. The mid-point of the line segment joining the points P(x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

7. The point that divides each median in the ratio 2 : 1 is the centroid of a triangle.

8. The centroid of a triangle is the point of intersection of its medians. Hence the coordinates of the centroid are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

9. The area of the triangle formed by the points (x_1, y_1) (x_2, y_2) and (x_3, y_3) is the numerical value of the expression $\frac{1}{2}\,|x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)|$

10. Area of a triangle formula 'Heron's Formula'

$$A = \sqrt{S(S-a)\ (S-b)\ (S-c)} \qquad \because\ S = \frac{a+b+c}{2}$$

(a, b, c are three sides of ΔABC)

11. Slope of the line containing the points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$