There is a tall tree in the backyard of Snigdha’s house. She wants to find out the height of that tree but she is not sure about how to find it. Meanwhile, her uncle arrives at home. Snigdha requests her uncle to help her with the height. He thinks for a while and then ask her to bring a mirror. He places it on the ground at a certain distance from the base of the tree. He then asked Snigdha to stand on the otherside of the mirror at such a position from where she is able to see the top of the tree in that mirror.

When we draw the figure from (AB) girl to the mirror (C) and mirror to the tree (DE) as above, we observe triangles ABC and DEC. Now, what can you say about these two triangles? Are they congruent? No, because although they have the same shape but their sizes are different. Do you know what we call the geometrical figures which have the same shape, but are not necessarily of the same size? They are called similar figures.

Can you guess how the heights of trees, mountains or distances of far-away, objects such as the Sun have been found out? Do you think these can be measured directly with the help of a measuring tape? The fact is that all these heights and distances have been found out using the idea of indirect measurements which is based on the principle of similarity of figures.

8.2 Similar Figures

Observe the object (car) in the previous figure.

If its breadth is kept the same and the length is doubled, it appears as in fig.(ii).
If the length in fig. (i) is kept the same and its breadth is doubled, it appears as in fig. (iii).

Now, what can you say about fig. (ii) and (iii)? Do they resemble fig. (i)? We find that the figure is distorted. Can you say that they are similar? No, they have same shape, yet they are not similar.

Think what a photographer does when she prints photographs of different sizes from the same film (negative)? You might have heard about stamp size, passport size and post card size photographs. She generally takes a photograph on a small size film, say 35 mm., and then enlarges it into a bigger size, say 45 mm (or 55 mm). We observe that every line segment of the smaller photograph is enlarged in the ratio of 35 : 45 (or 35 : 55). Further, in the two photographs of different sizes, we can see that the angles are equal. So, the photographs are similar.

Similarly in geometry, two polygons of the same number of sides are similar if their corresponding angles are equal and their corresponding sides are in the same ratio or proportion.

A polygon in which all sides and angles are equal is called a regular polygon.

The ratio of the corresponding sides is referred to as scale factor (or representative factor). In real life, blue prints for the construction of a building are prepared using a suitable scale factor.

Can you give some more examples from your daily life where scale factor is used.

All regular polygons having the same number of sides are always similar. For example, all squares are similar, all equalateral triangles are similar and so on.

Circles with same radius are congruent and those with different radii are not congruent. But, as all circles have same shape, they are all similar.

We can say that all congruent figures are similar but all similar figures need not be congruent.
To understand the similarity of figures more clearly, let us perform the following activity.

**Activity**

Place a table directly under a lighted bulb, fitted in the ceiling in your classroom. Cut a polygon, say ABCD, from a plane cardboard and place it parallel to the ground between the bulb and the table. Then, a shadow of quadrilateral ABCD is cast on the table. Mark the outline of the shadow as quadrilateral $A'B'C'D'$.

Now this quadrilateral $A'B'C'D'$ is enlargement or magnification of quadrilateral ABCD. Further, $A'$ lies on ray OA where ‘O’ is the bulb, $B'$ on $\overline{OB}$, $C'$ on $\overline{OC}$ and $D'$ on $\overline{OD}$. Quadrilaterals ABCD and $A'B'C'D'$ are of the same shape but of different sizes.

$A'$ corresponds to vertex A and we denote it symbolically as $A' \leftrightarrow A$. Similarly $B' \leftrightarrow B$, $C' \leftrightarrow C$ and $D' \leftrightarrow D$.

By actually measuring angles and sides, you can verify

(i) $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$, $\angle D = \angle D'$

(ii) $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{DA}{D'A'}$.

This emphasises that two polygons with the same number of sides are similar if

(i) All the corresponding angles are equal and
(ii) All the corresponding sides are in the same ratio (or in proportion)

Is a square similar to a rectangle? In both the figures, corresponding angles are equal but their corresponding sides are not in the same ratio. Hence, they are not similar. For similarity of polygons only one of the above two conditions is not sufficient, both have to be satisfied.

**Think - Discuss**

Can you say that a square and a rhombus are similar? Discuss with your friends. Write why the conditions are not sufficient.
1. Fill in the blanks with similar / not similar.
   (i) All squares are ..................
   (ii) All equilateral triangles are ..................
   (iii) All isosceles triangles are ..................
   (iv) Two polygons with same number of sides are ................... if their corresponding angles are equal and corresponding sides are equal.
   (v) Reduced and Enlarged photographs of an object are ..................
   (vi) Rhombus and squares are ........................ to each other.

2. Write the True / False for the following statements.
   (i) Any two similar figures are congruent.
   (ii) Any two congruent figures are similar.
   (iii) Two polygons are similar if their corresponding angles are equal.

3. Give two different examples of pair of
   (i) Similar figures    (ii) Non similar figures

8.3 **Similarity of Triangles**

In the example we had drawn two triangles, those two triangles showed the property of similarity. We know that, two triangles are similar if their

(i) Corresponding Angles are equal and

(ii) Corresponding sides are in the same ratio (in proportion)

   In ΔABC and ΔDEC in the introduction,

   \[ \angle A = \angle D, \quad \angle B = \angle E, \quad \angle ACB = \angle DCE \]

   Also \[ \frac{DE}{AB} = \frac{EC}{BC} = \frac{DC}{AC} = K \] (scale factor)

   then ΔABC is similar to ΔDEC

   Symbolically we write ΔABC ~ ΔDEC

   (Symbol ‘~’ is read as “Is similar to”)

   As we have stated K is a scale factor, So

   if \( K > 1 \) we get enlarged figures,
   \( K = 1 \) We get congruent figures and

   \( K < 1 \) gives reduced (or diminished) figures
Further, in triangles ABC and DEC, corresponding angles are equal. So they are called equiangular triangles. The ratio of any two corresponding sides in two equiangular triangles is always the same. For proving this, Basic Proportionality theorem is used. This is also known as Thales Theorem.

To understand Basic proportionality theorem or Thales theorem, let us do the following activity.

**Activity**

Take any ruled paper and draw a triangle on that with base on one of the lines. Several lines will cut the triangle ABC. Select any one line among them and name the points where it meets the sides AB and AC as P and Q.

Find the ratio of \( \frac{AP}{PB} \) and \( \frac{AQ}{QC} \). What do you observe?

The ratios will be equal. Why? Is it always true? Try for different lines intersecting the triangle. We know that all the lines on a ruled paper are parallel and we observe that every time the ratios are equal.

So in \( \triangle ABC \), if \( PQ \parallel BC \) then \( \frac{AP}{PB} = \frac{AQ}{QC} \).

This is known as the result of basic proportionality theorem.

**8.3.1 Basic Proportionality Theorem (Thales Theorem)**

**Theorem-8.1**: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

**Given**: In \( \triangle ABC \), DE \parallel BC which intersects sides AB and AC at D and E respectively.

**RTP**: \( \frac{AD}{DB} = \frac{AE}{EC} \)

**Construction**: Join B, E and C, D and then draw

\( DM \perp AC \) and \( EN \perp AB \).

**Proof**: Area of \( \triangle ADE = \frac{1}{2} \times AD \times EN \)

Area of \( \triangle BDE = \frac{1}{2} \times BD \times EN \)
So \[
\frac{\text{ar}(\triangle \text{ADE})}{\text{ar}(\triangle \text{BDE})} = \frac{\frac{1}{2} \times \text{AD} \times \text{EN}}{\frac{1}{2} \times \text{BD} \times \text{EN}} = \frac{\text{AD}}{\text{BD}} \quad \text{...(1)}
\]

Again Area of \(\triangle \text{ADE} = \frac{1}{2} \times \text{AE} \times \text{DM}\)

Area of \(\triangle \text{CDE} = \frac{1}{2} \times \text{EC} \times \text{DM}\)

\[
\frac{\text{ar}(\triangle \text{ADE})}{\text{ar}(\triangle \text{CDE})} = \frac{\frac{1}{2} \times \text{AE} \times \text{DM}}{\frac{1}{2} \times \text{EC} \times \text{DM}} = \frac{\text{AE}}{\text{EC}} \quad \text{...(2)}
\]

Observe that \(\triangle \text{BDE} \) and \(\triangle \text{CDE} \) are on the same base \(\text{DE} \) and between same parallels \(\text{BC} \) and \(\text{DE} \).
So \(\text{ar}(\triangle \text{BDE}) = \text{ar}(\triangle \text{CDE}) \quad \text{...(3)}\)

From (1) (2) and (3), we have

\[
\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}
\]

Hence proved.

Is the converse of the above theorem also true? To examine this, let us perform the following activity.

**Activity**

Draw an angle \(\text{XAY} \) on your note book and on ray \(\text{AX} \), mark points \(\text{B}_1, \text{B}_2, \text{B}_3, \text{B}_4\) and \(\text{B} \) such that

\[\text{AB}_1 = \text{B}_1\text{B}_2 = \text{B}_2\text{B}_3 = \text{B}_3\text{B}_4 = \text{B}_4\text{B} = 1 \text{ cm (say)}\]

Similarly on ray \(\text{AY} \), mark points \(\text{C}_1, \text{C}_2, \text{C}_3, \text{C}_4\) and \(\text{C} \) such that

\[\text{AC}_1 = \text{C}_1\text{C}_2 = \text{C}_2\text{C}_3 = \text{C}_3\text{C}_4 = \text{C}_4\text{C} = 2 \text{ cm (say)}\]

Join \(\text{B}_1, \text{C}_1\) and \(\text{B}, \text{C}\).

Observe that \(\frac{\text{AB}_1}{\text{B}_1\text{B}} = \frac{\text{AC}_1}{\text{C}_1\text{C}} = \frac{1}{4}\) and \(\text{B}_1\text{C}_1 \parallel \text{BC}\).
Similarly, joining $B_2C_2, B_3C_3$ and $B_4C_4$, you see that

\[
\frac{AB_2}{B_2B} = \frac{AC_2}{C_2C} = \frac{2}{3} \quad \text{and} \quad B_2C_2 \parallel BC
\]

\[
\frac{AB_3}{B_3B} = \frac{AC_3}{C_3C} = \frac{3}{2} \quad \text{and} \quad B_3C_3 \parallel BC
\]

\[
\frac{AB_4}{B_4B} = \frac{AC_4}{C_4C} = \frac{4}{1} \quad \text{and} \quad B_4C_4 \parallel BC
\]

From this we obtain the following theorem called converse of the Thales theorem

**Theorem-8.2:** If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.

**Given:** In $\triangle ABC$, a line $DE$ is drawn such that \[\frac{AD}{DB} = \frac{AE}{EC}\]

**RTP:** $DE \parallel BC$

**Proof:** Assume that $DE$ is not parallel to $BC$ then draw the line $DE'$ parallel to $BC$

So \[\frac{AD}{DB} = \frac{AE'}{E'C'} \quad \text{(why ?)}\]

\[\therefore \quad \frac{AE}{EC} = \frac{AE'}{E'C'} \quad \text{(why ?)}\]

Adding 1 to both sides of the above, you can see that $E$ and $E'$ must coincide (why ?)

**Try This**

1. E and F are points on the sides PQ and PR respectively of $\triangle PQR$. For each of the following, state whether. $EF \parallel QR$ or not?

   (i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

   (ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm.

   (iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 1.8 cm and PF = 3.6 cm
2. In the following figures $DE \parallel BC$.

(i) Find $EC$

(ii) Find $AD$

**Construction: Division of a line segment (using Thales theorem)**

Madhuri drew a line segment. She wants to divide it in the ratio of 3 : 2. She measured it by using a scale and divided it in the required ratio. Meanwhile her elder sister came. She saw this and suggested Madhuri to divide the line segment in the given ratio without measuring it. Madhuri was puzzled and asked her sister for help to do it. Then her sister explained. You may also do it by the following activity.

**Activity**

Take a sheet of paper from a lined notebook. Number the lines by 1, 2, 3, ... starting with the bottom line numbered ‘0’.

Take a thick cardboard paper (or file card or chart strip) and place it against the given line segment $AB$ and transfer its length to the card. Let $A_1$ and $B_1$ denote the points on the file card corresponding to $A$ and $B$.

Now place $A_1$ on the zeroeth line of the lined paper and rotate the card about $A_1$ until point $B_1$ falls on the 5th line ($3 + 2$).

Mark the point where the third line touches the file card, by $P_1$.

Again place this card along the given line segment and transfer this point $P_1$ and denote it with ‘$P$’.

So $P$ is required point which divides the given line segment in the ratio 3:2.

Now let us learn how this construction can be done.

Given a line segment $AB$. We want to divide it in the ratio $m : n$ where $m$ and $n$ are both positive integers. Let us take $m = 3$ and $n = 2$.

**Steps**:

1. Draw a ray $AX$ through $A$ making an acute angle with $AB$. 
2. With ‘A’ as centre and with any length draw an arc on ray AX and label the point A1.

3. Using the same compass setting and with A1 as centre draw another arc and locate A2.

4. Like this locate 5 points (=m + n) A1, A2, A3, A4, A5 such that AA1 = A1A2 = A2A3 = A3A4 = A4A5

5. Join A5B. Now through point A3(m = 3) draw a line parallel to A5B (by making an angle equal to A A5 B) intersecting AB at C and observe that AC : CB = 3 : 2.

Now let us solve some examples on Thales theorem and its converse.

Example-1. In ΔABC, DE || BC and \( \frac{AD}{DB} = \frac{3}{5} \).

AC = 5.6. Find AE.

Solution : In ΔABC, DE || BC

\[
\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} \quad \text{(by B.P.T)}
\]

but \( \frac{AD}{DB} = \frac{3}{5} \) So \( \frac{AE}{EC} = \frac{3}{5} \)

Given AC = 5.6 and AE : EC = 3 : 5.

\[
\frac{AE}{AC} = \frac{3}{5}
\]

\[
\frac{AE}{5.6 - AE} = \frac{3}{5} \quad \text{(cross multiplication)}
\]

5AE = (3 × 5.6) − 3AE

8AE = 16.8

AE = \( \frac{16.8}{8} \) = 2.1cm.
Example-2. In the given figure LM \parallel AB

\[ AL = x - 3, \quad AC = 2x, \quad BM = x - 2 \]
and \( BC = 2x + 3 \) find the value of \( x \)

**Solution:** In \( \Delta ABC \), LM \parallel AB

\[ \frac{AL}{LC} = \frac{BM}{MC} \quad \text{(by B.P.T)} \]

\[ \frac{x - 3}{2x - (x - 3)} = \frac{x - 2}{(2x + 3) - (x - 2)} \]

\[ \frac{x - 3}{x + 3} = \frac{x - 2}{x + 5} \quad \text{(cross multiplication)} \]

\[ (x - 3)(x + 5) = (x - 2)(x + 3) \]

\[ x^2 + 2x - 15 = x^2 + x - 6 \]

\[ 2x - x = -6 + 15 \]

\[ x = 9 \]

---

**Do This**

1. What value(s) of \( x \) will make DE \parallel AB, in the given figure?
   
   \[ AD = 8x + 9, \quad CD = x + 3 \]
   
   \[ BE = 3x + 4, \quad CE = x. \]

2. In \( \Delta ABC \), DE \parallel BC.  \( AD = x, DB = x - 2, \)
   \( AE = x + 2 \) and EC = \( x - 1. \)
   
   Find the value of \( x \).

---

Example-3. The diagonals of a quadrilateral ABCD intersect each other at point ‘O’ such that

\[ \frac{AO}{BO} = \frac{CO}{DO}. \]

Prove that ABCD is a trapezium.

**Solution:** Given: In quadrilateral ABCD, \( \frac{AO}{BO} = \frac{CO}{DO}. \)

**RTP:** ABCD is a trapezium.

**Construction:** Through ‘O’ draw a line parallel to AB which meets DA at X.

**Proof:** In \( \Delta DAB \), XO \parallel AB \quad \text{(by construction)}

\[ \Rightarrow \frac{DX}{XA} = \frac{DO}{OB} \quad \text{(by basic proportionality theorem)} \]
\[
\frac{AX}{XD} = \frac{BO}{OD} \quad \ldots (1)
\]

Again,
\[
\frac{AO}{BO} = \frac{CO}{DO} \quad \text{(given)}
\]

\[
\frac{AO}{CO} = \frac{BO}{OD} \quad \ldots (2)
\]

From (1) and (2)
\[
\frac{AX}{XD} = \frac{AO}{CO}
\]

In \(\triangle ADC\), \( XO \) is a line such that \(\frac{AX}{XD} = \frac{AO}{OC}\)
\(\Rightarrow\) \( XO \parallel DC \) \quad \text{(by converse of the basic proportionality theorem)}
\(\Rightarrow\) \( AB \parallel DC \)

In quadrilateral \(ABCD\), \(AB \parallel DC\)
\(\Rightarrow\) \(ABCD\) is a trapezium \quad \text{(by definition)}
Hence proved.

**Example-4.** In trapezium \(ABCD\), \(AB \parallel DC\). \(E\) and \(F\) are points on non-parallel sides \(AD\) and \(BC\) respectively such that \(EF \parallel AB\). Show that \(\frac{AE}{ED} = \frac{BF}{FC}\).

**Solution:** Let us join \(AC\) to intersect \(EF\) at \(G\).
\(AB \parallel DC\) and \(EF \parallel AB\) \(\text{(given)}\)
\(\Rightarrow\) \(EF \parallel DC\) \(\text{(Lines parallel to the same line are parallel to each other)}\)

In \(\triangle ADC\), \(EG \parallel DC\)
\(\Rightarrow\) \(\frac{AE}{ED} = \frac{AG}{GC}\) \(\text{(by BPT)}\) \(\ldots (1)\)

Similarly, In \(\triangle CAB\), \(GF \parallel AB\)
\(\Rightarrow\) \(\frac{CG}{GA} = \frac{CF}{FB}\) \(\text{(by BPT)}\) \(\text{i.e.,} \frac{AG}{GC} = \frac{BF}{FC}\) \(\ldots (2)\)

From (1) & (2) \(\frac{AE}{ED} = \frac{BF}{FC}\).
1. In \( \triangle PQR \), ST is a line such that \( \frac{PS}{SQ} = \frac{PT}{TR} \) and also \( \angle PST = \angle PRQ \).
Prove that \( \triangle PQR \) is an isosceles triangle.

2. In the given figure, \( LM \parallel CB \) and \( LN \parallel CD \)
Prove that \( \frac{AM}{AB} = \frac{AN}{AD} \).

3. In the given figure, \( DE \parallel AC \) and \( DF \parallel AE \)
Prove that \( \frac{BF}{FE} = \frac{BE}{EC} \).

4. In the given figure, \( AB \parallel CD \parallel EF \) given \( AB = 7.5 \text{ cm} \) \( DC = y \text{ cm} \)
\( EF = 4.5 \text{ cm} \), \( BC = x \text{ cm} \).
Calculate the values of \( x \) and \( y \).

5. Prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side (Using basic proportionality theorem).

6. Prove that a line joining the midpoints of any two sides of a triangle is parallel to the third side. (Using converse of basic proportionality theorem)

7. In the given figure, \( DE \parallel OQ \) and \( DF \parallel OR \). Show that \( EF \parallel QR \).
8. In the adjacent figure, A, B and C are points on OP, OQ and OR respectively such that AB \parallel PQ and AC \parallel PR.
Show that BC \parallel QR.

9. ABCD is a trapezium in which AB \parallel DC and its diagonals intersect each other at point ‘O’.
Show that \frac{AO}{BO} = \frac{CO}{DO}.

10. Draw a line segment of length 7.2 cm and divide it in the ratio 5 : 3. Measure the two parts.

**Think - Discuss and Write**

Discuss with your friends that in what way similarity of triangles is different from similarity of other polygons?

### 8.4 Criteria for Similarity of Triangles

We know that two triangles are similar if corresponding angles are equal and corresponding sides are proportional. For checking the similarity of two triangles, we should check for the equality of corresponding angles and equality of ratios of their corresponding sides. Let us make an attempt to arrive at certain criteria for similarity of two triangles. Let us perform the following activity.

**Activity**

Use a protractor and ruler to draw two non congruent triangles so that each triangle has a 40° and 60° angle. Check the figures made by you by measuring the third angles of two triangles.

It should be each 80° (why?)

Measure the lengths of the sides of the triangles and compute the ratios of the lengths of the corresponding sides.

Are the triangles similar?

This activity leads us to the following criterion for similarity of two triangles.
8.4.1 AAA Criterion for Similarity of Triangles

**Theorem-8.3**: In two triangles, if the angles are equal, then the sides opposite to the equal angles are in the same ratio (or proportional) and hence the two triangles are similar.

**Given**: In triangles ABC and DEF,

- \( \angle A = \angle D \)
- \( \angle B = \angle E \)
- \( \angle C = \angle F \)

**RTP**: \( \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \)

**Construction**: Locate points P and Q on DE and DF respectively, such that AB = DP and AC = DQ. Join PQ.

**Proof**: \( \triangle ABC \cong \triangle DPQ \) (why ?)

This gives \( \angle B = \angle P = \angle E \) and PQ \( \parallel \) EF (How ?)

\[ \therefore \quad \frac{DP}{PE} = \frac{DQ}{QF} \quad \text{(why ?)} \]

\[ \text{i.e.,} \quad \frac{AB}{DE} = \frac{AC}{DF} \quad \text{(why ?)} \]

Similarly \( \frac{AB}{DE} = \frac{BC}{EF} \) and So \( \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \)

Hence proved.

**Note**: If two angles of a triangle are respectively equal to the two angles of another triangle, then by the angle sum property of a triangle, third angles will also be equal.

So AA similarity criterion is stated as if two angles of one triangle are respectively equal to the two angles of another triangle, then the two triangles are similar.

What about the converse of the above statement?

If the sides of a triangle are respectively proportional to the sides of another triangle, is it true that their corresponding angles are equal?

Let us exercise it through an activity.

**Activity**

Draw two triangles ABC and DEF such that AB = 3 cm, BC = 6 cm, CA = 8 cm, DE = 4.5 cm, EF = 9 cm and FD = 12 cm.
8.4.2. SSS Criterion for Similarity of Triangles

**Theorem-8.4**: If in two triangles, the sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the triangles are similar.

**Given**: △ABC and △DEF are such that

\[
\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} < 1
\]

**RTP**: \(\angle A = \angle D, \angle B = \angle E, \angle C = \angle F\)

**Construction**: Locate points P and Q on DE and DF respectively such that AB = DP and AC = DQ. Join PQ.

**Proof**: \(\frac{DP}{PE} = \frac{DQ}{QF}\) and PQ \parallel EF (why?)

So \(\angle P = \angle E\) and \(\angle Q = \angle F\) (why?)

\[
\therefore \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}
\]

So \(\frac{DP}{DE} = \frac{DQ}{DF} = \frac{BC}{EF}\) (why?)

So \(BC = PQ\) (Why?)

△ABC \cong △DPQ (why?)

So \(\angle A = \angle D, \angle B = \angle E\) and \(\angle C = \angle F\) (How?)

We studied that for similarity of two polygons any one condition is not sufficient. But for the similarity of triangles, there is no need for fulfillment of both the conditions as one automatically implies the other. Now let us look for SAS similarity criterion. For this, let us perform the following activity.
Activity

Draw two triangles ABC and DEF such that \( AB = 2 \text{ cm} \), \( \angle A = 50^0 \), \( AC = 4 \text{ cm} \), \( DE = 3 \text{ cm} \), \( \angle D = 50^0 \), and \( DF = 6 \text{ cm} \).

\( \angle A = \angle D = 50^0 \).

Observe that \( \frac{AB}{DE} = \frac{AC}{DF} = \frac{2}{3} \) and \( \angle A = \angle D = 50^0 \).

Now measure \( \angle B \), \( \angle C \), \( \angle E \), \( \angle F \) also measure BC and EF.

Observe that \( \angle B = \angle E \) and \( \angle C = \angle F \) also \( \frac{BC}{EF} = \frac{2}{3} \).

So, the two triangles are similar. Repeat the same for triangles with different measurements, which gives the following criterion for similarity of triangles.

8.4.3 SAS Criterion for Similarity of Triangles

Theorem-8.5: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

Given: In \( \triangle ABC \) and \( \triangle DEF \)

\[
\frac{AB}{DE} = \frac{AC}{DF} \quad (\leq 1) \quad \text{and} \quad \angle A = \angle D
\]

RTP: \( \triangle ABC \sim \triangle DEF \)

Construction: Locate points P and Q on DE and DF respectively such that \( AB = DP \) and \( AC = DQ \). Join PQ.

Proof: \( PQ \parallel EF \) and \( \triangle ABC \equiv \triangle DPQ \) (How ?)

So \( \angle A = \angle D \), \( \angle B = \angle P \), \( \angle C = \angle Q \)

\( \therefore \triangle ABC \sim \triangle DEF \) (why ?)
1. Are the triangles similar? If so, name the criterion of similarity. Write the similarity relation in symbolic form.

(i) (ii)

(iii) (iv)

(v) (vi)

(vii) (viii)

2. Explain why the triangles are similar and then find the value of $x$. 

(i) (ii)

(iii) (iv)

(v) (vi)
Construction: To construct a triangle similar to a given triangle as per given scale factor.

a) Construct a triangle similar to a given triangle ABC with its sides equal to \( \frac{3}{4} \) of corresponding sides of \( \triangle ABC \) (scale factor \( \frac{3}{4} \)).

Steps:
1. Draw a ray BX, making an acute angle with BC on the side opposite to vertex A.
2. Locate 4 points B₁, B₂, B₃, and B₄ on BX so that BB₁ = B₁B₂ = B₂B₃ = B₃B₄.
3. Join B₄C and draw a line through B₃ parallel to B₄C intersecting BC at C'.
4. Draw a line through C' parallel to CA to intersect AB at A'.

So \( \triangle A'B'C' \) is the required triangle.

Let us take some examples to illustrate the use of these criteria.

Example-5. A person 1.65m tall casts 1.8m shadow. At the same instance, a lamp-post casts a shadow of 5.4 m. Find the height of the lamp-post.

Solution: In \( \triangle ABC \) and \( \triangle PQR \)

\[
\angle B = \angle Q = 90^0.
\]

\[
\angle C = \angle R \quad (AC \parallel PR, \text{ all sun's rays are parallel at any instance})
\]

\( \triangle ABC \sim \triangle PQR \) (by AA similarity)

\[
\frac{AB}{PQ} = \frac{BC}{QR} \quad \text{(cpst, corresponding parts of Similar triangles)}
\]
\[ \frac{1.65}{\text{PQ}} = \frac{1.8}{5.4} \]

\[ \text{PQ} = \frac{1.65 \times 5.4}{1.8} = 4.95 \text{m} \]

The height of the lamp post is 4.95 m.

**Example 6.** A man sees the top of a tower in a mirror which is at a distance of 87.6 m from the tower. The mirror is on the ground facing upwards. The man is 0.4 m away from the mirror and his height is 1.5 m. How tall is the tower?

**Solution:** In \( \triangle ABC \) & \( \triangle EDC \)

\( \angle ABC = \angle EDC = 90^\circ \)

\( \angle BCA = \angle DCE \) (angle of incidence and angle of reflection are same)

\( \triangle ABC \sim \triangle EDC \) (by AA similarity)

\[ \frac{AB}{ED} = \frac{BC}{CD} \implies \frac{1.5}{h} = \frac{0.4}{87.6} \]

\[ h = \frac{1.5 \times 87.6}{0.4} = 328.5 \text{m} \]

Hence, the height of the tower is 328.5 m.

**Example 7.** Gopal is worrying that his neighbour can see into his living room from the top floor of his house. He has decided to build a fence that is high enough to block the view from their top floor window. What should be the height of the fence? The measurements are given in the figure.

**Solution:** In \( \triangle ABD \) & \( \triangle ACE \)

\( \angle B = \angle C = 90^\circ \)

\( \angle A = \angle A \) (common angle)

\( \triangle ABD \sim \triangle ACE \) (by AA similarity)

\[ \frac{AB}{AC} = \frac{BD}{CE} \implies \frac{2}{8} = \frac{BD}{1.2} \]

\[ BD = \frac{2 \times 1.2}{8} = \frac{2.4}{8} = 0.3 \text{m} \]

Total height of the fence required is 1.5 m. + 0.3 m. = 1.8 m to block the neighbour’s view.
**Exercise - 8.2**

1. In the given figure, \( \angle ADE = \angle B \)
   
   (i) Show that \( \triangle ABC \sim \triangle ADE \)
   
   (ii) If \( AD = 3.8 \text{ cm}, \ AE = 3.6 \text{cm} \)
        \( BE = 2.1 \text{ cm}, \ BC = 4.2 \text{ cm} \)
        find \( DE \).

2. The perimeters of two similar triangles are 30 cm and 20 cm respectively. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

3. A girl of height 90 cm is walking away from the base of a lamp post at a speed of 1.2 m/sec. If the lamp post is 3.6m above the ground, find the length of her shadow after 4 seconds.

4. CM and RN are respectively the medians of \( \triangle ABC \) and \( \triangle PQR \).
   
   Prove that
   
   (i) \( \triangle AMC \sim \triangle PNR \)
   
   (ii) \( \frac{CM}{RN} = \frac{AB}{PQ} \)
   
   (iii) \( \triangle CMB \sim \triangle RNQ \)

5. Diagonals AC and BD of a trapezium ABCD with AB \parallel DC intersect each other at the point ‘O’. Using the criterion of similarity for two triangles, show that \( \frac{OA}{OC} = \frac{OB}{OD} \).

6. AB, CD, PQ are perpendicular to BD.
   
   \( AB = x, \ CD = y \) and \( PQ = Z \)
   
   prove that \( \frac{1}{x} + \frac{1}{y} = \frac{1}{z} \).

7. A flag pole 4m tall casts a 6 m., shadow. At the same time, a nearby building casts a shadow of 24m. How tall is the building?

8. CD and GH are respectively the bisectors of \( \angle ACB \) and \( \angle EGF \) such that D and H lie on sides AB and FE of \( \triangle ABC \) and \( \triangle FEG \) respectively. If \( \triangle ABC \sim \triangle FEG \) then show that

   (i) \( \frac{CD}{GH} = \frac{AC}{FG} \)        (ii) \( \triangle DCB \sim \triangle HGE \)        (iii) \( \triangle DCA \sim \triangle HGF \)

---

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9. AX and DY are altitudes of two similar triangles ΔABC and ΔDEF. Prove that AX : DY = AB : DE.

10. Construct a triangle shadow similar to the given ΔABC, with its sides equal to \( \frac{5}{3} \) of the corresponding sides of the triangle ABC.

11. Construct a triangle of sides 4cm, 5 cm and 6 cm. Then, construct a triangle similar to it, whose sides are \( \frac{2}{3} \) of the corresponding sides of the first triangle.

12. Construct an Isosceles triangle whose base is 8cm and altitude is 4 cm. Then, draw another triangle whose sides are \( 1 \frac{1}{2} \) times the corresponding sides of the isosceles triangle.

### 8.5 Areas of Similar Triangles

For two similar triangles, ratio of their corresponding sides is the same. Do you think there is any relationship between the ratio of their areas and the ratio of their corresponding sides? Let us do the following activity to understand this.

**Activity**

Make a list of pairs of similar polygons in this figure.

Find

(i) The ratio of similarity and
(ii) The ratio of areas.

You will observe that ratio of areas is the square of the ratio of their corresponding sides.

Let us prove it like a theorem.

**Theorem-8.6**: The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

*Given*: ΔABC ~ ΔPQR
RTP: \[
\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2.
\]

**Construction:** Draw AM \(\perp BC\) and PN \(\perp QR\).

**Proof:**
\[
\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{1}{2} \times BC \times AM \quad \frac{1}{2} \times QR \times PN = \frac{BC \times AM}{QR \times PN} \quad \text{...(1)}
\]

In \(\Delta ABM \& \Delta PQN\)
\[
\angle B = \angle Q \quad (\because \Delta ABC \sim \Delta PQR)
\]
\[
\angle M = \angle N = 90^\circ
\]
\[
\therefore \Delta ABM \sim \Delta PQN \quad \text{(by AA similarity)}
\]
\[
\frac{AM}{PN} = \frac{AB}{PQ} \quad \text{...(2)}
\]

Also \(\Delta ABC \sim \Delta PQR \quad \text{(given)}\)
\[
\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad \text{...(3)}
\]
\[
\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB \times AB}{PQ \times PQ} \quad \text{from (1), (2) and (3)}
\]
\[
= \left(\frac{AB}{PQ}\right)^2.
\]

Now by using (3), we get
\[
\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2
\]

Hence proved.

Now let us see some examples.
Example-8. Prove that if the areas of two similar triangles are equal, then they are congruent.

Solution: \( \Delta ABC \sim \Delta PQR \)

So \( \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \)

But \( \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1 \) (\( \because \) areas are equal)

\[ \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 = 1 \]

So \( AB^2 = PQ^2 \)
\( BC^2 = QR^2 \)
\( AC^2 = PR^2 \)

From which we get \( AB = PQ \)
\( BC = QR \)
\( AC = PR \)

\( \therefore \) \( \Delta ABC \cong \Delta PQR \) (by SSS congruency)

Example-9. \( \Delta ABC \sim \Delta DEF \) and their areas are respectively 64 cm\(^2\) and 121 cm\(^2\).

If \( EF = 15.4 \) cm., then find \( BC \).

Solution: \( \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \left(\frac{BC}{EF}\right)^2 \)

\[ \frac{64}{121} = \left(\frac{BC}{15.4}\right)^2 \]

\[ 8 = \frac{BC}{15.4} \Rightarrow BC = \frac{8 \times 15.4}{11} = 11.2 \text{ cm} \]

Example-10. Diagonals of a trapezium \( ABCD \) with \( AB \parallel DC \), intersect each other at the point ‘\( O \)’. If \( AB = 2CD \), find the ratio of areas of triangles \( AOB \) and \( COD \).

Solution: In trapezium \( ABCD \), \( AB \parallel DC \) also \( AB = 2CD \).

In \( \Delta AOB \) and \( \Delta COD \)

\( \angle AOB = \angle COD \) (vertically opposite angles)
\( \angle OAB = \angle OCD \) (alternate interior angles)
\[ \Delta AOB \sim \Delta COD \text{ (by AA similarity)} \]

\[
\frac{\text{ar}(\Delta AOB)}{\text{ar}(\Delta COD)} = \frac{AB^2}{DC^2}
\]

\[
= \frac{(2DC)^2}{(DC)^2} = \frac{4}{1}
\]

\[
\therefore \text{ar}(\Delta AOB) : \text{ar}(\Delta COD) = 4 : 1.
\]

**Exercise - 8.3**

1. Equilateral triangles are drawn on the three sides of a right angled triangle. Show that the area of the triangle on the hypotenuse is equal to the sum of the areas of triangles on the other two sides.

2. Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangles described on its diagonal.

3. D, E, F are mid points of sides BC, CA, AB of \(\triangle ABC\). Find the ratio of areas of \(\triangle DEF\) and \(\triangle ABC\).

4. In \(\triangle ABC\), XY || AC and XY divides the triangle into two parts of equal area. Find the ratio of \(\frac{AX}{XB}\).

5. Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

6. \(\triangle ABC \sim \triangle DEF\). BC = 3cm EF = 4cm and area of \(\triangle ABC\) = 54 cm\(^2\). Determine the area of \(\triangle DEF\).

7. ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm. and BP = 3cm, AQ = 1.5 cm, CQ = 4.5 cm.

   Prove that (area of \(\triangle APQ\)) = \(\frac{1}{16}\) (area of \(\triangle ABC\)).

8. The areas of two similar triangles are 81 cm\(^2\) and 49 cm\(^2\) respectively. If the attitude of the bigger triangle is 4.5 cm. Find the corresponding attitude for the smaller triangle.

**8.6 Pythagoras Theorem**

You are familiar with the Pythagoras theorem, you had verified this theorem through some activities. Now we shall prove this theorem using the concept of similarity of triangles. For this, we make use of the following result.
**Theorem-8.7**: If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

**Proof**: ABC is a right triangle, right angled at B. Let BD be the perpendicular to hypotenuse AC.

In $\triangle ADB$ and $\triangle ABC$

$\angle A = \angle A$

And $\angle ADB = \angle ABC$ (why?)

So $\triangle ADB \sim \triangle ABC$ (How?) ...(1)

Similarly, $\triangle BDC \sim \triangle ABC$ (How?) ...(2)

So from (1) and (2), triangles on both sides of the perpendicular BD are similar to the whole triangle ABC.

Also since $\triangle ADB \sim \triangle ABC$

$\triangle BDC \sim \triangle ABC$

So $\triangle ADB \sim \triangle BDC$

This leads to the following theorem.

**Think - Discuss**

For a right angled triangle with integer sides at least one of its measurements must be an even number. Why? Discuss this with your friends and teachers.

### 8.6.1 Pythagoras Theorem (Baudhayana Theorem)

**Theorem-8.8**: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

**Given**: $\triangle ABC$ is a right triangle right angled at B.

**RTP**: $AC^2 = AB^2 + BC^2$

**Construction**: Draw $BD \perp AC$.

**Proof**: $\triangle ADB \sim \triangle ABC$

\[
\frac{AD}{AB} = \frac{AB}{AC} \quad \text{(sides are proportional)}
\]

$AD \cdot AC = AB^2 \quad \text{...(1)}$

Also, $\triangle BDC \sim \triangle ABC$
\[ \Rightarrow \frac{CD}{BC} = \frac{BC}{AC} \]

CD . AC = BC^2 \hspace{1cm} (2)

On adding (1) & (2)

\[ AD . AC + CD . AC = AB^2 + BC^2 \]

\[ AC (AD + CD) = AB^2 + BC^2 \]

\[ AC^2 = AB^2 + BC^2 \]

The above theorem was earlier given by an ancient Indian mathematician Baudhayan (about 800 BC) in the following form.

“The diagonal of a rectangle produces by itself the same area as produced by its both sides (i.e. length and breadth).” So sometimes, this theorem is also referred to as the Baudhayan theorem.

What about the converse of the above theorem?

We prove it like a theorem, as done earlier also.

**Theorem-8.9**: In a triangle if square of one side is equal to the sum of squares of the other two sides, then the angle opposite to the first side is a right angle and the triangle is a right angled triangle.

**Given**: In \( \triangle ABC \),

\[ AC^2 = AB^2 + BC^2 \]

**RTP**: \( \angle B = 90^0 \).

**Construction**: Construct a right angled triangle \( \triangle PQR \) right angled at Q such that PQ = AB and QR = BC.

**Proof**: In \( \triangle PQR \), \( PR^2 = PQ^2 + QR^2 \) (Pythagoras theorem as \( \angle Q = 90^0 \))

\[ PR^2 = AB^2 + BC^2 \] \hspace{1cm} (by construction) \hspace{1cm} (1)

but \( AC^2 = AB^2 + BC^2 \) \hspace{1cm} (given) \hspace{1cm} (2)

\[ \therefore \ AC = PR \] from (1) & (2)

Now In \( \triangle ABC \) and \( \triangle PQR \)

\[ AB = PQ \] \hspace{1cm} (by construction)

\[ BC = QE \] \hspace{1cm} (by construction)

\[ AC = PR \] \hspace{1cm} (proved)
\[\therefore \triangle ABC \cong \triangle PQR \text{ (by SSS congruency)}\]

\[\therefore \angle B = \angle Q \text{ (by cpct)}\]

but \[\angle Q = 90^\circ \text{ (by construction)}\]

\[\therefore \angle B = 90^\circ.\]

Hence proved.

Now let us take some examples.

**Example-11.** A ladder 25m long reaches a window of building 20m above the ground. Determine the distance of the foot of the ladder from the building.

**Solution:** In \(\triangle ABC\), \[\angle C = 90^\circ\]

\[\Rightarrow AB^2 = AC^2 + BC^2 \text{ (by Pythagorean theorem)}\]

\[25^2 = 20^2 + BC^2\]

\[BC^2 = 625 - 400 = 225\]

\[BC = \sqrt{225} = 15m\]

Hence, the foot of the ladder is at a distance of 15m from the building.

**Example-12.** BL and CM are medians of a triangle ABC right angled at A. Prove that \[4(BL^2 + CM^2) = 5BC^2.\]

**Solution:** BL and CM are medians of \(\triangle ABC\) in which \[\angle A = 90^\circ.\]

In \(\triangle ABC\)

\[BC^2 = AB^2 + AC^2 \text{ (Pythagorean theorem)} \ldots (1)\]

In \(\triangle ABL\), \[BL^2 = AL^2 + AB^2\]

So \[BL^2 = \left(\frac{AC}{2}\right)^2 + AB^2 \quad (\because \text{L is the midpoint of AC})\]

\[BL^2 = \frac{AC^2}{4} + AB^2\]

\[\therefore 4BL^2 = AC^2 + 4AB^2 \ldots (2)\]

In \(\triangle CMA\), \[CM^2 = AC^2 + AM^2\]

\[CM^2 = AC^2 + \left(\frac{AB}{2}\right)^2 \quad (\because \text{M is the mid point of AB})\]
\[ CM^2 = AC^2 + \frac{AB^2}{4} \]
\[ 4CM^2 = 4AC^2 + AB^2 \quad \text{...(3)} \]

On adding (2) and (3), we get
\[ 4(BL^2 + CM^2) = 5(AC^2 + AB^2) \]
\[ \therefore 4(BL^2 + CM^2) = 5BC^2 \quad \text{from (1).} \]

**Example-13.** ‘O’ is any point inside a rectangle ABCD.

Prove that \( OB^2 + OD^2 = OA^2 + OC^2 \)

**Solution:** Through ‘O’ draw PQ \( \parallel \) BC so that P lies on AB and Q lies on DC.

Now PQ \( \parallel \) BC

\[ \therefore \quad PQ \perp AB \quad \text{&} \quad PQ \perp DC \quad (\because \angle B = \angle C = 90^\circ) \]

So, \( \angle BPQ = 90^\circ \) & \( \angle CQP = 90^\circ \)

\[ \therefore \quad \text{BPQC and APQD are both rectangles.} \]

Now from \( \triangle OPB, \quad OB^2 = BP^2 + OP^2 \quad \text{...(1)} \]

Similarly from \( \triangle OQD, \quad \text{we have} \quad OD^2 = OQ^2 + DQ^2 \quad \text{...(2)} \]

From \( \triangle OQC, \quad \text{we have} \quad OC^2 = OQ^2 + CQ^2 \quad \text{...(3)} \]

And from \( \triangle OAP, \quad OA^2 = AP^2 + OP^2 \)

Adding (1) & (2)

\[ OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2 \]

\[ = CQ^2 + OP^2 + OQ^2 + AP^2 \quad (\because BP = CQ \text{ and } DQ = AP) \]

\[ = CQ^2 + OQ^2 + OP^2 + AP^2 \]

\[ = OC^2 + OA^2 \quad \text{(from (3) & (4))} \]

**Do This**

1. In \( \triangle ACB, \quad \angle C = 90^\circ \) and \( CD \perp AB \)

Prove that \( \frac{BC^2}{AC^2} = \frac{BD}{AD} \).

2. A ladder 15m long reaches a window which is 9 m above the ground on one side of a street. Keeping its foot at the same point, the ladder is turned to other side of the street to reach a window 12m high. Find the width of the street.
3. In the given fig. if AD \perp BC

Prove that \(AB^2 + CD^2 = BD^2 + AC^2\).

**Example-14.** The hypotenuse of a right triangle is 6m more than twice of the shortest side. If the third side is 2 m., less than the hypotenuse, find the sides of the triangle.

**Solution :** Let the shortest side be \(x\) m.

Then hypotenuse = \((2x + 6)\)m and third side = \((2x + 4)\)m.

by Pythagorean theorem, we have

\[(2x + 6)^2 = x^2 + (2x + 4)^2\]

\[4x^2 + 24x + 36 = x^2 + 4x^2 + 16x + 16\]

\[x^2 - 8x - 20 = 0\]

\[(x - 10)(x + 2) = 0\]

\[x = 10 \text{ or } x = -2\]

but \(x\) can’t be negative as side of a triangle.

\[\therefore x = 10\]

Hence, the sides of the triangle are 10m, 26m and 24m.

**Example-15.** \(ABC\) is a right triangle right angled at \(C\). Let \(BC = a\), \(CA = b\), \(AB = c\) and let \(p\) be the length of perpendicular from \(C\) on \(AB\). Prove that (i) \(pc = ab\) (ii) \(\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}\).

**Solution :**

(i) \(CD \perp AB\) and \(CD = p\).

Area of \(\Delta ABC\) = \(\frac{1}{2} \times AB \times CD\)

\[= \frac{1}{2} \times cp\]

Also area of \(\Delta ABC\) = \(\frac{1}{2} \times BC \times AC\)

\[= \frac{1}{2} \times ab\]
\[ \frac{1}{2} \text{cp} = \frac{1}{2} \text{ab} \]
\[ \text{cp} = \text{ab} \] ...(1)

(ii) Since \( \triangle ABC \) is a right triangle right angled at \( C \).
\[ AB^2 = BC^2 + AC^2 \]
\[ c^2 = a^2 + b^2 \]
\[ \left( \frac{ab}{p} \right)^2 = a^2 + b^2 \]
\[ \frac{1}{p^2} = \frac{a^2 + b^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2}. \]

**Exercise - 8.4**

1. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

2. \( \triangle ABC \) is a right triangle right angled at \( B \). Let \( D \) and \( E \) be any points on \( AB \) and \( BC \) respectively. Prove that \( AE^2 + CD^2 = AC^2 + DE^2 \).

3. Prove that three times the square of any side of an equilateral triangle is equal to four times the square of the altitude.

4. \( \triangle PQR \) is a triangle right angled at \( P \) and \( M \) is a point on \( QR \) such that \( PM \perp QR \).
   Show that \( PM^2 = QM \cdot MR \).

5. \( \triangle ABD \) is a triangle right angled at \( A \) and \( AC \perp BD \)
   Show that (i) \( AB^2 = BC \cdot BD \).
   (ii) \( AC^2 = BC \cdot DC \)
   (iii) \( AD^2 = BD \cdot CD \).

6. \( \triangle ABC \) is an isosceles triangle right angled at \( C \). Prove that \( AB^2 = 2AC^2 \).

7. ‘\( O \)’ is any point in the interior of a triangle \( ABC \).
   \( OD \perp BC, OE \perp AC \) and \( OF \perp AB \), show that
   (i) \( OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2 \)
   (ii) \( AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2 \).
8. A wire attached to vertical pole of height 18m is 24m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

9. Two poles of heights 6m and 11m stand on a plane ground. If the distance between the feet of the poles is 12m find the distance between their tops.

10. In an equilateral triangle ABC, D is a point on side BC such that \(BD = \frac{1}{3}BC\). Prove that \(9AD^2 = 7AB^2\).

11. In the given figure, ABC is a triangle right angled at B. D and E are points on BC trisect it. Prove that \(8AE^2 = 3AC^2 + 5AD^2\).

12. ABC is an isosceles triangle right angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of \(\triangle ABE\) and \(\triangle ACD\).

8.7 Different Forms of Theoretical Statements

1. Negation of a statement:

We have a statement and if we add “Not” after the statement, we will get a new statement; which is called negation of the statement.

For example take a statement “\(\triangle ABC\) is a equilateral”. If we denote it by “\(p\)”, we can write like this.

\(p\) : Triangle ABC is equilateral and its negation will be “Triangle ABC is not equilateral”. Negation of statement \(p\) is denoted by \(~p\); and read as negotiation of \(p\). the statement \(~p\) negates the assertion that the statement \(p\) makes.

When we write the negation of the statements we would be careful that there should no confusion; in understanding the statement.

Observe this example carefully

\(P\) : All irrational numbers are real numbers. We can write negation of \(P\) like these ways.
i) \( \sim p \): All irrational numbers are not real numbers.

ii) \( \sim p \): Not all the irrational are real numbers.

How do we decide which negation is correct? We use the following criterion “Let \( p \) be a statement and \( \sim p \) its negation. Then \( \sim p \) is false whenever \( p \) is true and \( \sim p \) is true whenever \( p \) is false.

For example  
\[
\begin{align*}
\text{s} & : 2 + 2 = 4 \text{ is True} \\
\sim \text{s} & : 2 + 2 \neq 4 \text{ is False}
\end{align*}
\]

2. **Converse of a statement:**

A sentence which is either true or false is called a simple statement. If we combine two simple statements then we will get a compound statement. Connecting two simple statements with the use of the words “If and then” will give a compound statement which is called implication (or) conditional.

Combining two simple statements \( p \) \& \( q \) using if and then, we get \( p \) implies \( q \) which can be denoted by \( p \Rightarrow q \). In this \( p \Rightarrow q \), suppose we interchange \( p \) and \( q \) we get \( q \Rightarrow p \). This is called its converse.

Example : \( p \Rightarrow q \): In \( \triangle ABC \), if \( AB = AC \) then \( \angle C = \angle B \)

Converse \( q \Rightarrow p \): In \( \triangle ABC \), if \( \angle C = \angle B \) then \( AB = AC \)

3. **Proof by contradiction:**

In this proof by contradiction, we assume the negation of the statement as true; which we have to prove. In the process of proving we get contradiction somewhere. Then, we realize that this contradiction occur because of our wrong assumption which is negation is true. Therefore we conclude that the original statement is true.

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**Optional Exercise**

[This exercise is not meant for examination]

1. In the given figure,

\[
\frac{QT}{PR} = \frac{QR}{QS} \quad \text{and} \quad \angle 1 = \angle 2
\]

prove that \( \triangle PQS \sim \triangle TQR \).
2. Ravi is 1.82m tall. He wants to find the height of a tree in his backyard. From the tree’s base he walked 12.20 m. along the tree’s shadow to a position where the end of his shadow exactly overlaps the end of the tree’s shadow. He is now 6.10m from the end of the shadow. How tall is the tree?

3. The diagonal AC of a parallelogram ABCD intersects DP at the point Q, where ‘P’ is any point on side AB. Prove that CQ×PQ = QA×QD.

4. △ABC and △AMP are two right triangles right angled at B and M respectively. Prove that (i) △ABC ~ △AMP (ii) \[ \frac{CA}{PA} = \frac{BC}{MP} \]

5. An aeroplane leaves an airport and flies north at a speed of 1000 kmph. At the same time another aeroplane leaves the same airport and flies due west at a speed of 1200 kmph. How far apart will the two planes be after 1\(\frac{1}{2}\) hour?

6. In a right triangle ABC right angled at C. P and Q are points on sides AC and CB respectively which divide these sides in the ratio of 2 : 1. Prove that (i) \[ 9AQ^2 = 9AC^2 + 4BC^2 \] (ii) \[ 9BP^2 = 9BC^2 + 4AC^2 \] (iii) \[ 9(AQ^2 + BP^2) = 13AB^2 \]

**What We Have Discussed**

1. Two figures having the same shape but not necessarily of the same size are called similar figures.
2. All the congruent figures are similar but the converse is not true.
3. Two polygons of the same number of sides are similar
   (i) their corresponding angles are equal and
   (ii) Their corresponding sides are in the same ratio (ie proposition)
   For similarity of polygons either of the above two condition is not sufficient.
4. If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points then the other two sides are divided in the same ratio.
5. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

6. If in two triangles, angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (AAA similarity).

7. If two angles of a triangle are equal to the two angles of another triangle, then third angles of both triangles are equal by angle sum property of a triangle.

8. If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar. (SSS similar)

9. If one angle of triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio then the triangle are similar. (SAS similarity)

10. The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

11. If a perpendicular is drawn from the vertex of a right triangle on both sides of the perpendicular are similar to the whole triangle and also to each other.

12. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Pythagorean Theorem).

13. In a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

**Puzzle**

Draw a triangle. Join the mid-points of the sides of the triangle. You get 4 triangles. Again join the mid-points of these triangles. Repeat this process. All the triangles drawn are similar triangles. Why? Think and discuss with your friends.