

Ratio and Proportion

11.1 INTRODUCTION

In our day to day life, we compare quantities in different ways. In the market, we compare which vegetables look fresh, which are expensive, which are reasonably priced etc. Let us consider some examples.

Every day Satya and Madhukar drink milk before going to school. Satya adds 2 spoons of sugar to 1 cup of milk. Madhukar adds 1 spoon of sugar in same sized cup of milk.



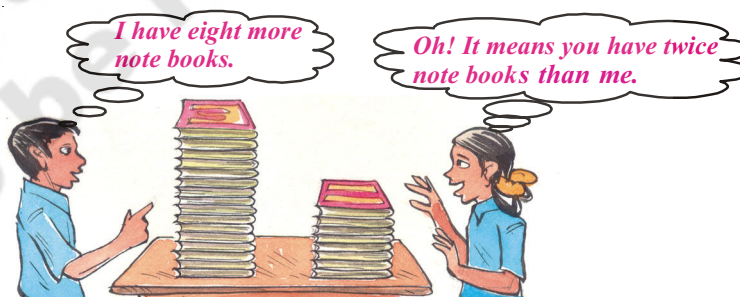
Which milk will be sweeter? Can we say without tasting the milk?

Sharada has 3 spoons of sugar in 2 cups of milk. How do we compare the sweetness of milk in the three cases?

Consider the following situation.

Siri has 8 note books and Ravi has 16 note books.

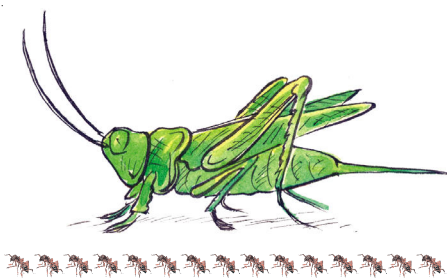
To compare the books, Ravi compared them by finding their difference. And Siri compared them by division. One quantity is how much more or how much less than the other quantity is Comparison by Subtraction. And one



quantity is how many times more or less than the other is Comparison by Division.

Give 3 instances where we compare quantities by subtraction and by division.

If we wish to compare the length of an ant and a grasshopper. The difference in length does not express the comparison. The grasshopper's length, which is approximately 4 cm to 5 cm, is too long in comparison to an ant's length which is just a few mm. The difference in length would be around 4cm only. That by itself does not appear to be a big difference. Comparison will be better if we try to find out how many ants can be placed one behind the other to match the length of the grasshopper. So we may say that 15 to 20 ants together have the same length of a grasshopper.



Consider another example.

The cost of a car is ₹2,50,000 and that of a motorbike is ₹50,000. If we compare the cost by taking the difference between the costs, the difference is ₹2,00,000. This does not help us understand the extent of difference. If we compare by division; then $\frac{2,50,000}{50,000} = \frac{5}{1}$ which tells us that for cost of every car we can buy 5 motorcycles.

Thus, in certain situations, comparison by division makes better sense than comparison by taking the difference.

Let us consider one more example.

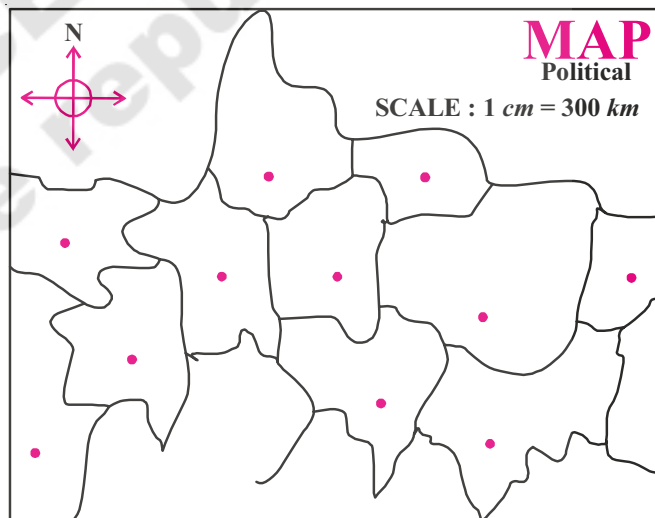
Latha is 3 years old and Kareem is 18 years old. We can say that Kareem is 15 years older than Latha. Compare this with Rahim being 65 years old and Reshma 50 years. In both the cases the difference in the ages is 15 years. It is obvious however, the difference in Latha and Kareem is of a different kind. It is much better to say that Kareem is six times older than Latha.

This type of the comparison where we compare things by division is called ratio. In this chapter we will learn about ratio in detail.

Another example where we use such comparison is in making maps.

Look at the map:

The places on a map are very close by as compared to the actual distance between them. The scale of map tells us that the comparison between the actual distance and the distance shown in the map. For example, if for the a map of a street or a market, the scale says one centimetre is equal to one hundred metres then we know that the distance on the map is ten thousandth of the actual distance. In other words actual distance is 10,000 times the distance on the map. If we compare the distance on the map to the actual distance



represented we would say that 5 cm represents 500 meters. Comparison by subtraction would tell us that the actual distance is 499 meters 95 cm more than the distance on the map. Compared to the statement that the actual distance is 10,000 times the distance on the map, this statement does not convey much.

In the first example ratio of Siri's books to Ravi's books is $16/8 = 2/1 = 2 : 1$

We read it as 2 is to 1.

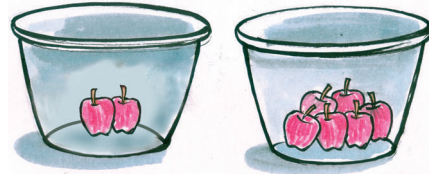
If we change the comparison order and ask for the ratio of Ravi's books to Siri's books, the answer will be $8/16 = 1/2 = 1 : 2$

When we compare two quantities, we have to take care of order of the quantities.

TRY THESE



Observe the example and fill in the blanks.



S. No	First Quantity	Second Quantity	Comparing statement	Ratio	Comparison by changing the order	Ratio
1	2 apples	6 apples	Apples in the first basket are one-third of the apples in the second basket.	1 : 3	Apples in the second basket are 3 times the apples in the first basket.	3 : 1
2	500g of Copper	1000g of Iron				
3	Cost of T-Shirt ₹200	Cost of a Coat ₹1000				

11.2 Comparing quantities with different units

The height of a tree is 13m and its picture in the book is 26cm long. Can we say that the height of the tree in the picture is twice that of the actual tree?

Obviously not, as we know actual tree is taller than the tree shown in the picture.

Height of the tree is 13m i.e. 1300 cm and the height of tree in the picture is just 26 cm

Now the ratio between two heights $1300/26 = 50 : 1$

So we can say that the actual tree's actual height is 50 times than that of the picture in the book.

When we compare two quantities, they must be in the same units.

In general the ratio of two quantities a and b is written as **a : b** and we read it as **a is to b**

The two quantities **a** and **b** are called terms of the ratio. First quantity 'a' is called **first term** or **antecedent** and second quantity 'b' is called **second term** or **consequent**.



Example-1. Rafi has 16 red marbles and 4 blue marbles. Find the ratio of red marbles to blue marbles Rafi has?

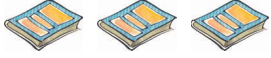
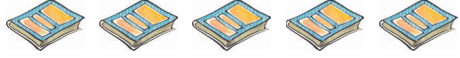








Solution: Red marbles : Blue marbles = 16 : 4
= 4 : 1

Number of red marbles are 4 times than that of blue marbles.



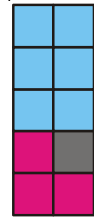
EXERCISE - 11.1

1. Complete the following table.

S. No.	First quantity	Second quantity	ratio
(i)			3 : 5
(ii)			
(iii)			
(iv)			
(v)			

2. Compare:

- Number of blue coloured squares _____ times the number of red colour squares.
- Number of red coloured squares is _____ of the number of blue coloured squares.
- Find the ratio of number of blue squares to the number of red squares.



3. Solve the following:

- A milk man adds 250 ml of water to 1 litre of milk. Find the ratio of water to milk.
- Satya's mother bought 4 kg pulses and 50g chilli powder. Find the ratio of weights of chilli powder to pulses. What is the ratio of weights of the pulses to chilli powder?
- Rani takes 30 minutes to reach school from home. Ismail takes $\frac{1}{2}$ an hour to cover the same distance. Find the ratio of time taken by Rani to the time taken by Ismail.

11.3 Ratio in different situation

Sloka is in class VI. She has ₹50 with her. Mahesh is also in the same class. He has ₹100 with him. Both of them decided to save their amounts in 'Sanchayika', a savings programme in their school. After depositing their amounts, they came to know that total money saved by their class students is ₹2000. They want to compare their amount to the total amount saved.

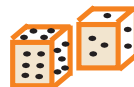
The ratio of savings of Sloka to savings of Mahesh = 50:100

The ratio of savings of Sloka to the total money saved = 50:2000

The ratio of savings of Mahesh to the total money saved. = 100: 2000

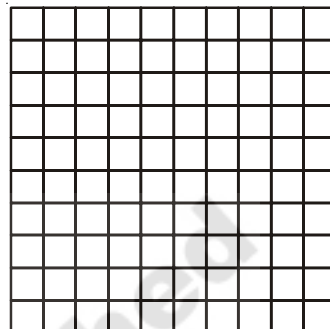
ACTIVITY

Take a square ruled paper. Throw a die and note the number on the die. Fill as many squares as the number noted with your favorite colour. Ask your friend to throw a dice and colour as many squares with some other colour.



1. Find the ratio of number of squares coloured by you to the number of squares coloured by your friend?

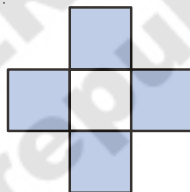
2. Find the ratio of number of squares coloured by you to the total number of squares coloured? _____
3. Find the ratio of number of squares coloured by your friend to the total number of squares coloured?
_____.
4. Can you find any other ratios in this activity? Think and discuss with your friend.



TRY THESE

In the given figure, find the ratio of

- (i) Shaded parts to unshaded parts
- (ii) Shaded parts to total parts
- (iii) Unshaded parts to total parts



11.4 Same ratio in different situations

Consider the following:

- Length of a room is 30 m and its breadth is 20 m. So, the ratio of length of the room to the breadth of the room = $\frac{30}{20} = \frac{3}{2}$ is same as 3 : 2
- There are 24 girls and 16 boys going for a picnic. Ratio of the number of girls to the number of boys = $\frac{24}{16} = \frac{3}{2}$ is the same as 3 : 2

The ratio in both the examples is 3 : 2.

- Note, the ratio 30 : 20 and 24 : 16, in lowest form are same as 3 : 2. These are **equivalent ratios**.

Can you think of some more examples having the ratio 3 : 2?

It is fun to write situations that give rise to a certain ratio. For example, write situations that give the ratio 2 : 3. We have given two examples, you write 3 more.

- Ratio of the breadth of a table to the length of the table is 2 : 3.
- Sheena has 2 marbles and her friend Shabnam has 3 marbles.

Example-2. In a mathematics class there are 16 boys and 20 girls. Find the ratio of number of boys to the number of girls and express it in the simplest form.

Solution: Ratio of number of boys to number of girls = 16: 20

$$= \frac{16}{20} = \frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 5} = \frac{4}{5}$$

The simplest form is 4 : 5

A ratio is said to be in the **simplest form** or in the **lowest terms** when it is written in terms of whole numbers with no common factors other than 1.

TRY THESE



1. Complete the following tables

Ratio	1:2	2:3	5:7		
1 time	1:2				
2 times		4:6			
3 times			15:21		
4 times				12:16	
5 times					20:25

2. Complete the following table.

Fill this first

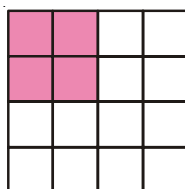
1 rupee = paisa
 1000g = kg
 1 hour = min
 100cm = m
 1 minute = sec
 10 mm = cm

S. No.	First quantity	Second quantity	ratio	Simplified ratio
1.	20 paise	₹ 1		
2.	800 g	1 kg		
3.	1 hr	30 min		
4.	2 m	125 cm		
5.	3 min	45 sec		
6.	30 mm	1 cm		

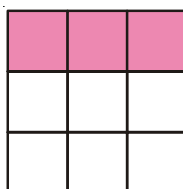
Remember:

Ratio is comparison of quantities expressed in same units

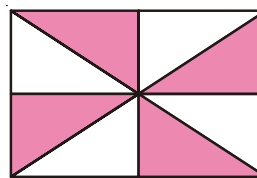
3. In the following figures, express the ratio of shaded parts to unshaded parts in the simplest terms.



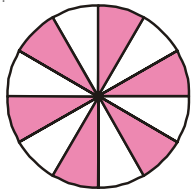
(i)



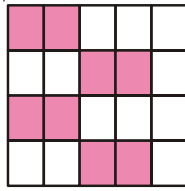
(ii)



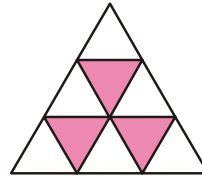
(iii)



(iv)



(v)



(vi)



EXERCISE - 11.2

- Express the following ratios in their simplest form.
 - 2:3
 - 16:20
 - 5:6
 - 20:60
 - 8:15
 - 19 : 2
- A bag contains 20kg of rice and another bag contains 60 kg of wheat. Find the ratio of the amount of rice to that of wheat? What is the ratio of rice to the total weight?
- Suppose there are 32 students in a class of which 12 are girls. Find:
 - The ratio of number of boys to number of girls
 - The ratio of number of boys to total number of students
 - The ratio of number of girls to total number of students.
- Draw a four sided closed figure and divide it in to some number of equal parts. Shade the figure with any colour so that the ratio of shaded parts to unshaded parts 1 : 3. Draw two more different figures and do the same.
- Imran bought 2 liters of oil and Vijay bought 500ml of oil. Find the ratio of quantities of oil bought by Imran to oil bought by Vijay.
- Weight of Abraham is 20 kg and his father's weight is 60kg. Find the ratio of weight of Abraham and his father. Express it in the simplest form.
- Ramu spent $\frac{2}{5}$ th of his money on a story book. Find the ratio of money spent to the money with him at the beginning.

11.5 Division of a given quantity in a given ratio

Example-3. On Snigdha's birthday, her father brought a flower bouquet that contain 18 flowers in all. If the ratio of red flowers to yellow is 1 : 2. Find their number.

Solution: Ratio of red flowers to yellow flowers = 1 : 2

Total parts = $1+2 = 3$ parts

Total number of flowers = 18 flowers

3 parts = 18 flowers

Each part = $18/3=6$ flowers

Red flowers = 1 part = 1×6 flowers = 6 flowers

Yellow flowers = 2 parts = 2×6 flowers = 12 flowers.

Example-4. A goldsmith mixes gold and copper in the ratio 7:2 to prepare an ornament. If the ornament weighs 45gms, find the weight of gold and copper in it.

Solution: Ratio of gold and copper = 7: 2

Sum of the ratio terms = $7+2 = 9$

Weight of 9 parts = 45 gm

Weight of Each part = $45 \div 9 = 5$ gm

Part of gold weighs = 7 parts $\times 5$ gm = 35 gm

Part of copper weighs = 2 parts $\times 5$ gm = 10 gm

Example-5. Line segment AB is divided into five equal parts.



- (i) In what ratio does X divides line segment AB?
- (ii) If the length of the line segment AB is 15 cm, find the length of the line segments AX and XB.

Solution: (i) X divides AB in the ratio 3:2

- (ii) Total parts = $3 + 2 = 5$ parts
Length of line segment AB = 15 cm
Length of 5 parts = 15 cm

$$\text{Length of each part} = \frac{15}{5} = 3 \text{ cm}$$

$$\text{Length of line segment AX} = 3 \text{ parts} = 3 \times 3 \text{ cm} = 9 \text{ cm}$$

$$\text{Length of line segment XB} = 2 \text{ parts} = 2 \times 3 \text{ cm} = 6 \text{ cm}$$

TRY THESE

Make a pattern with squared tiles using black and white tiles in the ratio 2 : 5. There are many possible ways.



Example-6. Hari and Teja won a Lottery, which they agreed to share in the ratio of 5:3. If Teja received ₹150, how much did Hari receive? Also find the total amount.

Solution: Ratio of Teja's amount to Hari's amount is = 5 : 3

Teja has 3 parts, if 3 parts = 150

$$1 \text{ part} = \frac{150}{3} = 50$$

$$\text{and 5 parts} = 50 \times 5 = 250$$

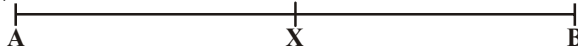
$$\text{Hari will receive ₹250 and the other amount is} = 250 + 150 = \text{₹400}$$



EXERCISE - 11.3

1. A bag of 25 marbles is shared between Rahul and Kiran in the ratio 2: 3
 - (i) How many marbles does Kiran receive?
 - (ii) How many marbles does Rahul receive?
2. A point X on AB = 14 cm divides it in the ratio 3 : 4. Find the length of AX and XB.

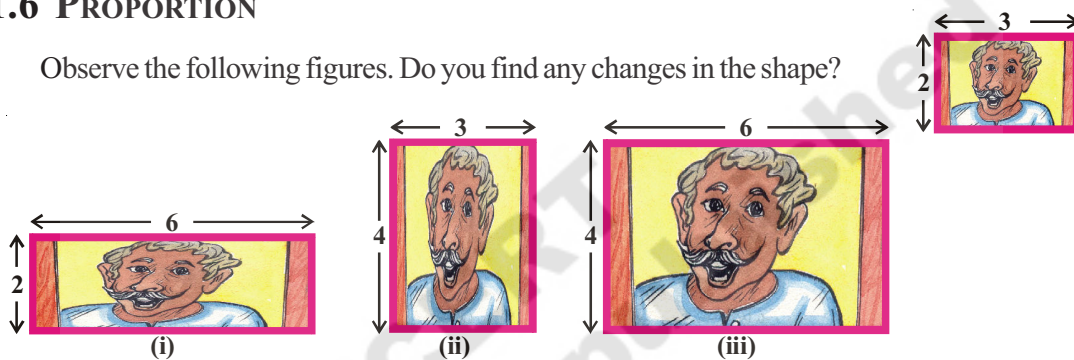
- Geetha and Laxmi won ₹ 1050 in a game. They agreed to share the amount in the ratio of 3:4. How much does each person receive?
- Divide ₹ 3600 between Satya and Vishnu in the ratio of 3:5.
- Two numbers are in the ratio 5:6. If the sum of the numbers is 132, find the two numbers.
- Estimate the ratio in which X divides AB and then check your estimation by measuring it.



- The income and savings of an employee are in the ratio 11:2. If his expenditure is ₹5346, then find his income and savings.

11.6 PROPORTION

Observe the following figures. Do you find any changes in the shape?



What difference do you find in the figures? The figures (i) and (ii) look different and their shapes have changed. Figure (iii) is enlarged but it does not look different. This is because there is a change in the size, but not in the shape.

Let us find the ratio of length and breadth in all the three cases

Ratio of length and breadth in the original picture = 3:2

Ratio of length and breadth in picture (i) = 6 : 2 i.e. simplest form is 3 : 1

Ratio of length and breadth in picture (ii) = 4 : 3 i.e. simplest form is 4 : 3

Ratio of length and breadth in picture (iii) = 6 : 4 i.e. simplest form is 3:2 which is as same as the ratio in the original picture. We can say picture (iii) is proportionate to the original picture and that's why their ratios are same. This equality of ratios is Proportion.

In general if the ratio of 'a' and 'b' is equal to the ratio of 'c' and 'd', we say that they are in proportion. This is represented as $a : b :: c : d$.

Consider another example.

Bhavika has 28 marbles and Vinila has 180 flowers. They want to share these among themselves. Bhavika gave 14 marbles to Vinila, and Vinila gave 90 flowers to Bhavika. But Vinila was not satisfied. She felt that she had given more flowers to Bhavika and the marbles given by Bhavika to her were much less.

What do you think? Is Vinila correct?

To solve this, both went to Vinila's mother Pooja.

Pooja explained that out of 28 marbles Bhavika gave 14 marbles to Vinila.

Therefore, ratio is $14 : 28 = 1 : 2$

And out of 180 flowers, Vini had given 90 flowers to Bhavika.

Therefore, ratio is $90 : 180 = 1 : 2$.

Since both the ratios are the same, the distribution is fair. Do you agree with Pooja's version?

Example-7. Raju and Bharath add their money and bought 20 pencils. Raju contributed ₹12 and Bharath ₹18. They wanted to distribute the pencils between them.

- (i) Bharath said 10 pencils for each
- (ii) Raju said 12 pencils for Bharath and 8 pencils for him.

Who is correct? justify your answer.

Solution: Ratio of amounts given by Raju and Bharath = $12 : 18$

$$= 12 \div 6 : 18 \div 6$$

$$= 2 : 3$$

According to Bharath, Ratio of pencils

$$= 10 : 10$$

$$= 10 \div 10 : 10 \div 10$$

$$= 1 : 1$$

Equal distribution of pencils is not proportional to the amount contributed.

According to Raju, Ratio of pencils

$$= 8 : 12$$

$$= 8 \div 4 : 12 \div 4 \text{ [HCF=4]}$$

$$= 2 : 3$$

For fair distribution the ratio of number of pencils should be the same as the ratio of amounts contributed. So it can be said that Raju is correct and 8 pencils should go to Raju and 12 pencils to Bharath.

TRY THESE

In the given square rule paper with 5 squares, colour 3 squares red and 2 squares green.

If 10 squares are given, find how many are to be red and how many of them are to be green so that it becomes proportionate to the figure.

If there are 15 squares then colour them accordingly.

11.7 Unitary Method

Consider the following:

Ravi went to purchase 3 kg of tomatoes. One shop keeper told him that the cost of tomatoes is ₹ 40 for 5 kg. Another shop keeper gave the price as ₹ 42 for 6 kg

What should Ravi do? Should he purchase tomatoes from the first shop keeper or from the second? How much he has to pay for 3 kg in each case?

Sreedevi helped him. She said, "Find the price of one kg of tomatoes in each shops and compare".

In the first shop 1 kg of tomatoes cost ₹ $40/5 = ₹ 8$ per kg

In the second shop 1 kg of tomatoes cost ₹ $42/6 = ₹ 7$ per kg

She advised Ravi to purchase tomatoes from the second shop-keeper unless the tomatoes are worse than in the other shop.

Do you agree with her?

Price of 1 kg of tomato in this shop is ₹ 7.

Then price of 3 kg of tomatoes = ₹ 7×3
= ₹ 21.

The method in which we first find the value of one unit and then the value of the required number of units is known as the **unitary method**.

Example-8. If the cost of 12 pencils is ₹ 24, then find the cost of 10 pencils.

Solution: First we find the cost of 1 pencil by dividing ₹ 24 by 12.

Cost of 12 pencils = ₹ 24

Cost of 1 pencil = $24 \div 12 = ₹ 2$

Cost of 10 pencils = $2 \times 10 = ₹ 20$

Cost of 10 pencils is ₹ 20.

Example-9. If the cost of 6 bottles of juice is ₹ 210, then what will be the cost of 4 bottles of juice?

Solution: Cost of 6 bottles of juice = ₹ 210

Cost of 1 bottle of juice = $210 \div 6 = ₹ 35$

Cost of 1 bottle of juice is ₹ 35

To find the cost of 4 bottles of juice, multiply the cost of 1 bottle of juice by 4

Cost of 1 bottle of juice = ₹ 35

Cost of 4 bottles of juice = $4 \times 35 = ₹ 140$

Cost of 4 bottles of juice is ₹ 140.



EXERCISE - 11.4

1. If three apples cost ₹ 45, how much would five apples cost?
2. Laxmi bought 7 books for a total of ₹ 56. How much would she pay for just 3 books?
3. Reena wants to prepare vegetable pulao, she needs 300 grams of rice. If she has to feed 4 people. How much of rice be needed if the same pulao is prepared for 7 people?
4. The cost of 16 chairs is ₹ 3600. Find the number of chairs that can be purchased for ₹ 4500.

5. A train moving at a constant speed covers a distance of 90 km. in 2 hours. Find the time taken by the train to cover a distance of 540 km at the same speed.
6. The income of Kumar for 3 months is ₹ 15000. If he earns the same amount for a month.
 - (i) How much will he earn in 5 months?
 - (ii) In how many months will he earn ₹ 95000?
7. If the cost of 7 meters of cloth is Rs 294, find the cost of 5m of cloth.
8. A farmer has sheep and cows in the ratio 8 : 3.
 - (i) How many sheep has the farmer, if he has 180 cows?
 - (ii) Find the ratio of the number of sheep to the total number of animals the farmer has.
 - (iii) Find the ratio of the total number of animals with the farmer to the number of cows with him.
9. Are 3, 5, 15, 9 in proportion? If we change their order, can we think of proportional pairs? Write as many proportionality statements as you can for the above example.?
10. The temperature has dropped by 15 degree Celsius in the last 30 days. If the rate of temperature drop remains the same, how much more will the temperature drop in the next 10 days?
11. Fill in the following blanks:

$$\frac{15}{18} = \frac{\square}{6} = \frac{10}{\square} = \frac{\square}{30}$$

12. (i) Ratio of breadth and length of a hall is 2: 5. Complete the following table that shows some possible breadths and lengths of the hall

Breadth of the hall(in m)	10	?	40			
Length of the hall(in m)	25	50	?			

Add 3 more of your choice.

- (ii) Find is the ratio of length to breadth of your classroom.
13. Geetha earns ₹ 12000 a month, out of which she saves ₹ 3000. Find the ratio of her
 - (i) Expenditure to savings
 - (ii) Savings to her income
 - (iii) Expenditure to her income.
14. There are 45 persons working in an office. The number of females is 25 and the remaining are males. Find the ratio of
 - (i) The number of females to number of males
 - (ii) The number of males to the number of females.
15. A bag of sweets contain yellow and green sweets. For every 2 yellow sweets, there are 6 green sweets. Complete this table based on the above information.

Yellow		4	6		
Green	6	12		24	
Total Sweets	8		24		40

Now answer these questions.

- (i) What is the ratio of green to yellow sweets?
- (ii) If you have 8 yellow sweets, how many green sweets will you have?
- (iii) If there are 32 sweets in the medium sized bag. How many will be yellow?
- (iv) In the super fat size bag there are 40 sweets. How many will be green?
- (v) In the sweet bowl if there are 16 yellow sweets. How many total sweets are in the bowl?

16. In a school survey it was found that for every 4 girls there were 5 boys.

Fill in the following table.

Girls	4	8			
Boys			15	20	
Total					45

Now answer these questions:

- (i) What is the ratio of girls to boys?
- (ii) In a class of 27 children, how many would be girls?
- (iii) There are 54 children in a class. How many are boys?
- (iv) If 20 girls join in a year. How many boys would join?

WHAT HAVE WE DISCUSSED?

1. A Ratio is an ordered comparison of quantities of the same units
2. The ratio of two quantities 'a' and 'b' can be given in any one of the following ways.
 - (i) Symbolic form $a : b$
 - (ii) Fractional form $\frac{a}{b}$
 - (iii) Verbal form a is to b
3. The two quantities 'a' and 'b' are called terms of the ratio. First quantity 'a' is called first term or antecedent and second quantity 'b' is called second term or consequent.
4. A ratio is in the simplest form or in the lowest terms when it is written in terms of whole numbers having no common factors other than 1.
5. Equality of ratios is called Proportion.
6. The method in which we first find the value of one unit and then the value of the required number of units is known as unitary method.

