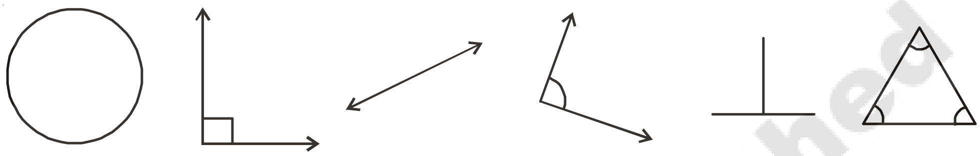
Practical Geometry

Chapter - 13

13.1 Introduction

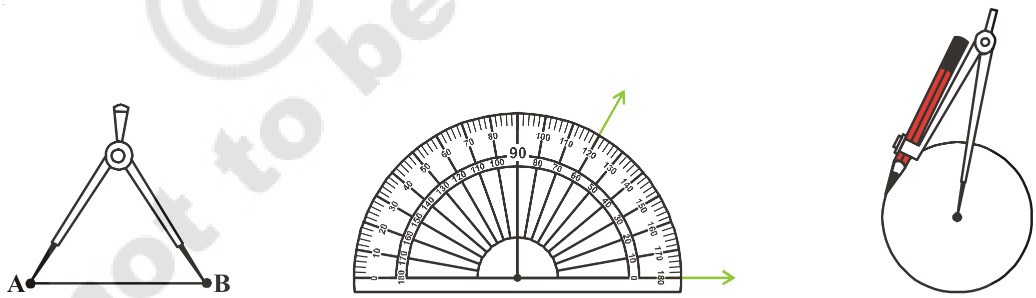
Copy the following shapes in your notebook with a pencil.



Do they look exactly the same? Measure their sides and angles by ruler and protractor.

What do you find? You will find their measures are not exactly the same. To make them exactly same we need to draw them of accurate sizes. For this we need to use tools. We will learn to construct such figure, in this chapter by using compasses, ruler and protractor. Ruler, compasses and protractor are our tools. These are all a part of our geometry box. Let us observe the geometry box.

What all is there in the geometry box? Besides the ruler, compasses and protractor we have a divider and set squares. The ruler is used for measuring lines, a comapsses for constructing, protractor measures angles and the divider is to make equal line segments or mark points on a line.



Divider Protractror Compass

13.2 A Line Segment

Let A and B be two points on a paper. Then the straight path from A to B is called a line segment AB, denoted by $\overline{AB}$.

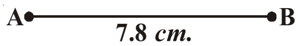
A ——— B

The distance between the points A and B is called the length of AB. Thus a line segment has a definite length, which can be measured.

13.2.1 Construction of a Line Segment of a given Length

We can construct a line segment of given length in two ways.

1. **By using ruler**: Suppose we want to draw a line segment of length 7.8cm



We can do it in this way

Place the ruler on paper and hold it firmly. Mark a point with a sharp edged pencil against 0 cm mark of the ruler. Name the point as A. Mark another point against 8 small divisions just after the 7 cm mark. Name this point as B. Join points A and B along the edge of the ruler. AB is the required line segment of length 7.8 cm

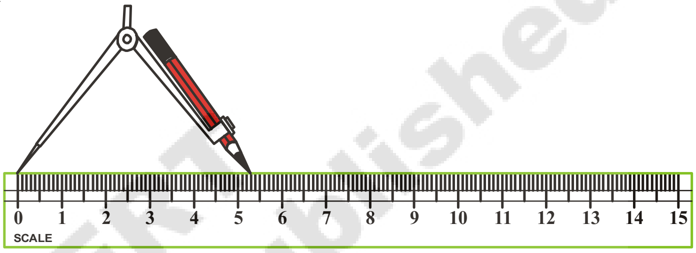
2. **By using Compasses:**

Suppose we want to draw a line segment of length 5.3 cm

Steps of Constructions:

***Step-1*:** Draw a line *l*. Mark a point A on the line *l*.

***Step-2*:** Place the metal pointer of the compasses on the zero mark of the ruler. Open the compasses so that pencil point touches the 5.3 cm mark on the ruler.



***Step-3*:** Place the pointer on A on the line *l* and draw an arc to cut the line. Mark the point where the arc cuts the line as B.

***Step-4*:** On the line l, we got the line segment AB of required length.

EXERCISE - 13.1

1. Construct a line segment of length 6.9 cm using ruler and compasses.
2. Construct a line segment of length 4.3 cm using ruler.
3. Construct a line segment MN of length 6cm. Mark any point O on it. Measure MO, ON and MN. What do you observe?
4. Draw a line segment $\overline{AB}$ of length 12 cm Mark a point C on the line segment $\overline{AB}$, such that $\overline{AC}$ = 5.6 cm. What should be the length of $\overline{CB}$? Measure the length of $\overline{CB}$.
5. Given that AB = 12 cm

(i) From the above figure measure the lengths of the following line segments.

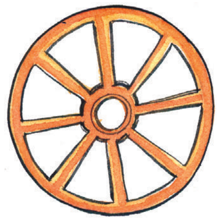
(a) $\overline{CD}$ (b) $\overline{DB}$ (c) $\overline{EA}$ (d) $\overline{AD}$

(ii) Verify $\overline{AE} - \overline{CE} = \overline{AC}$?

6. $\overline{AB}$ = 3.8 cm. Construct $\overline{MN}$ by compasses such that the length of $\overline{MN}$ is thrice that of $\overline{AB}$. Verify this with the help of a ruler.

13.3 Construction of a Circle

Look at the wheel shown here. Observe that every point on its boundary is at an equal distance from its centre.



Think of other such objects that are of this shape. Give 5 examples.

How to draw objects and figures having this shape. We can use many things like bangle, bowl top, plate and other things. These are however of a definite size. To draw a circle of given radius we use the compasses.

We use the following steps to construct a circle

Steps of Construction:

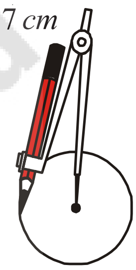
Step-1: Open the compasses for required radius. Let us say for example it is 3.7 *cm*

Step-2: Mark a point with sharp pencil. This is the centre. Mark it as O.

Step-3: Place the pointer of the compasses firmly at O.

Step-4: Without moving its metal point.

Now slowly rotate the pencil and till it come back to the starting point.

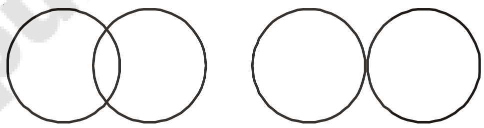


TRY THESE

Construct two circles with same radii (radius) in such a way that

(i) the circles intersects at two points

(ii) touch each other at one point only.

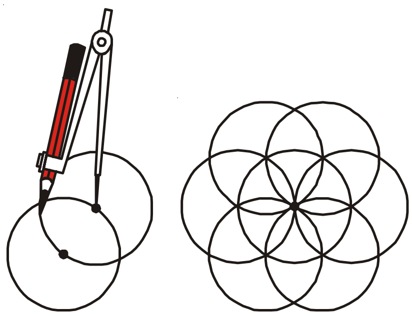


EXERCISE - 13.2

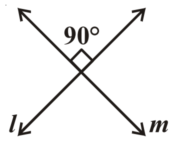
1. Construct a circle with centre M and radius 4 cm
2. Construct a circle with centre X and diameter 10 cm
3. Draw four circles of radius 2cm,3cm,4cm and 5cm with the same centre P.
4. Draw any circle and mark three points A, Band C such that
   (i) A is on the circle
   (ii) B is in the interior of the circle
   (iii) C is in the exterior of the circle.

ACTIVITY

Make a circle of desired radius in your note book. Make a point on it. Put compasses on it and make a circle without changing the radius. It will cut the circumference at two points. On both points repeat the process again, you will get a beautiful picture as shown . Colour it as you wish.

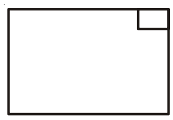


13.4 Perpendiculars



You know that two lines (or rays or segments) are said to be perpendicular if they intersect such that the angles formed between them are right angles.

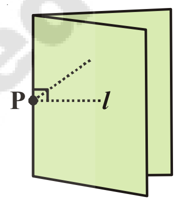
In the figure, the lines *l* and *m* are perpendicular.



The corners of a foolscap paper or your notebooks indicate lines meeting at right angles. Think other such objects where the lines meeting arc perpendicular. Give five examples.

1. Perpendicular through a Point on a given line

Activity



Take a tracing paper and draw a line *l* on it.

Mark a point P lying on this line. Now, we want to draw a perpendicular on *l* through P.

We simply fold the paper at point P such that the lines on both sides of the fold overlap each other.

When we unfold it, we find that the crease is perpendicular to *l*.

Think, Discuss and Write

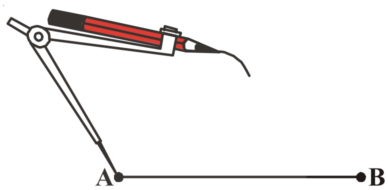
How would you check whether it is perpendicular or not? Note that it passes through P as required.

13.4.1 Constructing Perpendicular Bisector of the given Line Segment

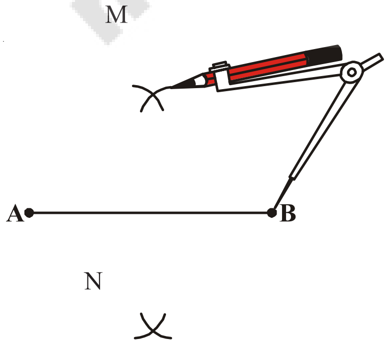
Steps of Construction:

***Step-1*:** Draw a line segment $\overline{AB}$.

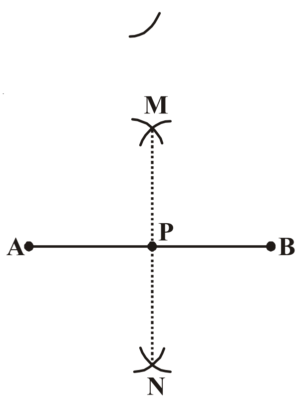
***Step-2*:** Set the compasses as radius more than half of the length of $\overline{AB}$.



***Step-3*:** With A as centre, draw arcs below and above the line segment.

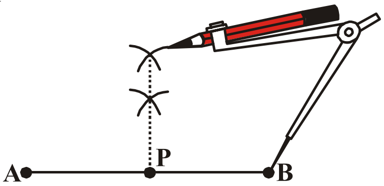


***Step-4*:** With the same radius and B as centre draw two arcs above and below the line segment to cut the previous arcs. Name the intersecting points of arcs as M and N.

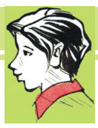


***Step-5*:** Join the points M and N. Then, the line *l* is the required perpendicular bisector of the line AB. Line *l* intersects line AB at P.

Observe the another method.



Do This



Measure the lengths of $\overline{AP}$ and $\overline{BP}$ in both the constructions. Are they equal ?

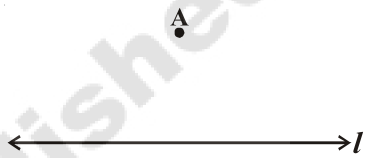
Think, Discuss and Write

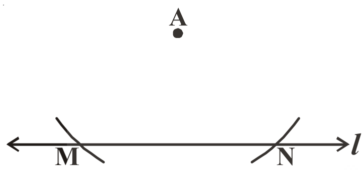
In the construction of perpendicular bisector in step 2. What would happen if we take the length of radius to be smaller than half the length of $\overline{AB}$?

2 Perpendicular to a Line, through a Point which is not on it

Steps of Construction:

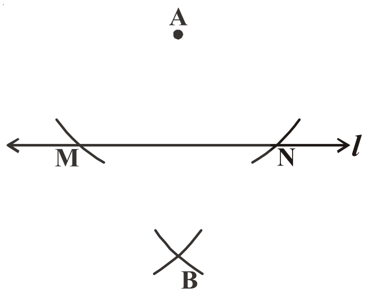
Step-1: Draw a line l and a point A not on it.

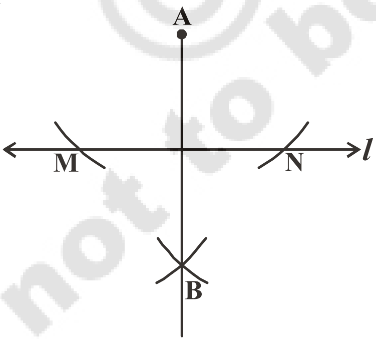




Step-2: With A as centre draw an arc which intersects the given line l at two points M and N.

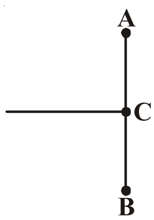
Step-3: Using the same radius and with M and N as centres construct two arcs that intersect at a point, say B on the other side of the line.





Step-4: Join A and B. AB is a perpendicular of the given line l.

Exercise - 13.3



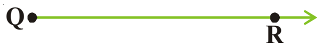
1. Draw a line segment PQ = 5.8cm and construct its perpendicular bisector using ruler and compasses.
2. Ravi made a line segment of length 8.6 cm. He constructed a bisector of AB on C. Find the length of AC & BC.
3. Using ruler and compasses, draw AB = 6.4 cm. Find it's mid point.

13.5 Construction of Angles using Protractor

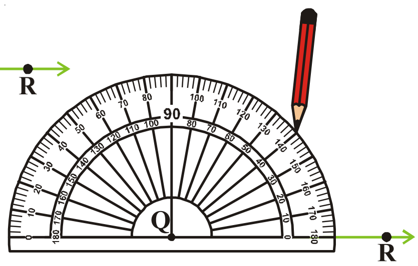
Let us construct $\angle PQR = 40°$.

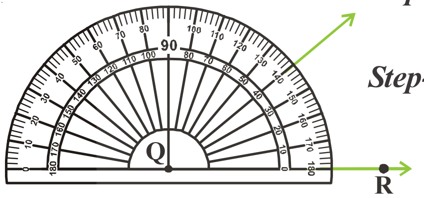
Steps of construction:

Step-1: Draw a ray QR of any length.



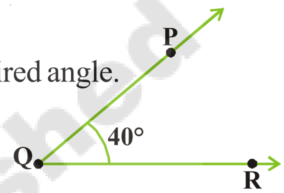
Step-2: Place the centre point of the protractor at Q and the line aligned with the $\overrightarrow{QR}$





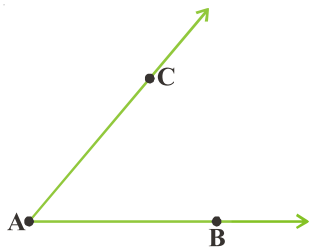
Step-3: Mark a point P at 40°.

Step-4: Join QP. $\angle RPQ$ is the required angle.



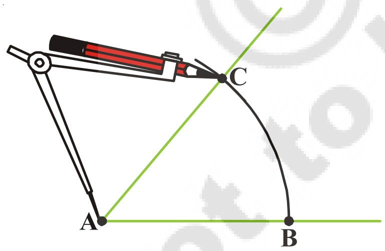
13.6 Constructing a copy of an angle of unknown measure

Suppose an angle (whose measure we do not know) is given and we want to replicate this angle.



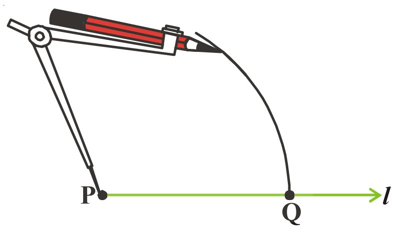
Let $\angle A$ is given, whose measure is not known.

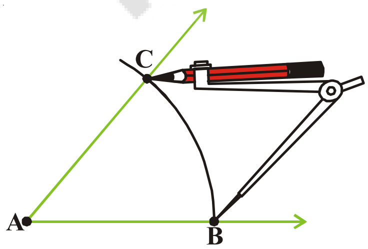
Step-1: Draw a line *l* and choose a point P on it.



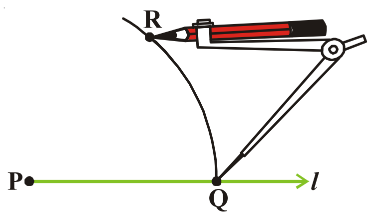
Step-2: Now place the compasses at A and draw an arc to cut the rays AC and AB.

Step-3: Use the same compasses setting to draw an arc with P as centre, cutting *l* at Q.

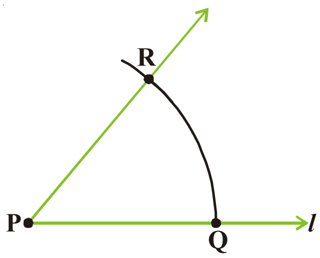




Step-4: Set your compasses with $\overline{BC}$ as the radius.



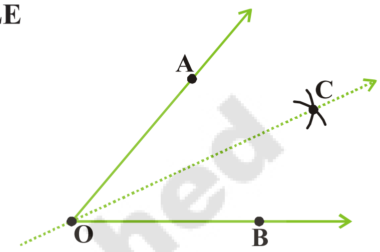
Step-5: Place the compasses pointer at Q and draw an arc to cut the existing arc at R.



***Step-6*:** Join PR. This gives us ∠RPQ. It has the same measure as ∠CAB.

This means ∠QPR has same measure as ∠BAC.

13.7 Construction to Bisect a given Angle

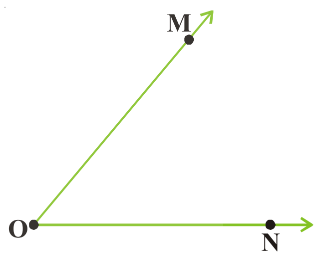


Take a tracing paper. Mark a point O on it. With O as initial point, draw two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$. You get ∠AOB. Fold the sheet through O such that the rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ coincide. Let $\overleftrightarrow{OC}$ be the crease of paper which is obtained after unfolding the paper.

$\overleftrightarrow{OC}$ is clearly a line of symmetry for ∠AOB.

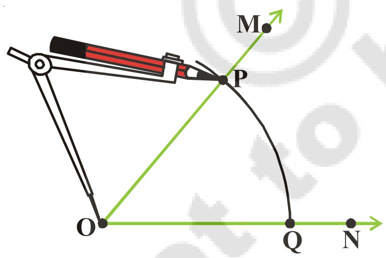
Measure ∠AOC and ∠COB. Are they equal? $\overleftrightarrow{OC}$ the line of symmetry, is therefore known as the angle bisector of ∠AOB.

Let an angle say ∠MON be given.

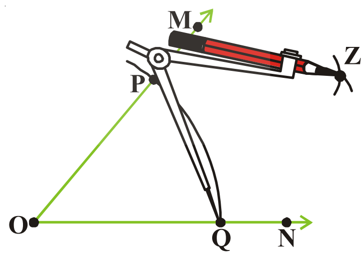


Steps of Construction:

***Step-1*:** With O as centre and any convenient radius, draw an arc $\overset{\frown}{PQ}$ cutting OM and ON at P and Q respectively.

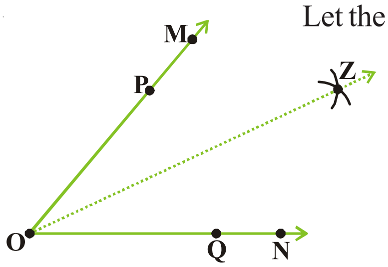


***Step-2*:** With P as centre and any radius slightly more than half of the length of PQ, draw an arc in the interior of the given angle.



***Step-3*:** With Q as centre and without altering radius (as in step 2) draw another arc in the interior of ∠MON.

Let the two arc intersect at Z.



***Step-4*:** Draw ray $\overrightarrow{OZ}$. Then $\overrightarrow{OZ}$ is the desired bisector of ∠MON.

Observe ∠MOZ = ∠ZON.

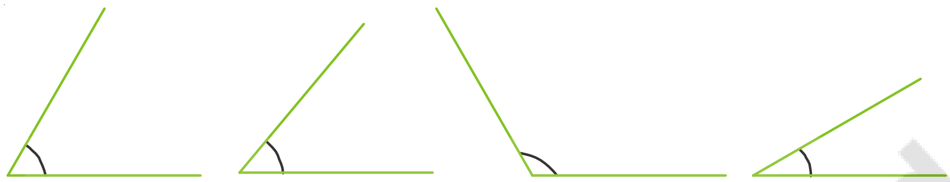
Exercise - 13.4

1. Construct the following angles with the help of a protractor .

   (i) $\angle ABC = 65°$ (ii) $\angle PQR = 136°$

   (iii) $\angle Y = 45°$ (iv) $\angle O = 172°$

2. Copy the following angles in your note book and find their bisector:



13.8 Constructing Angles of Special Measures

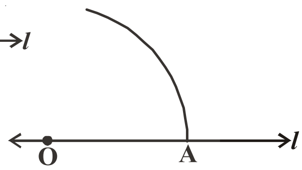
There are some elegant and accurate methods to construct some angles of special sizes which do not require the use of the protractor. A few have been discussed here.

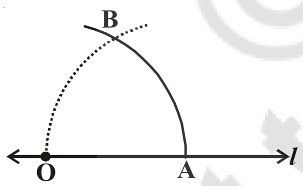
You learnt the construction of any given angle by using a protractor. Now we will learn construction of some angles by using compasses only.

13.8.1 Construction of 60° Angle

***Step-1*:** Draw a line *l* and mark a point O on it.

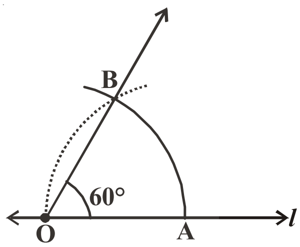
***Step-2*:** Place the pointer of the compasses at O and draw an arc of convenient radius which cuts the line *l* at a point say, A.





***Step-3*:** With the pointer at A (as centre) and the same radius as in the step-2. Now draw an arc that passes through O.

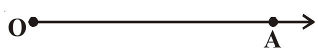
***Step-4*:** Let the two arcs intersect at B. Join OB. We get $\angle BOA$ whose measure is 60°.



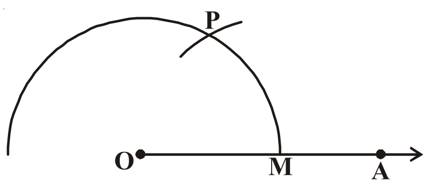
13.8.2 Construction of 120° Angle

An angle of 120° is nothing but twice of an angle of 60°. Therefore, it can be constructed as follows:

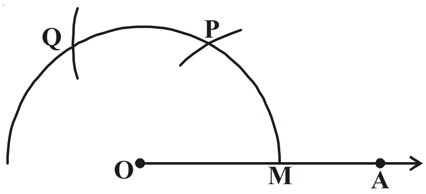
***Step-1*:** Draw any ray OA

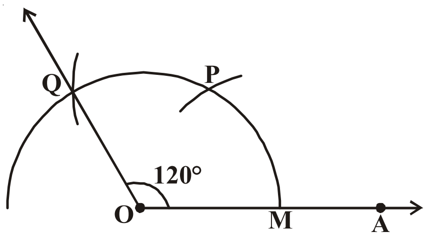


***Step-2*:** Place the pointer of the compasses at O. With O as centre and any convenient radius draw an arc cutting OA at M.



Step-3: With M as centre and without altering radius (as in step 2) draw an arc which cuts the first arc at P.





Step-4: With P as centre and without altering the radius (as in step 2) draw an arc which cuts the first arc at Q.

Step-5: Join OQ. Then ∠AOQ is the required angle.

Do This

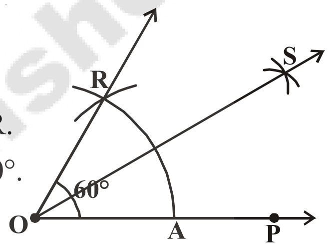
Construct angles of 180°, 240°, 300°.

13.8.3 Construction of 30°Angle

Steps of Construction:

Draw an angle of 60° as discussed above. Name it as ∠AOR.

Bisect this angle as shown earlier to get two angles each of 30°.



13.8.4 Construction of 90° Angle

Look at the given figure

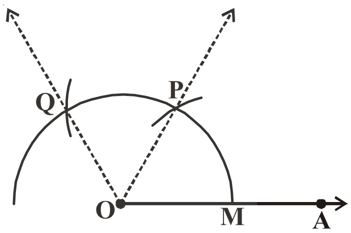
∠AOP = 60°, ∠POQ = 60° and

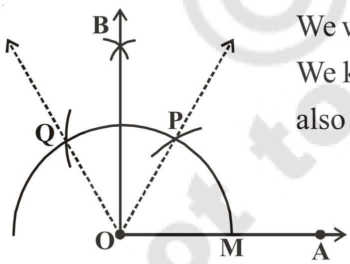
∠AOQ = 120°

We want to construct an angle of 90°.

We know that 90° = 60° + 30° and

also 90° = 120° - 30°





So, we need to bisect ∠POQ to get an angle of 30°.

∠BOP = 30° and ∠AOB = 90°

Think of one more way to construct a 90° angle.

Do This

Construct an angle of 45° by using compasses.

Exercise - 13.5

1. Construct ∠ABC = 60° without using protractor.
2. Construct an angle of 120° with using protractor and compasses.

3. Construct the following angles using ruler and compasses. Write the steps of construction in each case.

   (i) 75° (ii) 15° (iii.) 105°

4. Draw the angles given in Q.3 using a protractor.

5. Construct ∠ABC = 50° and then draw another angle ∠XYZ equal to ∠ABC without using a protractor.

6. Construct ∠DEF = 60°. Bisect it, measure each half by using a protractor.

WHAT HAVE WE DISCUSSED?

This chapter deals with methods of drawing geometrical shapes.

1. We use the followng geometrical instruments to construct shapes:

   (i) A graduated ruler (ii) The compasses

   (iii) The divider (iv) Set-squares (v) The protractor

2. Using the ruler and compasses, the following constructions can be made:

   (i) A circle, when the length of its radius is known.

   (ii) A line segment, if its length is given.

   (iii) A copy of a lines segment.

   (iv) A perpendicular to a line through a point

   (a) on the lines (b) not on the line

   (v) The perpendicular bisector of a line segment of given length.

   (vi) An angle of a given measure.

   (vii) A copy of an angle.

   (viii) The bisector of a given angle.

   (ix) Some angles of special measures such as:

   (a) 90° (b) 45° (c) 60° (d0 30° (e) 120° (f) 135°

Fun with curves

Mark 10 points at 1cm intervels on two lines at right angles, numbering them 1 to 10.
Join 1 to 10, 2 to 9, 3 to 8, . . . etc.so that the sum is 11 as shown in the figure. The result is a curve.
Make some pictures using this idea.