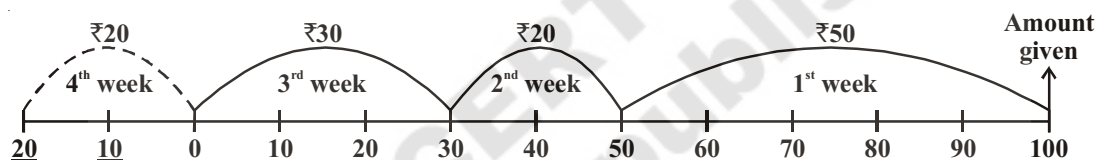


# Integers

## 6.1 INTRODUCTION

Rafi gets ₹ 100 as pocket money from his father every month. He gives this money to his mother and takes some amount from her when ever he required. His mother makes a note of the money given by and to Rafi.

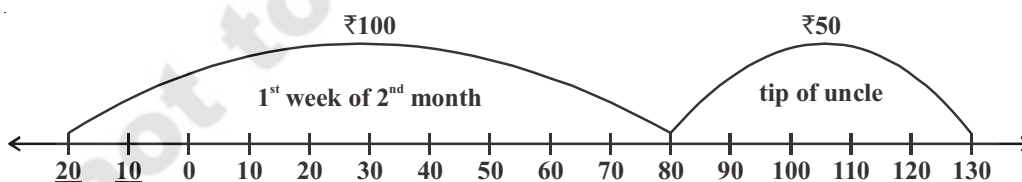
Rafi took ₹ 50 in the first week, ₹ 20 in the second week, ₹ 30 in the third week and wanted ₹ 20 in the last week. But Rafi's mother told him that he had taken the entire amount given to her. Rafi said that he would adjust the amount from next month's pocket money, but needs the money. She agreed and gave him ₹ 20 and recorded it as follows:



**Record of the money**

On the first day of the next month, Rafi got ₹ 100. He gave it to his mother. Can you say, how much money does Rafi have with his mother?

On the same evening his uncle gave him a tip of ₹ 50. He felt happy and gave the same to his mother to deposit, asking her to keep it and record the money. Can you find out, how much money did Rafi have with his mother then? Look at the record once again:



**Record of 2<sup>nd</sup> Month**

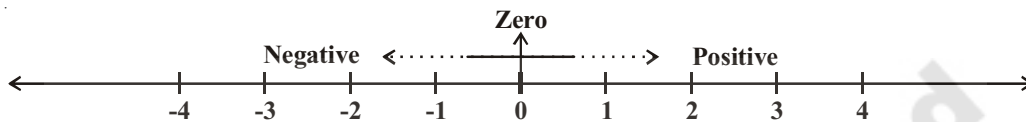
Now answer the following by using the record:

1. How much money does Rafi's father give him as pocket money every month?
2. How much money did Rafi spend in four weeks?
3. How much money did Rafi's mother lent him in the fourth week?
4. How did she mark the money she lent in the last week?
5. What is the difference between moving by ₹ 20 to the right of zero and by ₹ 20 to the left of zero?
6. Which side of line she has marked the money when Rafi gave ₹ 100 and ₹ 50 in the next month?

## 6.2 HOW NEGATIVE NUMBER'S ARISE?

You would have realized that ₹ 20 marked on the either sides of zero do not mean the same. The numbers on the left of zero are negative numbers and are less than zero. The numbers on the right are positive and are greater than zero.

There are several situations in our daily life where we use these numbers to represent loss and profit, past and future, low and high temperatures etc. The numbers on the left side of zero (i.e. less than zero) are called negative numbers. These are denoted as -1, -2, -3, ....., -10, -20, ... for easy understanding.



We use the negative numbers in daily life as:

- (i) The loss of ₹ 200 in a business is represented as (-200) and profit of ₹. 200 is represented as (+200)
- (ii) The temperature above  $0^{\circ}\text{C}$  is denoted as 'positive' and below  $0^{\circ}\text{C}$  is denoted as negative such as  $3^{\circ}\text{C}$  below  $0^{\circ}\text{C}$  is  $-3^{\circ}\text{C}$   
 $3^{\circ}\text{C}$  above  $0^{\circ}\text{C}$  is  $+3^{\circ}\text{C}$

### DO THIS

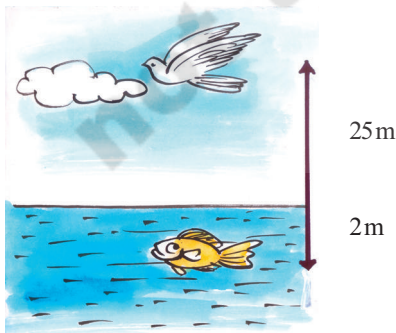


Manasa has borrowed ₹ 50 and Swetha has borrowed ₹ 20 from their mother. How will you represent this on the number line? Suppose their father gave them ₹ 100 each as pocket money, who will have more money after clearing the debit?

## 6.3 SOME USES OF NEGATIVE NUMBERS

Show the following using the ground level as zero with appropriate signs:

1. A bird is flying at a height of 25 meters above the sea level and a fish at a depth of 2 meters.



2. A flag is posted on top of a mountain at the height of 500m and another one placed on top of a tent made in the bed of a lake 25m below the ground.



3. The temperature on a cold night in Delhi was  $5^{\circ}\text{C}$  and in Kufri in Himachal Pradesh was 6 degree below zero.

## THINK, DISCUSS AND WRITE

Write some more models for positive and negative numbers in our daily life.



The numbers which are positive, zero and negative numbers together are called as 'Integers' and they are denoted by the letter 'I' (or  $\mathbb{Z}$ ).

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

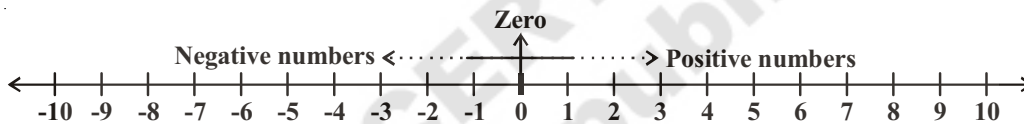
## TRY THESE

Collect information about temperatures recorded in various places in India in the month of January and write them using integers.



## 6.4 REPRESENTATION OF INTEGERS ON A NUMBER LINE

Now, Rafi understood how his mother is representing Integers on the number line.



The numbers which are on the right side of zero are positive numbers (natural numbers) and which are on the left side of zero are negative numbers. Do you agree? Why?

Now answer the following using number line:

1. Which is the nearest positive Integer to zero?
2. How many negative numbers you will find on left side of zero?
3. Which is greater (-2) or (-1)?
4. Which is smaller among 3 and -5? Why?
5. Which Integer is neither positive nor negative?

## DO THIS

Draw a vertical line and represent the following Integers on the number line:

-5, 4, -7, -8, -2, 9, 5, -6, 2.



## EXERCISE - 6.1

1. Represent the following statements using signs of Integers.
  - (i) An aeroplane is flying at a height of 3000 meters ( )
  - (ii) The fish is 10 meters below the water surface. ( )
  - (iii) The temperature in Hyderabad is  $35^{\circ}\text{C}$  above  $0^{\circ}\text{C}$ . ( )
  - (iv) Water freezes at  $0^{\circ}\text{C}$  temperature. ( )

- (v) The average temperature at the mount Everest in January is  $36^{\circ}\text{C}$  below zero degree. ( )
- (vi) The submarine is 500 meters below the surface of the sea. ( )
- (vii) The average temperature at Dargeeling in July is  $19^{\circ}\text{C}$  below zero degree. ( )
- (viii) The average low temperature in Vishakapatnam during January is  $18^{\circ}\text{C}$ . ( )

2. Write any five negative integers.

3. Write any five positive integers.

4. Mark the Integers on the number line given below: -4, 3, 2, 0, -1, 5

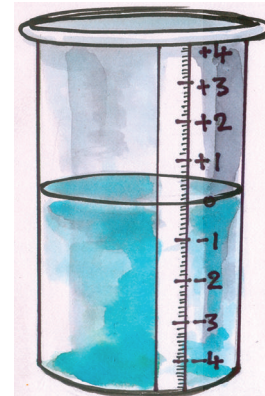


5. Write True or False. If the statement is false, correct the statement.

- (i) -7 is on the right side of -6 on the number line. ( )
- (ii) Zero is a positive number. ( )
- (iii) 9 is on the right side of zero on the number line. ( )
- (iv) -1 is an integer which lies between -2 and 0. ( )

## 6.5 ORDERING OF INTEGERS

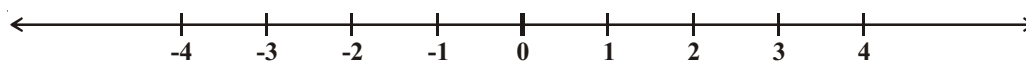
Pavan and Harish are friends and they noticed that the water level in the well of their village (where steps are observed) reduces during summer and rises during the rainy season. The level was shown by the steps made. They used the idea of the number of steps of the well and prepared a model of the well using a glass jar. They pasted a strip showing integers with steps below zero as -1, -2, -3 and steps above zero as 1, 2, 3, 4 and so on. They took zero as the levels of water on the steps, the day they observed.



They use this jar to depict the water level, taking out water when water level falls in the well and adding water when it rises. They were able to record the water level in the well as above the base step level as positive and below it as negative.

Now say 1. What happens when water is poured into the jar?

2. What happens when water is removed from the jar from the zero level?



Let us once again observe the integers which are represented on the number line.

We know that  $4 > 2$  and that 4 is to the right of 2 on the number line. Similarly,  $2 > 0$  and is to the right of 0. Now, since 0 is to the right of -3, we say  $0 > -3$ .

Thus, we see that on a number line, the number increases as we move to right and decreases as we move to the left. Therefore,  $-3 < -2$ ,  $-2 < -1$ ,  $-1 < 0$  and  $0 < 1$ ,  $1 < 2$ ,  $2 < 3$  so on.

## Do This



From the above understanding, fill in the boxes using  $<$  or  $>$  signs:

$0 \dots\dots\dots -1$

$-3 \dots\dots\dots -2$

$5 \dots\dots\dots 6$

$-4 \dots\dots\dots 0$



## EXERCISE - 6.2

- Put appropriate symbol  $>$  or  $<$  in the boxes given between the two integers:
 

(i)	-1	.....	0	(ii)	-3	.....	-7
(iii)	-10	.....	+10	(iv)	0	.....	-5
(v)	-100	.....	99	(vi)	0	.....	100
- Write the following integers in increasing and decreasing order:
 

(i)	-7, 5, -3	(ii)	-1, 3, 0
(iii)	1, 3, -6	(iv)	-5, -3, -1
- Write True or False, correct those that are false:
 

(i)	Zero is on the right of -3	( )
(ii)	-12 and +12 represent on the number line the same integer	( )
(iii)	Every positive integer is greater than zero	( )
(iv)	$-5 < 8$	( )
(v)	$(-100) > (+100)$	( )
(vi)	$-1 < -8$	( )
- Find all integers which lie between the given two integers. Also represent them on number line:
 

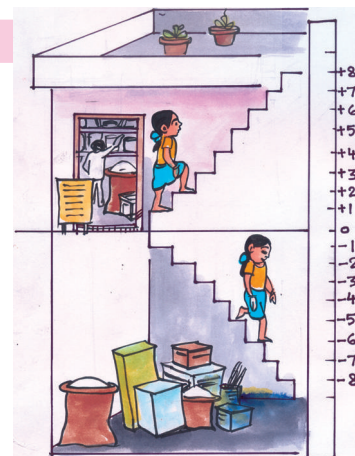
(i)	-1 and 1	(ii)	-5 and 0	(iii)	-6 and -8	(iv)	0 and -3
-----	----------	------	----------	-------	-----------	------	----------
- The temperature recorded in Shimla is  $-4^\circ\text{C}$  and in Kufri is  $-6^\circ\text{C}$  on the same day. Which place is colder on that day? How?

## Do This

Rajesh has a shop on the ground floor of a building. There are stairs going up to the terrace and stairs going down to the godown, where goods are stored.

Every day his daughter Hasini, after coming back from school goes up to the terrace to play. She helps father in arranging things in the godown at night.

Observe the picture and try to answer the questions using integers marked on the steps:





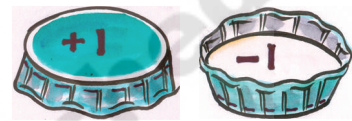
- (i) Go 7 steps up from the shop.
- (ii) Go 3 steps down from the ground floor.
- (iii) Go 5 steps up from the ground floor and then go 3 steps further up from there.
- (iv) Go 4 steps down from the ground floor and then further 3 steps from there.
- (v) Go down 5 steps down from the ground floor and 10 steps up from there.
- (vi) Go 8 steps up from the ground floor and come down 9 steps down from there.



Check your answers with your friend and discuss.

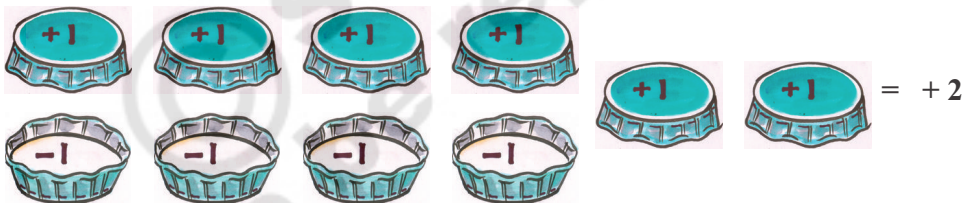
## 6.6 ADDITION AND SUBTRACTION OF INTEGERS

### Play a Game

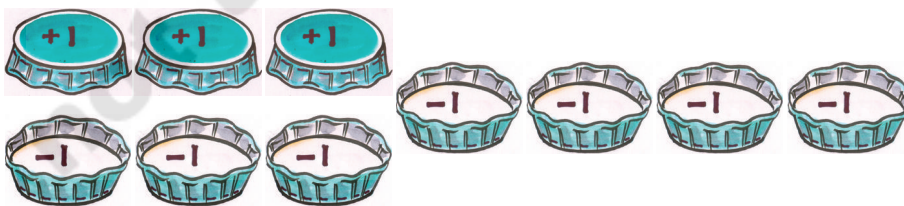
Take 10 identical caps of cool drink bottle. These bottle caps can be placed down words and upwards. Consider the top side of the cap to be (+1) and the bottom side to be (-1).



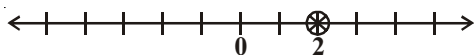
Ask your friend to throw 10 caps in a single move after shaking them vigorously. Look at the way the 10 caps lie. Which side of the cap is showing? Consider one up (+1) and one down (-1) to be a pair. Remove all the pairs like   i.e. (+1) and (-1). Are the remaining caps up or down? Count these caps. If there were 4 pairs made, two caps are left. As in the example below if these face up then it is +2 points.



If 3 pairs are formed and 4 caps are left facing down, then the points are '-4'.



Record the points in your note book using a number line. You can continue to play till any one of you get 10 points and wins the game.



**Your record**



**Your friend's record**


What happens if your friend has two down facing caps again?

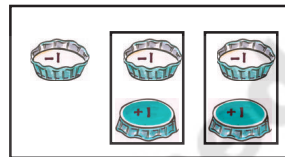
Does she move right or left on the line? Clearly she moves left two places starting from -4 and reaches -6 we say  $(-4) + (-2) = -6$ .


You add two positive integers like  $(+3) + (+1) = 4$ . You can also add two negative integers and the answer will take a minus sign (-) like  $(-3) + (-2) = -5$ .

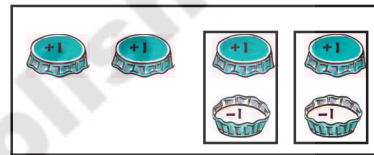
What happens when we have one positive integer and one negative integer. Let us take the help of caps. Place as many caps facing up as positive integer and as many caps facing down as negative integer. Remove caps in pairs i.e. an up cap with a down cap since  $(+1) + (-1) = 0$ .

Count the remaining caps.

$$\begin{aligned} \text{(i)} \quad (-3) + (+2) &= (-1) + [(-2) + (+2)] \\ &= -1 + 0 \\ &= -1 \end{aligned}$$




$$\begin{aligned} \text{(ii)} \quad (+4) + (-2) &= (+2) + [(+2) + (-2)] \\ &= (+2) + 0 \\ &= +2 \end{aligned}$$




Now you can play the game by adding scores easily.

## DO THIS

Find the values of the following:

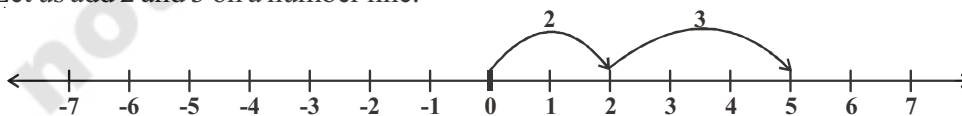
- (i)  $-7 + 8$       (ii)  $-3 + 5$       (iii)  $-3 - 2$       (iv)  $+7 - 10$



### 6.6.1 Addition of integers on the number line

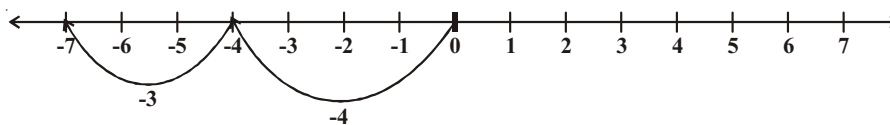
Let us see how we can add any two integers using a number line.

1. Let us add 2 and 3 on a number line.



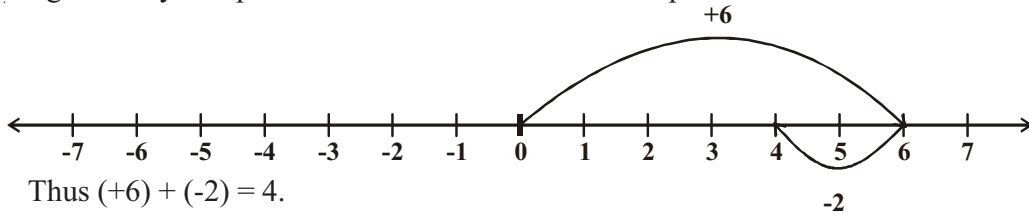
On the number line, we first move 2 steps to the right from 0 to reach 2, then we move 3 steps to the right of 2 and to reach 5. Thus we get  $2 + 3 = 5$ .

2. Let us add  $(-4)$  and  $(-3)$ .

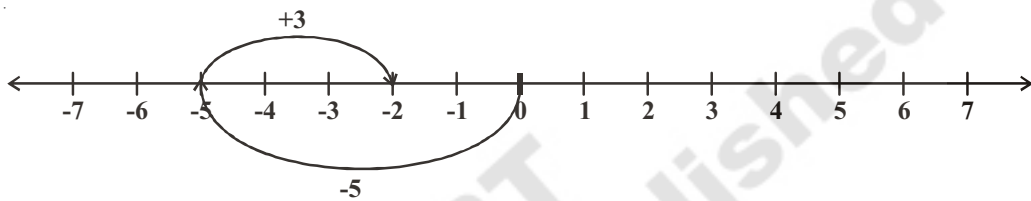


On the number line, we first move 4 steps to the left of 0 to reach -4, then we move 3 steps to the left of -4 and reach -7. Thus,  $(-4) + (-3) = -7$ .

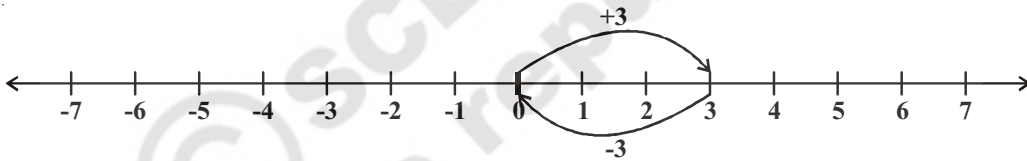
3. Suppose we wish to find the sum of  $(+6)$  and  $(-2)$  on the number line. First we move to the right of 0 by 6 steps to reach 6. Then we move 2 steps to the left of 6 to reach 4.



4. Similarly let us find the sum of  $(-5)$  and  $(+3)$  on the number line. First we move 5 steps to the left of 0 to reach  $-5$  and then from this point we move 3 steps to the right. We reach the point  $-2$ . Thus,  $(-5) + (+3) = -2$



5. Suneetha adds 3 and  $-3$ . She first moves from 0 to  $+3$  and then from  $+3$  she moves 3 points to the left. Where does she reach ultimately?



From the figure

$$3 + (-3) = 0$$

Similarly, if we add 1 and  $-1$ , 2 and  $-2$ , 3 and  $-3$  ..... so on we obtain the sum as zero. They are called additive inverse of each other i.e. **any two distinct numbers that give zero when added to each other are additive inverse of each other.**

What is additive inverse of 7?

What is additive inverse of  $-8$ ?

### TRY THESE

- Find the value following using a number line.
  - $(-3) + 5$
  - $(-5) + 3$

Make your own two new questions and solve them using the number line.
- Find the solution of the following:
  - $(+5) + (-5)$
  - $(+6) + (-7)$
  - $(-8) + (+2)$

Ask your friend five such questions and solve them.





Observe the following:

$$\begin{array}{lll} \text{(i)} & 3 + 2 = 5 & 20 + 6 = 26 & 30 + 22 = 52 \\ & 8 + 16 = 24 & 9 + 10 = 19 & 20 + 14 = 34 \end{array}$$

We can see that the sum of two positive integers is also a positive number.

Look at the following now:

$$\text{(ii)} \quad -4 + (-6) = -10 \qquad -8 + (-12) = -20 \qquad -3 + (-9) = -12$$

What do you learn from this? The sum of two negative integers is always a negative integer.

What happens if one integer is positive and the other negative? Let us see these:

$$\begin{array}{ll} \text{(iii)} & 15 + (-17) = -2 & -23 + 4 = -19 \\ & -11 + 16 = 5 & -12 + 12 = 0 \end{array}$$

From the above, we can conclude that when we add two integers one of which is positive and the other negative, then the sum may be either positive, negative or zero.

**Example-1.** Find the sum of  $(-10) + (+14) + (-5) + (+8)$

**Solution:** We can rearrange the numbers so that the positive integers and the negative integers groups together. We have

$$\begin{aligned} & (-10) + (+14) + (-5) + (+8) \\ &= (-10) + (-5) + (+14) + (+8) \\ &= -15 + 22 = 7. \end{aligned}$$

**Example-2.** Find the sum of  $(-20)$ ,  $(-82)$ ,  $(-28)$  and  $(-14)$ .

**Slution:**

$$\begin{aligned} & (-20) + (-82) + (-28) + (-14) \\ &= -144 \end{aligned}$$

**Example-3.** Find the sum of  $25 + (-21) + (-20) + (+17) + (-1)$

**Solution:**

$$\begin{aligned} 25 + (-21) + (-20) + (+17) + (-1) &= 25 + (+17) + (-21) + (-20) + (-1) \\ &= 42 - 42 = 0 \end{aligned}$$


### EXERCISE - 6.3

- Add the following integers using number line.
  - $7 + (-6)$
  - $(-8) + (-2)$
  - $(-6) + (-5) + (+2)$
  - $(-8) + (-9) + (+17)$
  - $(-3) + (-8) + (-5)$
  - $(-1) + 7 + (-3)$
- Add without using number line.
  - $10 + (-3)$
  - $-10 + (+16)$
  - $(-8) + (+8)$
  - $-215 + (+100)$
  - $(-110) + (-22)$
  - $17 + (-11)$
- Find the sum of:
  - 120 and -274
  - 68 and 28
  - 29, 38 and 190
  - 60, -100 and 300.

4. Simplify:

(i)  $(-6) + (-10) + 5 + 17$

(ii)  $30 + (-30) + (-60) + (-18)$

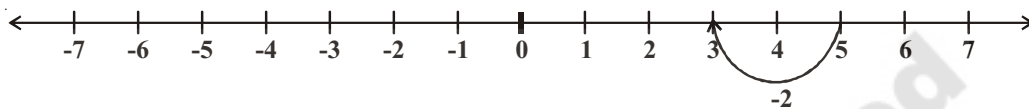
(iii)  $(-80) + (+40) + (-30) + (+6)$

(iv)  $70 + (-18) + (-10) + (-17)$

### 6.6.2 Subtraction of integers

We saw that to add 5 and  $(-2)$  on a number line we can start from 5 and then move 2 steps to the left of 5.

We reach at 3 so, we have  $5 + (-2) = 3$



Thus, we find that to add a positive integer we move towards the right on a number line and for adding a negative integer we move towards left.

We have also seen that while subtracting whole numbers on a number line, we would move towards left.

For example take  $5 - 2 = ?$

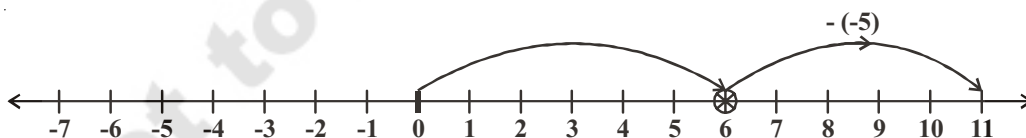
We start from 5 and take two steps to the left and end up at 3.

What does subtraction of a negative integer mean?

Let us observe the following example,

**Example-4.** Subtract -5 from 6.

**Solution:** To subtract -5 from 6, let us start at 6 and move 5 towards the right. For -5 we would have moved left but for  $(-5)$  we would move in the opposite direction. Moving 5 to the right, we reach 11.



We have  $6 - (-5) = 11$

i.e. To subtract -5 from 6 add 5 (the additive inverse of -5) to 6.

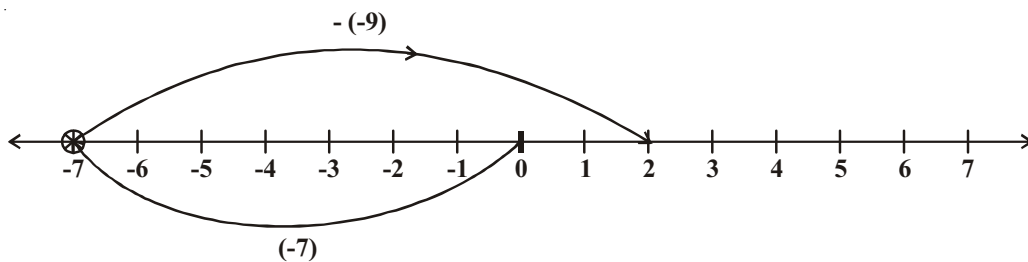
$$6 - (-5) = 6 + 5 = 11$$

What would we do for  $4 - (-2)$ ? Would you move towards the left on the number line or towards the right? If we move to the left then we reach 2. Then we have to say  $4 - (-2) = 2$ . This is not true because we know  $4 - 2 = 2$  and  $4 - (-2) \neq 4 - 2$ .

So for  $4 - (-2)$  we move two steps to the right of 4. This is opposite of what we would do for  $4 - (2)$ . We reach 6 in one case and reach 2 in the other.

**Example-5.** Find the value of  $(-7) - (-9)$  using number line.

**Solution:**



$(-7) - (-9)$  is equal to  $-7 + 9$  (Since  $-9$  is additive inverse of  $9$ ).

On the number line, start from  $-7$  and move  $9$  units to right, we will reach  $2$ .

So  $(-7) - (-9) = -7 + 9 = 2$ .

### DO THIS

- |                |                |                |
|----------------|----------------|----------------|
| a) $-5 - (-3)$ | b) $-7 - (+2)$ | c) $-7 - (-5)$ |
| d) $3 - (-4)$  | e) $5 - (+7)$  | f) $4 - (-2)$  |



### THINK, DISCUSS AND WRITE

$$3 - 3 = 0$$

$$3 - 2 = 1$$

$$3 - 1 = 2$$

$$3 - 0 = 3$$

$$3 - (-1) = 4$$

$$3 - (-2) = 5$$

$$3 - (-3) = 6$$

Observe that as the number we subtract from  $3$  is decreasing, the result obtained is increasing.

Do you say is it true for all Integers



**Example-6.** Subtract  $(-6)$  from  $(-13)$ .

**Solution:**  $(-13) - (-6) = (-13) + (\text{additive inverse of } -6)$   
 $= -13 + 6 = -7$ .

**Example-7.** Subtract  $(+8)$  from  $(-8)$

**Solution:**  $(-8) - (+8) = (-8) + (\text{additive inverse of } +8)$   
 $= -8 + (-8) = -16$

**Example-8.** Simplify:  $(-6) - (+7) - (-24)$

**Solution:**  $(-6) - (+7) - (-24) = (-6) + (\text{additive inverse of } +7) + (\text{additive inverse of } -24)$   
 $= -6 + (-7) + (+24) = -13 + 24 = 11$ .



## EXERCISE - 6.4

- Find:
  - $40 - (22)$
  - $84 - (98)$
  - $(-16) + (-17)$
  - $(-20) - (13)$
  - $(38) - (-6)$
  - $(-17) - (-36)$
- Fill in the blanks with  $>$ ,  $<$  or  $=$  sign:
  - $(-4) + (-5)$  \_\_\_\_\_  $(-5) - (-4)$
  - $(-16) - (-23)$  \_\_\_\_\_  $(-6) + (-12)$
  - $44 - (-10)$  \_\_\_\_\_  $47 + (-3)$
  - $(-21) + (-22)$  \_\_\_\_\_  $(-22) + (-21)$
- Fill in the blanks:
  - $(-13) +$  \_\_\_\_\_  $= 0$
  - $(-16) + 16 =$  \_\_\_\_\_
  - $(-5) +$  \_\_\_\_\_  $= -14$
  - \_\_\_\_\_  $- 16 = -22$
- Simplify:
  - $(-6) - (5) - (+2)$
  - $(-12) + 42 - 7 - 2$
  - $(-3) + (-6) + (-24)$
  - $40 - (-50) - (2)$

## WHAT HAVE WE DISCUSSED?

- We use negative numbers to represent debit, temperatures below the  $0^{\circ}\text{C}$ , past periods of time, depth below sea level.
- The collection of all the positive numbers  $\{1, 2, 3, \dots\}$ , negative numbers  $\{-1, -2, -3, \dots\}$  and zero put together, they are called integers. Set of integers is denoted by the letter 'I' or Z. And  $Z = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$ .
- We can show the addition and subtraction of integers on the number line.
- When two positive integers are added, we get a positive integer.
- When two negative integers are added, we get a negative integer.
- When one positive and one negative integer are added we subtract them and put the sign of the bigger integer.
- The subtraction of integers is the same as the addition of their additive inverse.

### Fun with Integers !

In its search for water a frog fell down into a 30m deep well!

Its progress out of the well was very erratic.

Each day it managed to climb up 3 m, but the following night, it slipped back 2 m.

How many days did it take to get out of the well?

