

UNDERSTANDING 3D AND 2D SHAPES

14.0 Introduction

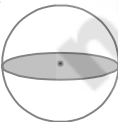

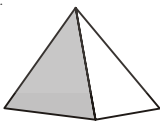
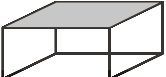
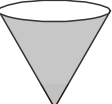
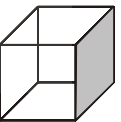
We have been introduced to various three-dimensional shapes in class VI. We have also identified their faces, edges and vertices. Let us first review what we have learnt in class VI.



Exercise - 1

- Given below are the pictures of some objects. Categorise and fill write their names according to their shape and fill the table with name of it.

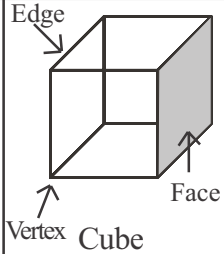
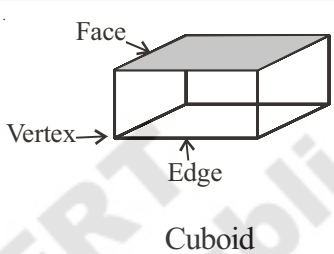
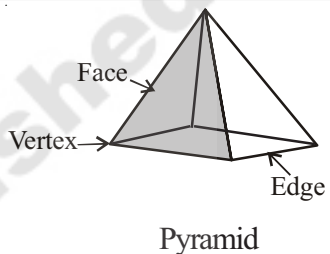


 Sphere	 Cylinder	 Pyramid	 Cuboid	 Cone	 Cube

2. Write names of at least 2 objects from day-to-day life, which are in the shape of the basic 3D shapes given below:

- (i) Cone ----- ----- ----- -----
- (ii) Cube ----- ----- ----- -----
- (iii) Cuboid ----- ----- ----- -----
- (iv) Sphere ----- ----- ----- -----
- (v) Cylinder ----- ----- ----- -----

3. Identify and state the number of faces, edges and vertices of the figures given below:

	 <p style="text-align: center;">Cube</p>	 <p style="text-align: center;">Cuboid</p>	 <p style="text-align: center;">Pyramid</p>
Faces			
Edges			
Vertices			

14.1 Nets of 3-D shapes

We now visualise 3-D shapes on 2-D surfaces, that is on a plain paper. This can be done by drawing the ‘nets’ of the various 3-D figures.

Take a cardboard box (cartoon of tooth paste or shoes etc.). Cut the edges to lay the box flat. You have now a net for that box. A net is a sort of skeleton-outline in 2-D (Figure 1), which, when folded (Figure 2), results in a 3-D shape (Figure 3).

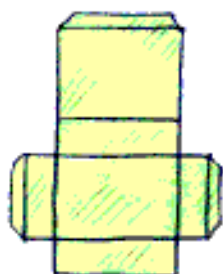


Figure 1



Figure 2

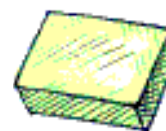


Figure 3

Here is a net pattern for a box. Trace it and paste it on a thick paper and try to make the box by suitably folding and gluing together. What is the shape of the box?

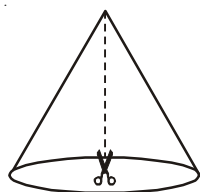
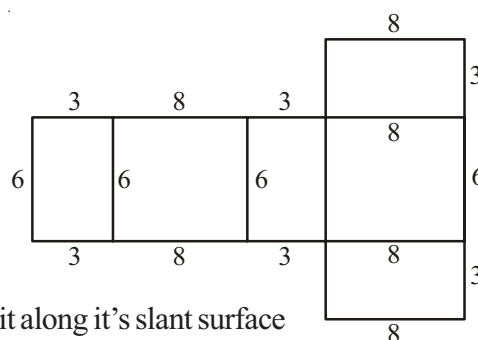


Figure 1



Figure 2

Similarly, take a cover of an ice-cream cone (or any like shape). Cut it along its slant surface as shown in Figure 1. You will get the net for the cone (Figure 2).



Try This

Take objects having different shapes (cylinder, cube, cuboid and cone) and cut them to get their nets with help of your teachers or friends.

You will come to know by the above activity that you have different nets for different shapes. Also, each shape can also have more than one net according to the way we cut it.



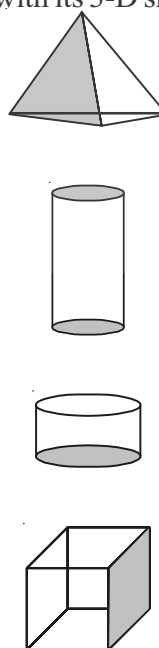
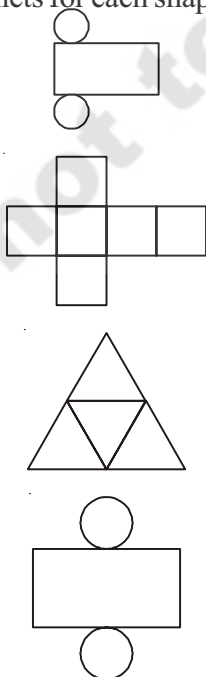
Exercise - 2

- Some nets are given below. Trace them and paste them on a thick paper. Try to make 3-D shapes by suitably folding them and gluing together. Match the net with its 3-D shape.

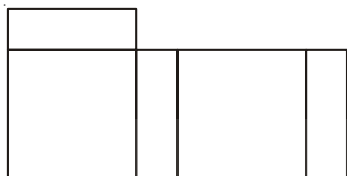
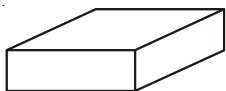
Nets

3D shapes

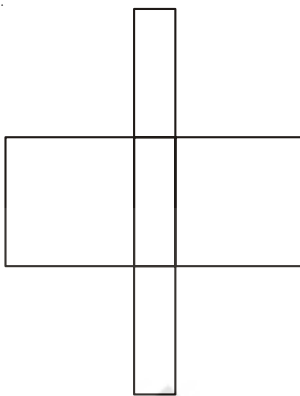
- Three nets for each shape are given here. Match the net with its 3-D shape.



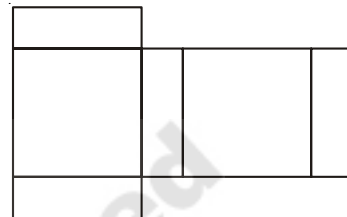
(i)



(a)

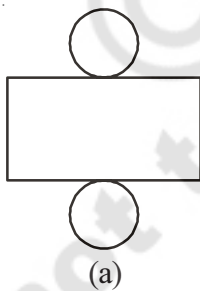


(b)



(c)

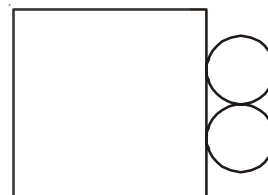
(ii)



(a)

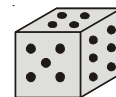


(b)

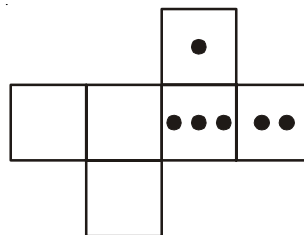
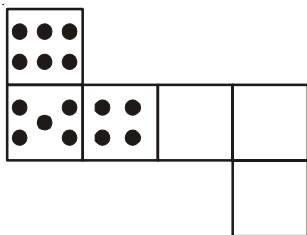


(c)

3. A dice is a cube with dots on each face. The opposite faces of a dice always have a total of seven dots on them.



Here are two nets to make dice. Insert the suitable number of dots in blanks.



Play This

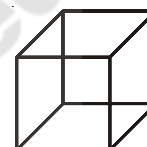
You and your friend sit back to back. One of you read out a net to make a 3-D shape, while the other copies it and sketches or builds the described object.

14.2 Drawing solids on a flat surface

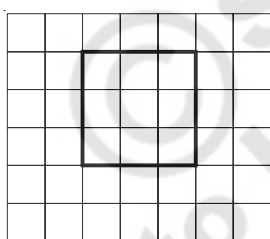
Our drawing surface is a paper, which is a flat surface. When you draw a solid shape, the images are somewhat distorted. It is a visual illusion. You will find here two techniques to help you to draw the 3-D shapes on a plane surface.

14.2.1 Oblique Sketches

Here is a picture of a cube. It gives a clear idea of how the cube looks, when seen from the front. You do not see all the faces as we see in reality. In the picture, all the lengths are not equal, as they are in a real cube. Still, you are able to recognise it as a cube. Such a sketch of a solid is called an oblique sketch.

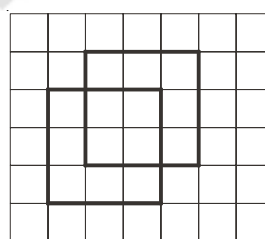


How can you draw such sketches? Let us attempt to learn the technique. You need a squared (lines or dots) paper. Initially practice to draw on these sheets and later on a plain sheet (without the aid of squared lines or dots!) Let us attempt to draw an oblique sketch of a $3 \times 3 \times 3$ cube (each edge is 3 units).



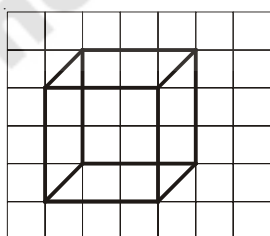
Step 1

Draw the front face.



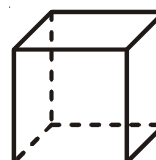
Step 2

Draw the opposite face. Sizes of the faces have to be same, but the sketch is somewhat off-set from step 1.



Step 3

Join the corresponding corners



Step 4

Redraw using dotted lines for hidden edges.
(It is a convention) The sketch is ready now.

In the above oblique sketch did you notice the following?

- (i) The sizes of the front face and its opposite face are same.
- (ii) The edges, which are all equal in a cube, appear so in the sketch, though the actual measures of edges are not taken so.

You could now try to make an oblique sketch of a cuboid (remember the faces in this case are rectangles).

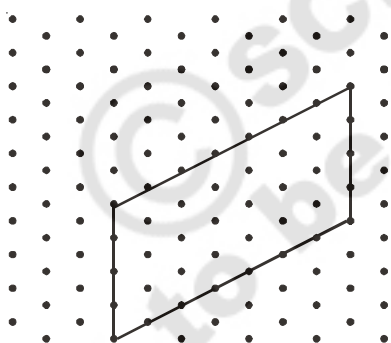
You can draw sketches in which measurements also agree with those of a given solid. To do this we need what is known as an isometric sheet.

Let us try to make a cuboid with dimensions 4 cm length, 3 cm breadth and 3 cm height on an isometric sheet.

14.2.2 Isometric Sketches

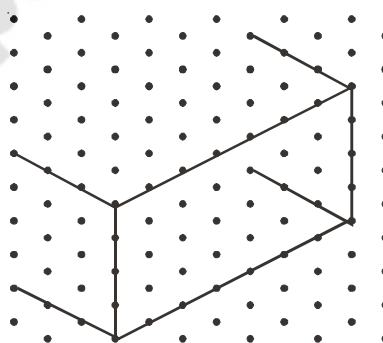
To draw sketches in which measurements also agree with those of the given solid, we can use isometric dot sheets. In such a sheet the paper is divided into small equilateral triangles made up of dots or lines.

Let us attempt to draw an isometric sketch of a cuboid of dimensions $4 \times 3 \times 3$ (which means the edges forming length, breadth and height are 4, 3, 3 units respectively).



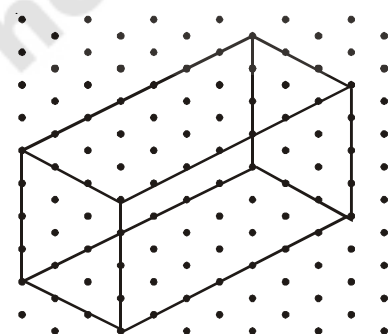
Step 1

Draw a rectangle to show the front face



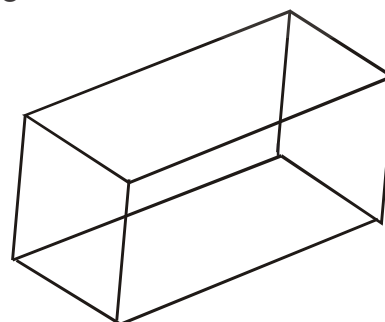
Step 2

Draw four parallel line segments of length 3 units starting from the four corners of the rectangle.



Step 3

Connect the matching corners with appropriate line segments.

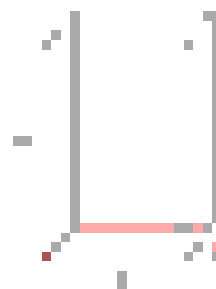


Step 4

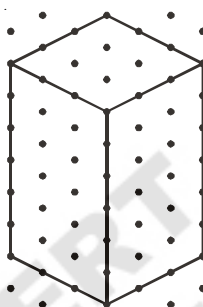
This is an isometric sketch of a cuboid.

Note that the measurements of the solid are of exact size in an isometric sketch; this is not so in the case of an oblique sketch.

Example 1 : Here is an oblique sketch of a cuboid. Draw an isometric sketch that matches this drawing.

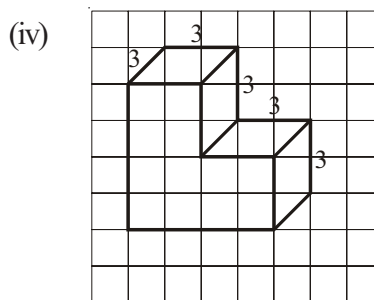
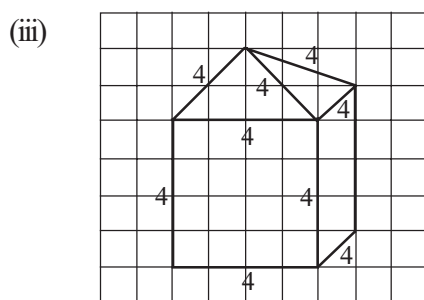
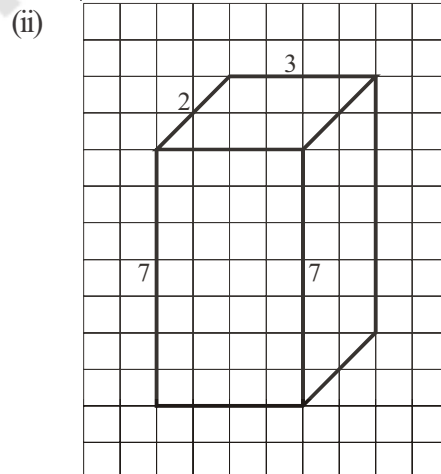
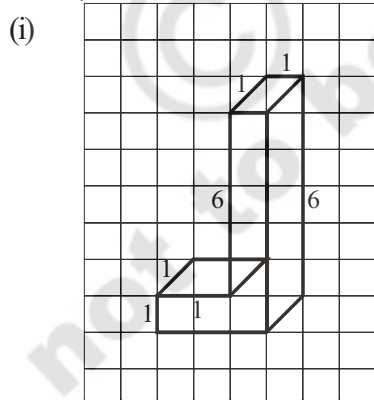


Solution : The length, breadth and height are 3, 3 and 6 units respectively

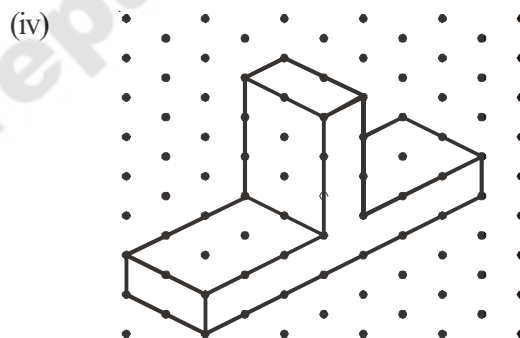
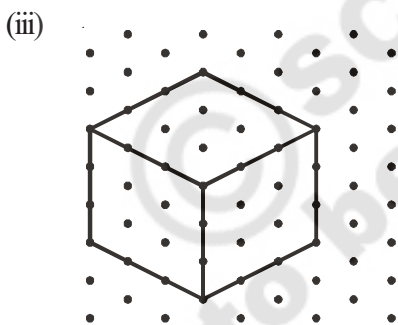
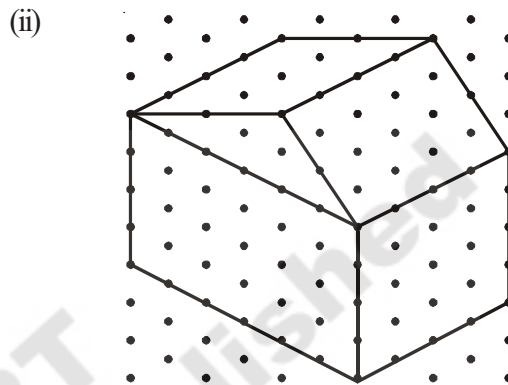
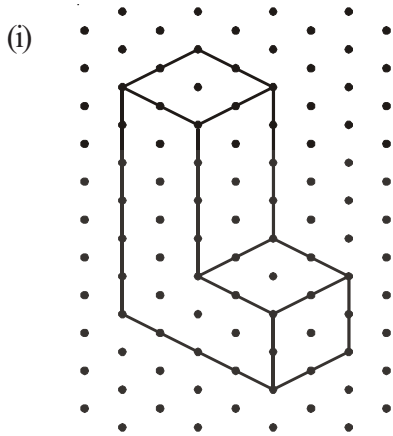


Exercise - 3

1. Use an isometric dot paper and make an isometric sketch for each one of the given shapes.



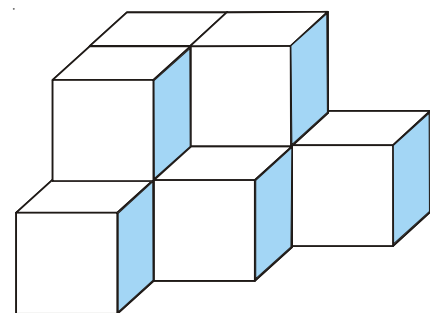
2. The dimensions of a cuboid are 5 cm, 3 cm and 2 cm. Draw three different isometric sketches of this cuboid.
3. Three cubes each with 2 cm edge are placed side by side to form a cuboid. Draw an oblique or isometric sketch of this cuboid.
4. Make an oblique sketch for each of the given isometric shapes.



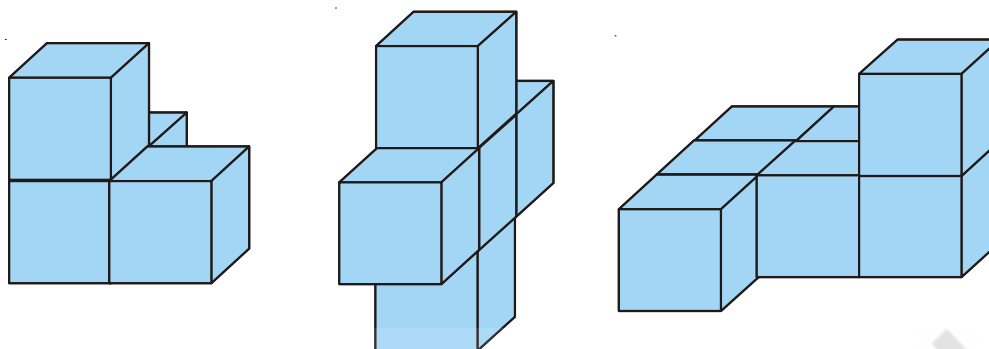
5. Give (i) an oblique sketch and (ii) an isometric sketch for each of the following:
 - (a) A cuboid of dimensions 5 cm, 3 cm and 2 cm. (Is your sketch unique?)
 - (b) A cube with an edge 4 cm long.

14.3 Visualising solid objects

Sometimes when you look at combined shapes, some of them may be hidden from your view.



Here are some activities to help you visualise some solid objects and how they look. Take some cubes and arrange them as shown below.

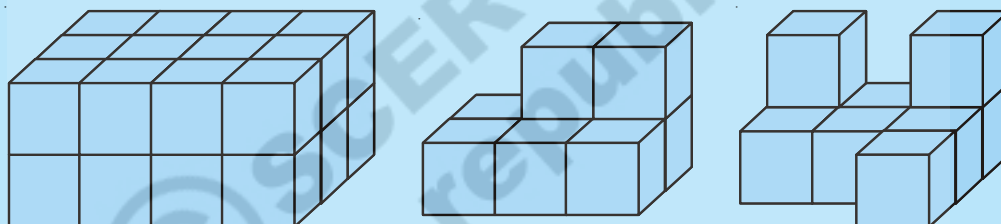


Now ask your friend to guess the total number of cubes in the following arrangements.



Try This

Estimate the number of cubes in the following arrangements.



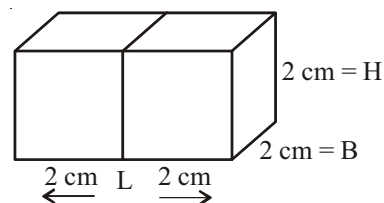
Such visualisations are very helpful.

Suppose you form a cuboid by joining cubes. You will be able to estimate what the length, breadth and height of the cuboid would be.

Example 2 : If two cubes of dimensions 2 cm by 2 cm by 2 cm are placed side by side, what would the dimensions of the resulting cuboid be?

Solution : As you can see when kept side by side, the length is the only measurement which increases, it becomes $2 + 2 = 4$ cm.

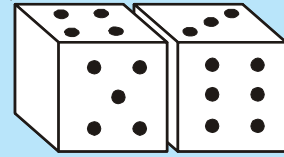
The breadth = 2 cm and the height = 2 cm.





Try This

- Two dice are placed side by side as shown. Can you say what the total would be on the faces opposite to them? (i) $5 + 6$ (ii) $4 + 3$



(Remember that in a dice the sum of numbers on opposite faces is 7)

- Three cubes each with 2 cm edge are placed side by side to form a cuboid. Try to make an oblique sketch and say what could be its length, breadth and height.

14.3.1 Viewing different sections of a solid

Now let us see how an object which is in 3-D can be viewed in different ways.

14.3.1a) One way to view an object is by cutting or slicing the object

Slicing game

Here is a loaf of bread. It is like a cuboid with a square faces. You ‘slice’ it with a knife.

When you give a ‘horizontal’ cut, you get several pieces, as shown in the figure. Each face of the piece is a square! We call this face a ‘cross-section’ of the whole bread. The cross section is nearly a square in this case.



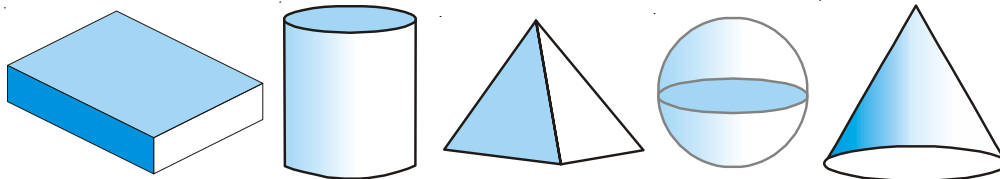
Beware! If your cut is ‘vertical’ you may get a different cross section! Think about it. The boundary of the cross-section you obtain is a plane curve. Do you notice it?

A kitchen play

Have you noticed cross-sections of some vegetables when they are cut for the purposes of cooking in the kitchen? Observe the various slices and get aware of the shapes that results as cross-sections.

Do This

- Make clay (or plasticine) models of the following solids and make vertical or horizontal cuts. Draw rough sketches of the cross-sections you obtain. Name them if possible.



2. What cross-sections do you get when you give a (i) vertical cut (ii) horizontal cut to the following solids?

(a) A brick (b) A round apple (c) A die (d) A circular pipe (e) An ice cream cone

14.3.1b) Another Way is by Shadow Play

A shadow play

Shadows are a good way to illustrate how three-dimensional objects can be viewed in two dimensions. Have you seen a shadow play? It is a form of entertainment using solid articulated figures in front of an illuminated backdrop to create the illusion of moving images. It makes some indirect use of ideas of Mathematics.

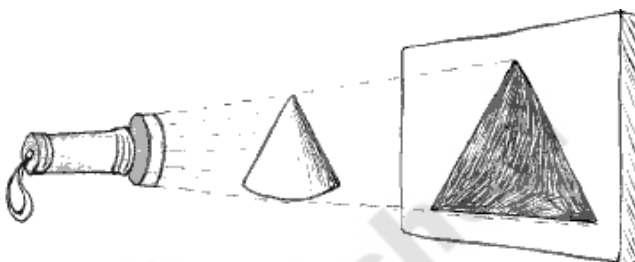


Figure 1

You will need a source of light and a few solid shapes for this activity. If you have an overhead projector, place the solid under the lamp and do these investigations.

Keep a torchlight, right in front of a cone. What type of shadow does it cast on the screen? (Figure 1).

The solid is three-dimensional; what about the shadow?

If, instead of a cone, you place a cube in the above game, what type of shadow will you get?

Experiment with different positions of the source of light and with different positions of the solid object. Study their effects on the shapes and sizes of the shadows you get.

Here is another funny experiment that you might have tried already:

Place a circular tumbler in the open when the sun at the noon time is just right above it as shown in the figure below. What is the shadow that you obtain?

Will it be same during (a) afternoon?

(b) evening?

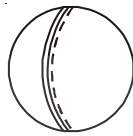


Study the shadows in relation to the position of the sun and the time of observation.



Exercise - 4

1. A bulb is kept burning just right above the following solids. Name the shape of the shadows obtained in each case. Attempt to give a rough sketch of the shadow. (You may try to experiment first and then answer these questions).



A ball

A cylindrical pipe

A book

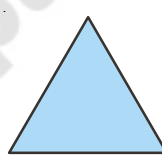
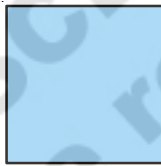
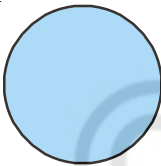
2. Here are the shadows of some 3D objects, when seen under the lamp of an overhead projector. Identify the solid(s) that match each shadow. (There may be many answers for these!)

A circle

A square

A triangle

A rectangle



(i)

(ii)

(iii)

(iv)

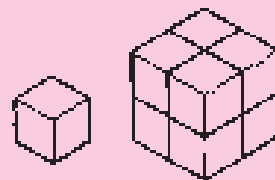


Looking Back

3D shapes can be visualised on 2D surfaces, that is on paper by drawing their nets. Oblique sketches and isometric sketches help in visualising 3D shapes on a plane surface.

Fun with a cube

A unit cube can be fitted together with 7 other identical cubes to make a larger cube with an edge of 2 units as shown in figure.



How many unit cubes are needed to make a cube with an edge of 3 units?

