

# CONGRUENCY OF TRIANGLES

## 8

### 8.0 Introduction

If we take a pile of one rupee coins and place them one on top of the other, they would match perfectly. Do you know why this happens? This is because all the coins have the same size and shape. In the same way papers of a blank note book have the same size and shape.



Look around you and find some examples of objects that share this kind of similarity i.e. they are identical in shape and size. Think of at least 5 such examples.

When we talk about objects of the same size and shape we say that the objects are congruent. A practical test of congruence is to place one object over the other and see if they superimpose exactly.

#### Activity:

Are all ten rupee notes congruent? How will you check?



Similarly, check if the five rupee note you find are congruent.



We see many examples of congruent objects all around us. Now, think of some shapes that are congruent.

**Do This**

1. Here are some shapes. See whether all the shapes given in a row are congruent to each other or not. You can trace the figures and check.

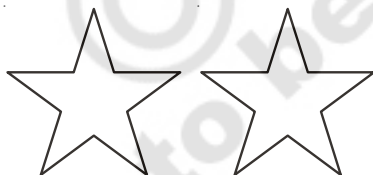


(i)

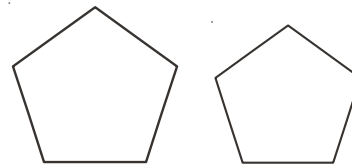
(ii)

(iii)

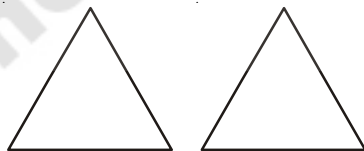
2. Which of the following pairs of figures are congruent?



(i)



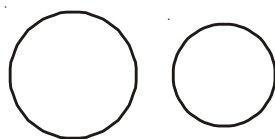
(ii)



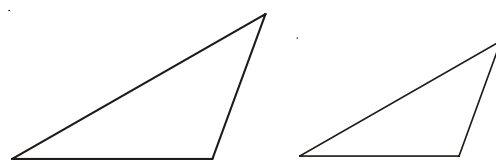
(iii)



(iv)



(v)



(vi)

## 8.1 Congruency of line segments

Observe the two pairs of line segments given below.

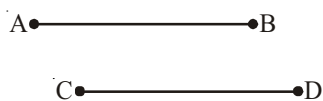


Figure 1

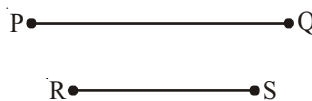


Figure 2

Copy the line segment AB on a tracing paper. Place it on CD. You will find that AB covers CD.

Hence the line segments are congruent. We write  $\overline{AB} \cong \overline{CD}$ .

Repeat this activity for the pair of line segments in Figure 2. What do you find? Are they congruent?

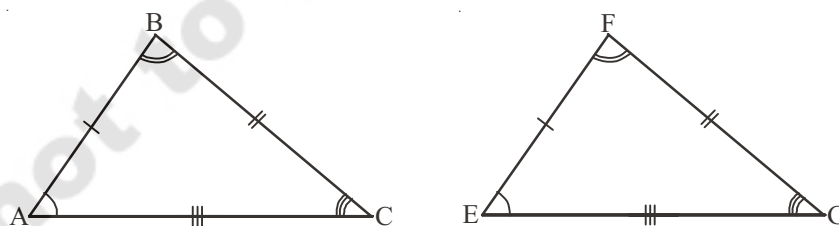
You will notice that the pair of line segments in Figure 1 match with each other because they have same length and this is not the case in Figure 2.

The line segment has only one dimension i.e., length. So if two line segments have the same length, they are congruent. Conversely, if two line segments are congruent, they have the same length.

When we write  $AB = CD$ , what we actually mean is  $\overline{AB} \cong \overline{CD}$ .

## 8.2 Congruency of triangles

We learnt that two line segments are congruent where one of them, is the copy of the other. We extend this idea to triangles. Two triangles are congruent if they are copies of one another and when superimposed, they cover each other exactly.



$\triangle ABC$  and  $\triangle EFG$  cover each other exactly i.e. they are of the same size and shape. They are congruent triangles. We express congruency of the two triangles as  $\triangle ABC \cong \triangle EFG$ .

If two triangles are congruent then all the corresponding six elements of the two triangles i.e. the three angles and three sides are congruent. We also say that if the corresponding parts of two triangles are congruent, then the triangles are congruent. This means that, when you place  $\triangle ABC$  on  $\triangle EFG$ , their corresponding corners fall on each other. A lies on E, B lies on F and C lies on G. Also  $\angle A$  falls on  $\angle E$ ,  $\angle B$  falls on  $\angle F$  and  $\angle C$  falls on  $\angle G$  and lastly AB falls on EF, BC falls on FG and AC falls on EG.

Thus, for two triangles that are congruent, their corresponding parts i.e. vertices, angles and sides match one another or are equal.

In  $\triangle ABC$  and  $\triangle EFG$

$A \rightarrow E$        $B \rightarrow F$        $C \rightarrow G$       (corresponding vertices)

$\angle A \cong \angle E$        $\angle B \cong \angle F$        $\angle C \cong \angle G$       (corresponding angles)

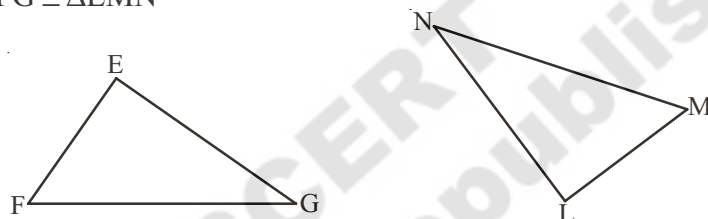
$\overline{AB} \cong \overline{EF}$        $\overline{BC} \cong \overline{FG}$        $\overline{AC} \cong \overline{EG}$       (corresponding sides)

The order of the alphabet in the names of congruent triangles displays the corresponding relationships. Thus, when we say that  $\triangle ABC \cong \triangle EFG$

Verify if  $\triangle ABC \cong \triangle EFG$ , their corresponding vertices, sides and angles.

**Do This**

1.  $\triangle EFG \cong \triangle LMN$

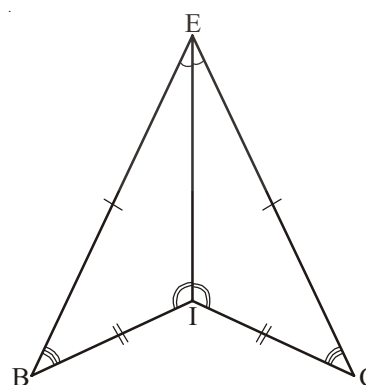
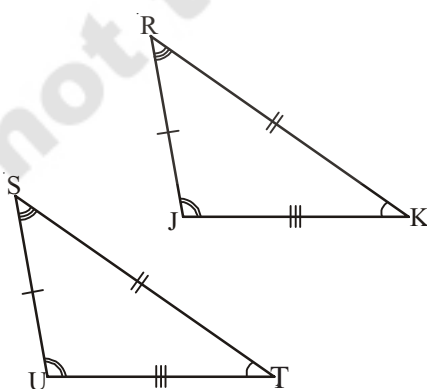


Write the corresponding vertices, angles and sides of the two triangles.

2. If  $\triangle ABC \cong \triangle DEF$  write the parts of  $\triangle ABC$  that correspond to-

(i)  $DE$       (ii)  $\angle E$       (iii)  $DF$       (iv)  $EF$       (v)  $\angle F$

3. Name the congruent triangles in each of the following pairs. Write the statement using ' $\cong$ '.



4. Name the congruent angles and sides for each pair of congruent triangles.

1.  $\triangle TUV \cong \triangle XYZ$

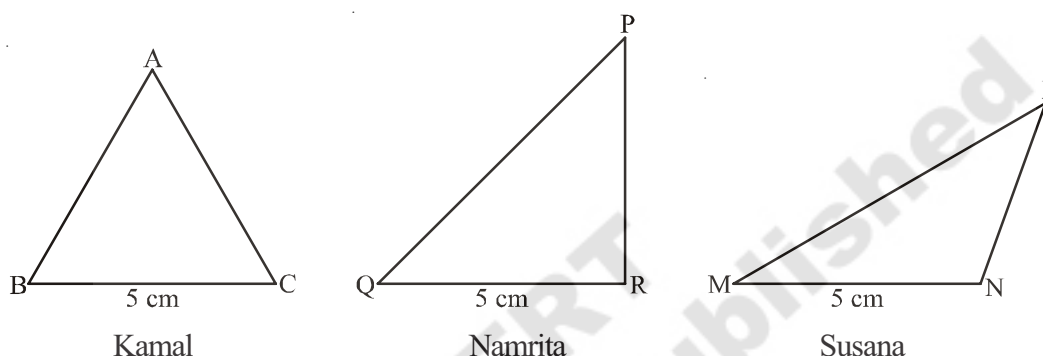
2.  $\triangle CDG \cong \triangle RSW$

### 8.3 Criterion for congruency of triangles

Is it necessary for congruency to check whether all the corresponding parts of two triangles are congruent? How can we check if the given triangles are congruent using a minimum number of measurements. Let us explore and find out.

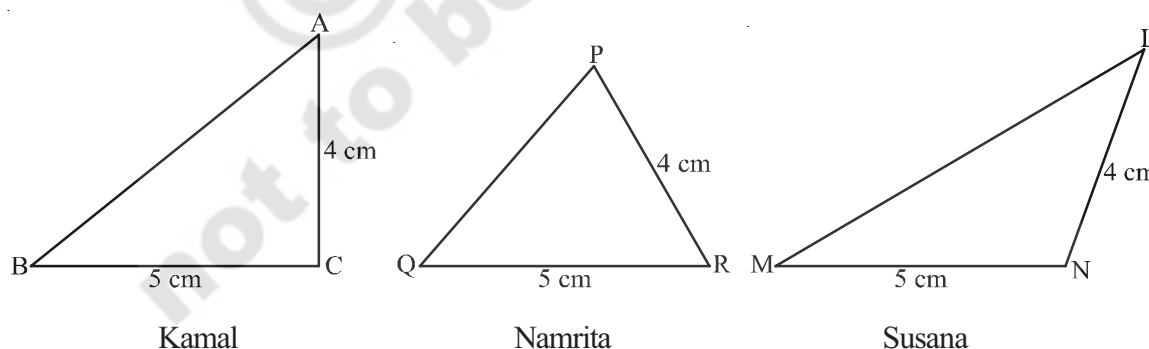
#### 8.3.1 Side-Side-Side congruency (SSS)

Will all of you draw the same triangle if you only knew that the measure of one side of the triangle is 5 cm? Kamal, Namrita and Susana have drawn them like this.

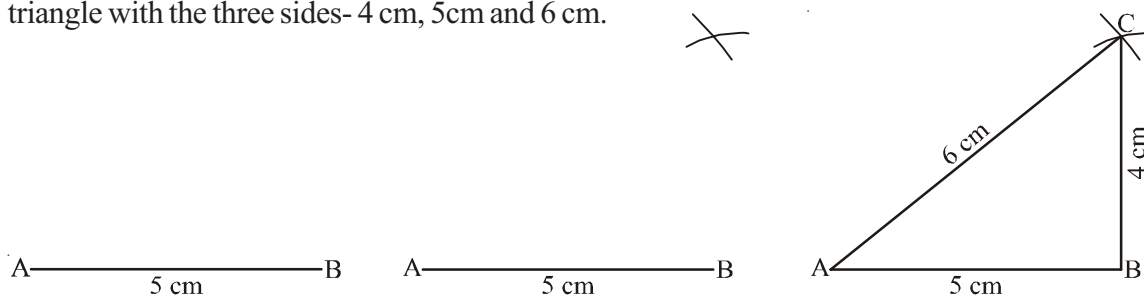


As you can see all the triangles are different. Kamal drew an equilateral triangle, Namrita drew a right-angled triangle and Susana drew an obtuse-angled triangle.

Now will all of you draw the same triangle, if you knew that the two sides of a triangle are say, 4 cm and 5 cm. Again Kamal, Namrita and Susana drew different triangles.



What if you know all the sides of the triangle? Kamal, Namrita and Susana all drew the same triangle with the three sides- 4 cm, 5 cm and 6 cm.



Thus, if we want to make a copy of  $ABC$  or a triangle congruent to  $ABC$ , we need the lengths of the three sides. This is referred to as the Side-Side-Side(SSS) criterion for congruency of triangles.

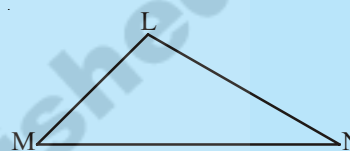
If two triangles are congruent because the lengths of their corresponding sides are equal, then will their angles also be equal?

**Side-Side-Side (SSS) criterion for congruence of triangles: If three sides of a triangle are equal to the corresponding three sides of another triangle, then the triangles are congruent.**

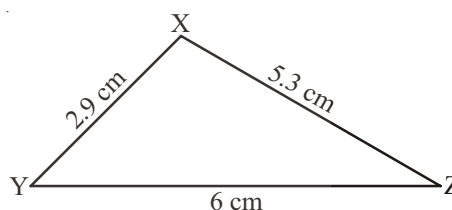
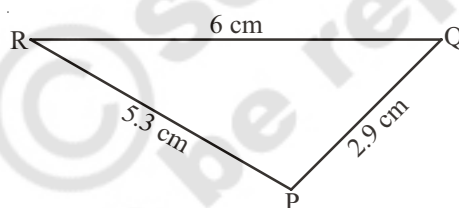


**Try This**

Measure the lengths of  $\triangle LMN$ . Now, construct a triangle with these measurements on a sheet of paper. Place this triangle over  $\triangle LMN$ . Are the triangles congruent? What criterion of congruency applies over here?



**Example 1 :** Is  $\triangle PQR \cong \triangle XYZ$ ? Also, write the corresponding angles of the two triangles.



**Solution :** According to the given figure of  $\triangle PQR$  and  $\triangle XYZ$ , we have

$$PQ = XY = 2.9 \text{ cm}$$

$$QR = YZ = 6 \text{ cm}$$

$$RP = ZX = 5.3 \text{ cm}$$

Therefore, by Side-Side-Side congruence criterion,  $\triangle PQR \cong \triangle XYZ$

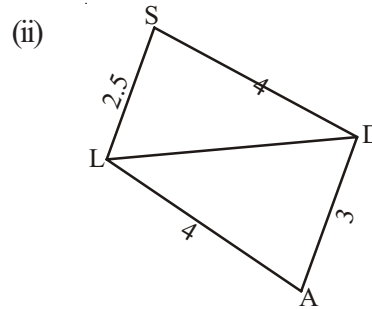
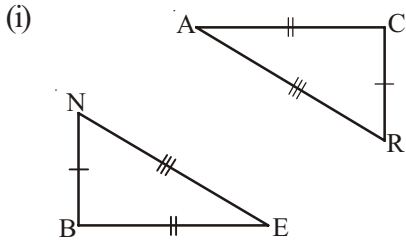
Clearly, the point P corresponds to point X, point Q corresponds to point Y and the point R corresponds to point Z.

So,  $\angle P, \angle X$ ;  $\angle Q, \angle Y$ ;  $\angle R, \angle Z$  are pairs of corresponding angles.

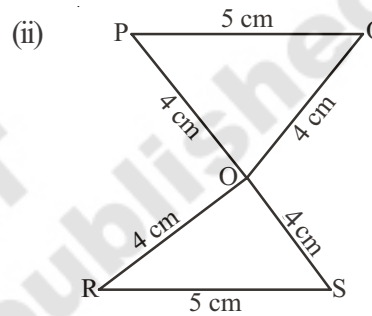
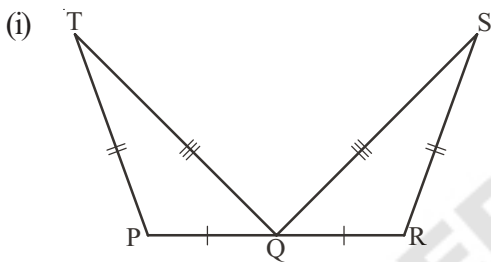


### Exercise - 1

1. Decide whether the SSS congruence is true with the following figures. Give reasons

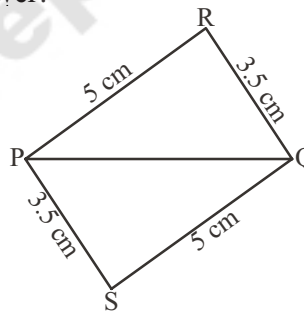


2. For the following congruent triangles, find the pairs of corresponding angles.

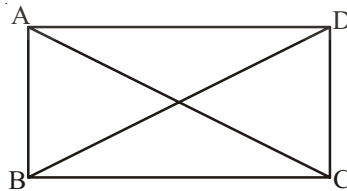


3. In adjacent figure, choose the correct answer!

- (i)  $\triangle PQR \cong \triangle PQS$
- (ii)  $\triangle PQR \cong \triangle QPS$
- (iii)  $\triangle PQR \cong \triangle SQP$
- (iv)  $\triangle PQR \cong \triangle SPQ$

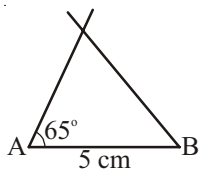


4. In the figure given below,  $AB = DC$  and  $AC = DB$ . Is  $\triangle ABC \cong \triangle DCB$ .

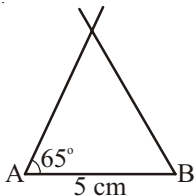


### 8.3.2 Side-Angle-Side Congruence

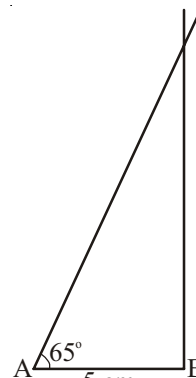
We have seen that it is not possible to draw congruent triangles, if we are given only the measurements of one side. Now, what if you were given one angle and one side? Kamal, Namrita and Susana were told to draw triangles with one side equal to 5 cm and one angle equal to  $65^\circ$ . They drew the following dissimilar triangles.



Kamal



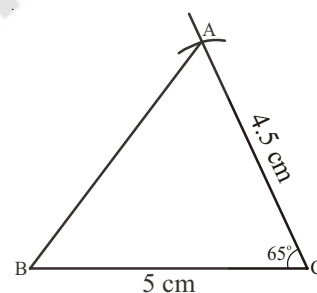
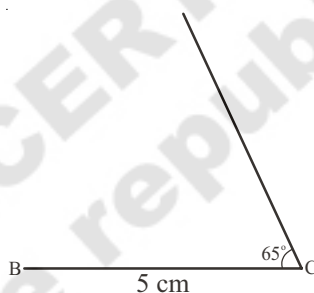
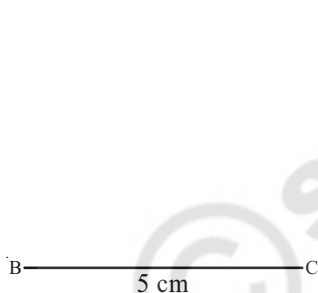
Namrita



Susana

Now, what if the three of them knew the two sides of the triangle and the angle included between these sides. The three children decided to draw triangles with sides 5 cm and 4.5 cm and the included angle of  $\angle 65^\circ$ .

Kamal drew  $\triangle ABC$ . He drew  $BC$  as the base = 5 cm. He then made  $\angle C = 65^\circ$  using a protractor and then marked point  $A$  at a length of 4.5 cm on the angular line. He then joined points  $A$  and  $B$ .



Can you draw the  $65^\circ$  angle at point  $B$  with side  $AB = 4.5$  cm. Will the triangle that is formed be congruent to Kamal's triangle? Can you take the base to be 4.5 cm, side = 5 cm and included angle =  $65^\circ$ ? Will the triangle that is formed be congruent to Kamal's triangle? You will find that the triangles formed in all these situations are congruent triangles.

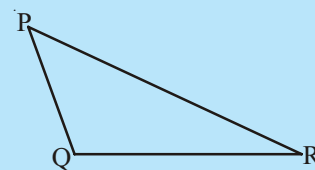
Therefore, if we want to make a copy of  $\triangle ABC$  or a triangle congruent to  $\triangle ABC$ , we need the lengths of the two sides and the measure of the angle between the two sides. This is referred to as the Side-Angle-Side(SAS) criterion for congruence of triangles.

Side-Angle-Side(SAS) criterion for congruence of triangles: If two sides and the angle included between the two sides of a triangle are congruent to the corresponding two sides and the included angle of another triangle, then the triangles are congruent.



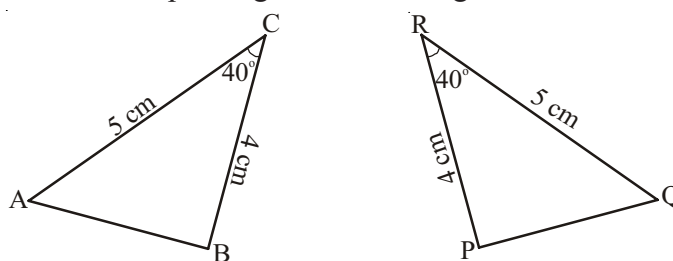
**Try This**

In  $\triangle PQR$  measure the lengths  $PQ$  and  $QR$  as well as  $\angle Q$ . Now, construct a triangle with these three measurements on a sheet of paper. Place this triangle over  $\triangle PQR$ . Are the triangles congruent? What criterion of congruency applies over here?





**Example 2 :** See the measurements of the triangles given below. Are the triangles congruent? Which are the corresponding vertices and angles?



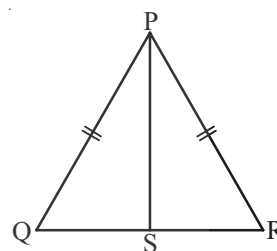
**Solution :** In  $\triangle ABC$  and  $\triangle PQR$   $AC = QR$  and  $BC = PR$  and included angle  $\angle C \cong \angle R$   
So,  $\triangle ABC \cong \triangle PQR$ .

The correspondence is as follows:

$A \leftrightarrow Q$ ,  $B \leftrightarrow P$  and  $C \leftrightarrow R$

Therefore,  $\angle A \cong \angle Q$ ,  $\angle B \cong \angle P$  and  $\angle C \cong \angle R$

**Example 3 :** In  $\triangle PQR$ ,  $PQ = PR$  and  $PS$  is angle bisector of  $\angle P$ .  
Are  $\triangle PQS$  and  $\triangle PRS$  congruent? If yes, give reason.



**Solution :** In  $\triangle PQS$  and  $\triangle PRS$

$PQ = PR$  ( given)

$PS = PS$  (common side in both the triangles)

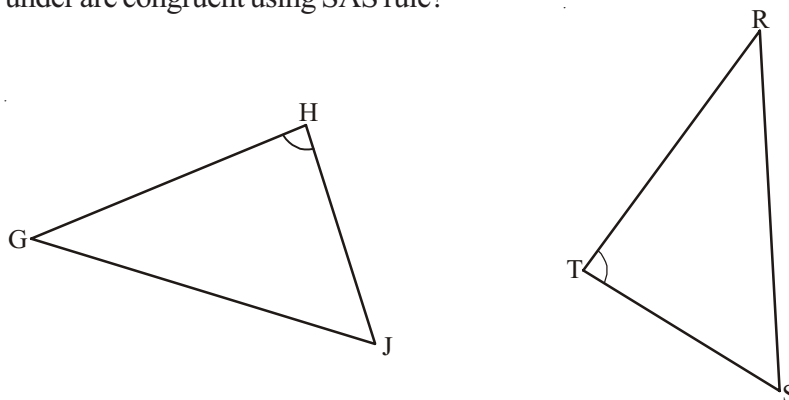
and included angle  $\angle QPS \cong \angle RPS$  ( $PS$  is the angle bisector)

Therefore,  $\triangle PQS \cong \triangle PRS$  (by SAS rule)

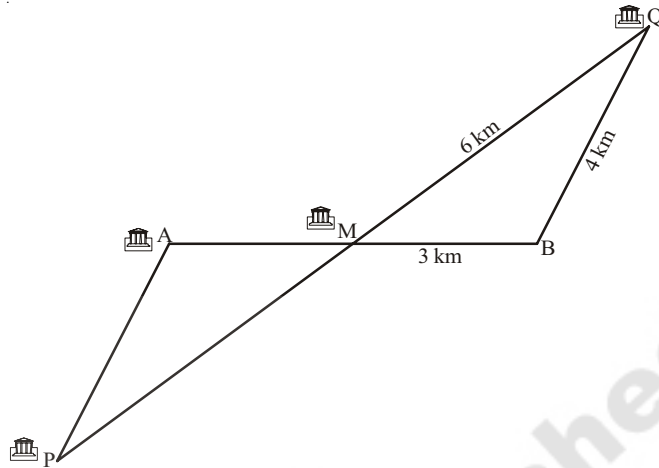


### Exercise - 2

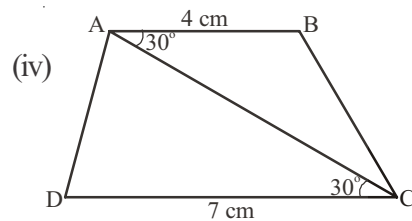
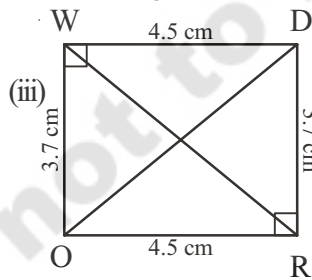
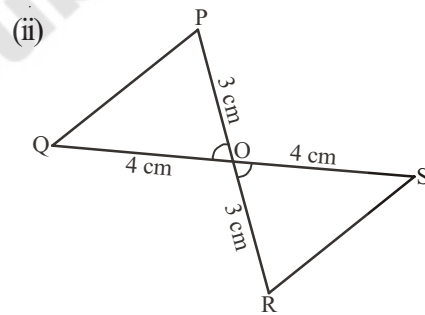
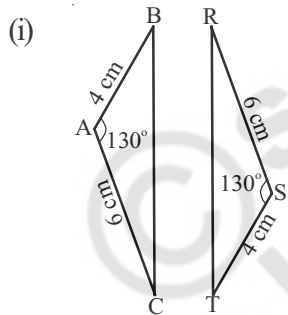
1. What additional information do you need to conclude that the two triangles given here under are congruent using SAS rule?



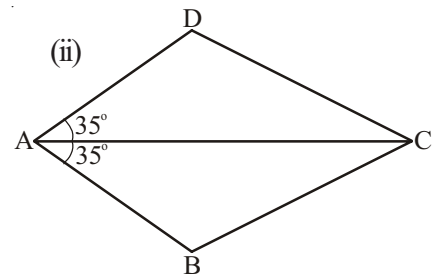
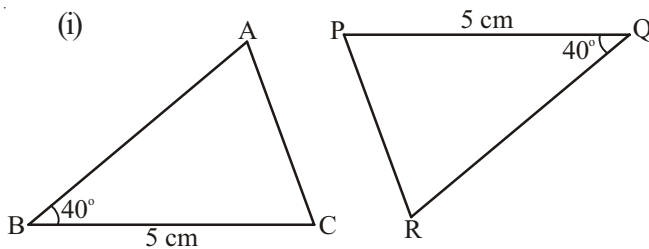
2. The map given below shows five different villages. Village M lies exactly halfway between the two pairs of villages A and B as well as and P and Q. What is the distance between village A and village P. (Hint: check if  $\triangle PAM \cong \triangle QBM$ )



3. Look at the pairs of triangles given below. Are they congruent? If congruent write the corresponding parts.



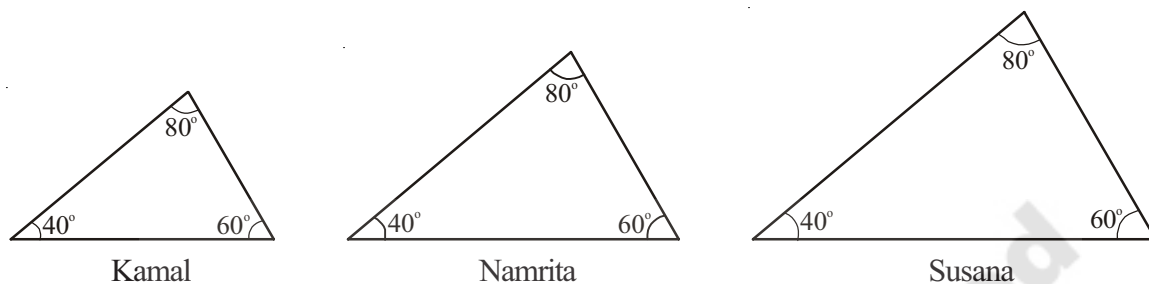
4. Which corresponding sides do we need to know to prove that the triangles are congruent using the SAS criterion?



### 8.3.3 Angle-Side-Angle congruency (ASA)

Can the children construct a triangle if they know only one angle of the triangle? What if they know two angles? Will children be able to draw congruent triangles if they know all the angles of the triangle?

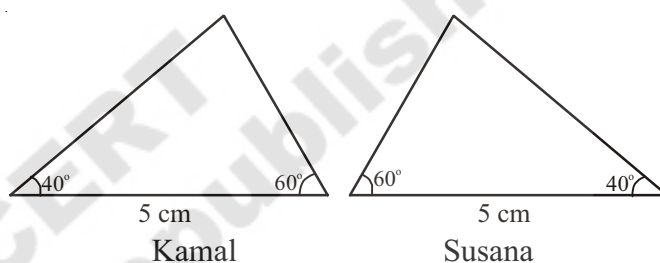
Kamal, Namrita and Susana drew the following triangles of angles-  $40^\circ$ ,  $60^\circ$  and  $80^\circ$ .



Therefore, though the angles of all the triangles are congruent, the lengths of their sides is not and they are not congruent.

Thus, we need to know the length of the sides, to draw congruent triangles.

What if we have two angles and one side? Kamal and Namrita drew the following triangles with angles  $60^\circ$  and  $40^\circ$  and side 5 cm. When both the



children constructed their triangles they made the given side, the included side.

We can conclude that if we want to make a copy of a triangle or a triangle congruent to another triangle, then we need to know two angles and the length of the side included between the two angles. This is referred to as the Angle-Side-Angle criterion of congruence.

Angle-Side-Angle criterion of congruence: If two angles and the included side of a triangle are congruent to the two corresponding angles and included side of another triangle then the triangles are congruent.



#### Try This

Teacher has asked the children to construct a triangle with angles  $60^\circ$ ,  $40^\circ$  and with a side 5 cm. Sushma calculated the third angle of the triangle as  $80^\circ$  using angle - sum property of triangle. Then Kamal, Sushma and Namratha constructed triangles differently using the following measurements.

Kamal:  $60^\circ$ ,  $40^\circ$  and 5cm side (as teacher said)

Sushma:  $80^\circ$ ,  $40^\circ$ , and 5 cm side

Namratha:  $60^\circ$ ,  $80^\circ$  and 5 cm side.

They cut these triangles and place them one upon the other. Are all of them congruent? You also try this.

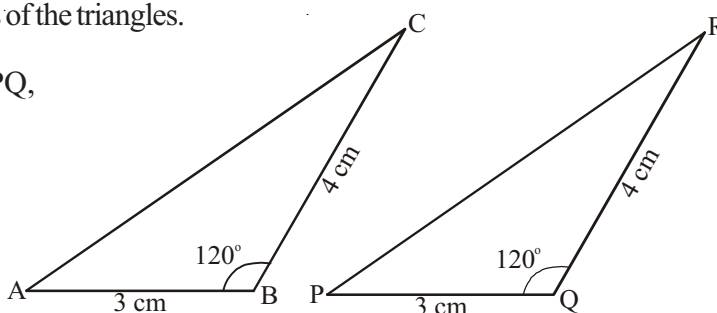
**Example 4 :** Two triangles  $\triangle CAB$  and  $\triangle RPQ$  are given below. Check whether the two are congruent? If they are congruent, what can you say about the measures of the remaining elements of the triangles.

**Solution :** In  $\triangle CAB$  and  $\triangle RPQ$ ,

$$BC = QR = 4 \text{ cm}$$

$$\angle B = \angle Q = 120^\circ$$

$$AB = PQ = 3 \text{ cm}$$



Thus, two sides and included angle of  $\triangle CAB$  are equal to the corresponding sides and included angle of  $\triangle RPQ$ .

Therefore, by Side-Angle-side criterion of congruency  $\triangle CAB \cong \triangle RPQ$

Thus, in the two triangles

$$AC \cong PR$$

$$\angle C \cong \angle R \text{ and } \angle A \cong \angle P$$

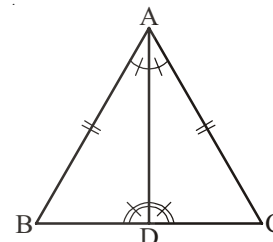
**Example-5 :** In the following picture, the equal angles in the two triangles are shown. Are the triangles congruent?

**Solution :** In  $\triangle ABD$  and  $\triangle ACD$

$$\angle BAD \cong \angle CAD \text{ (given in the question)}$$

$$\angle ADB \cong \angle ADC \text{ (given in the question)}$$

$$AD \cong AD \text{ (common side, seen in the figure)}$$

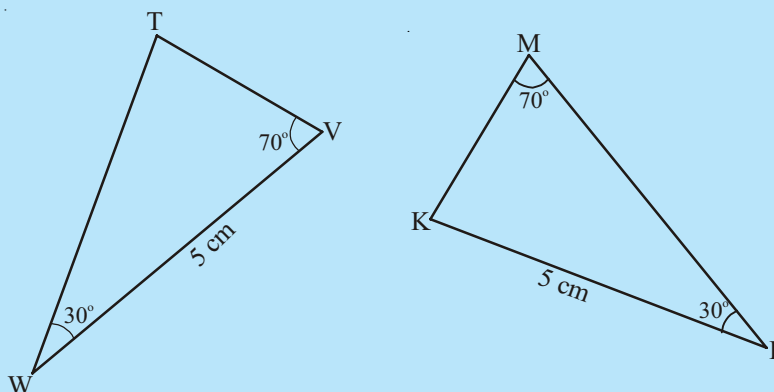


Thus, by Angle-side-Angle congruence criterion  $\triangle ABD \cong \triangle ACD$



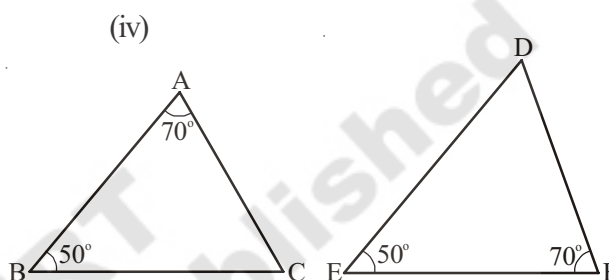
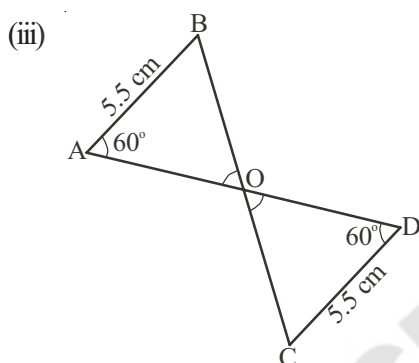
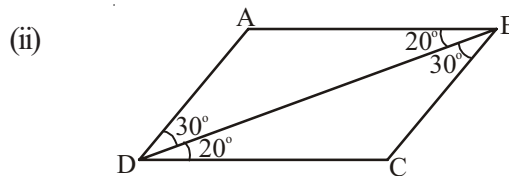
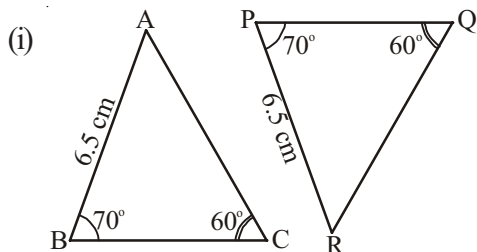
### Try This

Is the following pair of triangles congruent? Give reason to support your answer.

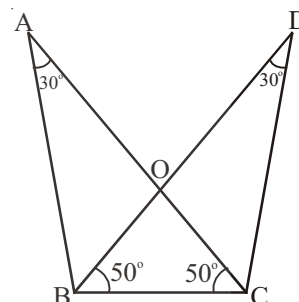


### Exercise - 3

1. In following pairs of triangles, find the pairs which are congruent? Also, write the criterion of congruence.



2. In the adjacent figure.
- Are  $\triangle ABC$  and  $\triangle DCB$  congruent?
  - Are  $\triangle AOB$  congruent to  $\triangle DOC$ ?



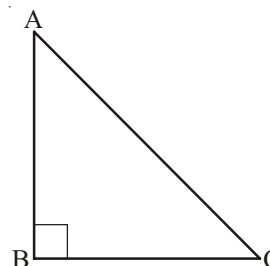
Also identify the relation between corresponding elements and give reason for your answer.

#### 8.3.4 Right-Angle Hypotenuse Side congruence

In right-angled triangles we already know that one of the angles is a right angle. So what else do we need to prove that the two triangles are congruent?

Let us take the example of  $\triangle ABC$  with  $\angle B = 90^\circ$ . Can we draw a triangle congruent to this triangle, if,

- only BC is known
- only  $\angle C$  is known
- $\angle A$  and  $\angle C$  are known
- AB and BC are known
- $\angle C$  and BC are known
- BC and the hypotenuse AC are known



When you try to draw the rough sketches of these triangles, you will find it is possible only in cases (iv), (v) and (vi).

The last of the situations is new to us and it is called the Right-Angle Hypotenuse Congruence Criterion.

**Right-Angle Hypotenuse Congruence Criterion:** If the hypotenuse and one side of a right angled triangle are equal to the corresponding hypotenuse and side of the other right angled triangle, then the triangles are congruent.

**Example 6 :** Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, using RHS congruence rule. In case of congruent triangles, write the result in symbolic form :

$\triangle ABC$

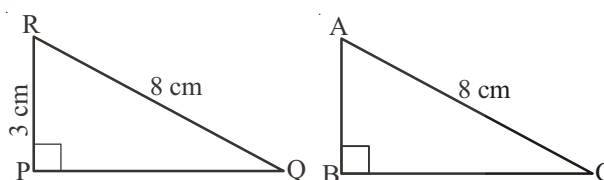
$\triangle PQR$

(i)  $\angle B = 90^\circ$ ,  $AC = 8$  cm,  $AB = 3$  cm       $\angle P = 90^\circ$ ,  $PR = 3$  cm,  $QR = 8$  cm

(ii)  $\angle A = 90^\circ$ ,  $AC = 5$  cm,  $BC = 9$  cm       $\angle Q = 90^\circ$ ,  $PR = 8$  cm,  $PQ = 5$  cm

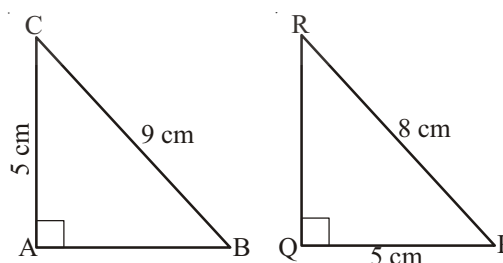
**Solution :**

- (i) Here,  $\angle B = \angle P = 90^\circ$   
hypotenuse,  $AC =$  hypotenuse,  $RQ (= 8$  cm) and  
side  $AB =$  side  $RP (= 3$  cm)



So,  $\triangle ABC \cong \triangle RPQ$  (By RHS Congruence rule).

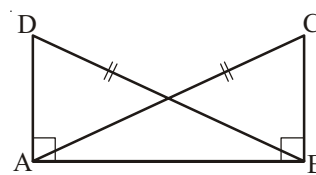
- (ii) Here,  $\angle A = \angle Q = 90^\circ$  and  
side  $AC =$  side  $PQ (= 5$  cm).



hypotenuse,  $BC \neq$  hypotenuse,  $PR$

So, the triangles are not congruent.

**Example 7 :** In the adjacent figure,  $\overline{DA} \perp \overline{AB}$ ,  $\overline{CB} \perp \overline{AB}$  and  $AC = BD$ .



State the three pairs of equal parts in  $\Delta ABC$  and  $\Delta DAB$ .

Which of the following statements is meaningful?

- (i)  $\Delta ABC \cong \Delta BAD$  (ii)  $\Delta ABC \cong \Delta ABD$

**Solution :** The three pairs of equal parts are :

$$\angle ABC = \angle BAD (= 90^\circ)$$

$$AC = BD \text{ (Given)}$$

$$AB = BA \text{ (Common side)}$$

$$\Delta ABC \cong \Delta BAD \text{ (By RHS congruence rule).}$$

From the above,

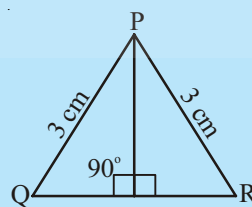
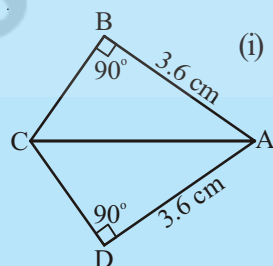
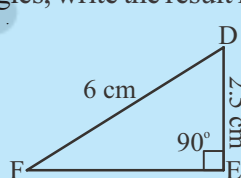
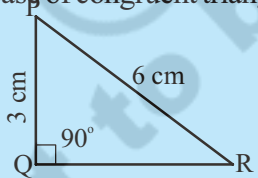
statement (i) is true;

statement (ii) is not meaningful, in the sense that the correspondence among the vertices is not satisfied.



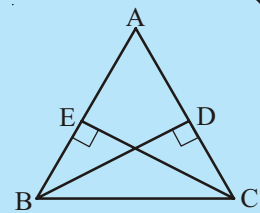
### Try This

- In the figures given below, measures of some parts of triangles are given. By applying RHS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.



- It is to be established by RHS congruence rule that  $\Delta ABC \cong \Delta RPQ$ . What additional information is needed, if it is given that  $\angle B = \angle P = 90^\circ$  and  $AB = RP$ ?

3. In the adjacent figure,  $\overline{BD}$  and  $\overline{CE}$  are altitudes of  $\triangle ABC$  such that  $BD = CE$ .

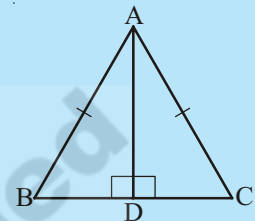


(i) State the three pairs of equal parts in  $\triangle CBD$  and  $\triangle BCE$ .

(ii) Is  $\triangle CBD \cong \triangle BCE$ ? Why or why not?

(iii) Is  $\angle DBC = \angle ECB$ ? Why or why not?

4.  $ABC$  is an isosceles triangle with  $\overline{AB} = \overline{AC}$  and  $\overline{AD}$  is one of its altitudes (fig ...).



(i) State the three pairs of equal parts in  $\triangle ADB$  and  $\triangle ADC$ .

(ii) Is  $\triangle ADB \cong \triangle ADC$ ? Why or why not?

(iii) Is  $\angle B \cong \angle C$ ? Why or why not?

(iv) Is  $BD \cong CD$ ? Why or why not?



### Exercise - 4

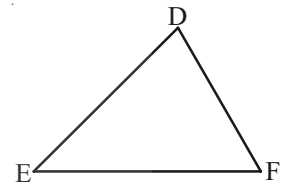
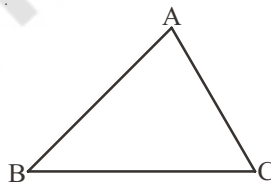
1. Which congruence criterion do you use in the following?

(i) Given :  $AC = DF$

$$AB = DE$$

$$BC = EF$$

So,  $\triangle ABC \cong \triangle DEF$

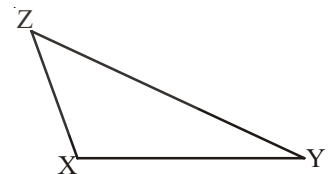
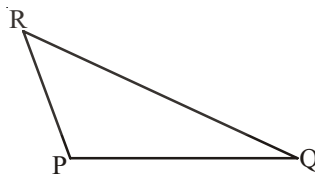


(ii) Given :  $ZX = RP$

$$RQ = ZY$$

$$\angle PRQ \cong \angle XZY$$

So,  $\triangle PQR \cong \triangle XYZ$

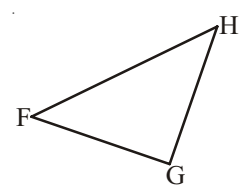
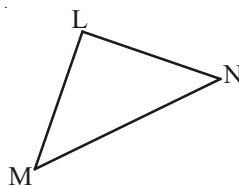


(iii) Given :  $\angle MLN \cong \angle FGH$

$$\angle NML \cong \angle GFH$$

$$ML = FG$$

So,  $\triangle LMN \cong \triangle GFH$



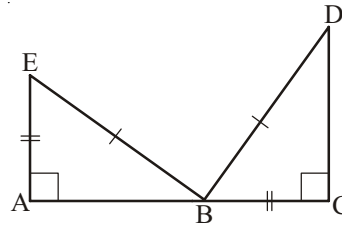


(iv) Given :  $EB = DB$

$AE = BC$

$\angle A = \angle C = 90^\circ$

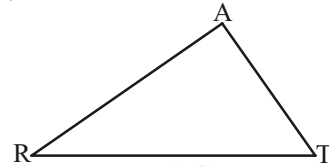
So,  $\triangle ABE \cong \triangle CDB$



2. You want to show that  $\triangle ART \cong \triangle PEN$ ,

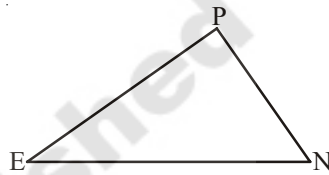
(i) If you have to use SSS criterion, then you need to show

(a)  $AR =$                       (b)  $RT =$                       (c)  $AT =$



(ii) If it is given that  $\angle T = \angle N$  and you are to use SAS criterion, you need to have

(a)  $RT =$                       and                      (ii)  $PN =$



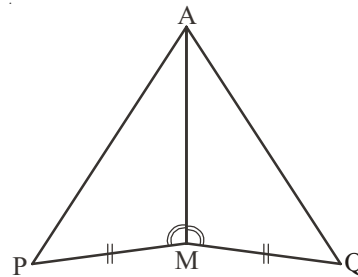
(iii) If it is given that  $AT = PN$  and you are to use ASA criterion, you need to have

(a) ?                                      (b) ?

3. You have to show that  $\triangle AMP \cong \triangle AMQ$ .

In the following proof, supply the missing reasons.

Steps	Reasons
(i) $PM = QM$	(i) .....
(ii) $\angle PMA \cong \angle QMA$	(ii) .....
(iii) $AM = AM$	(iii) .....
(iv) $\triangle AMP \cong \triangle AMQ$	(iv) .....

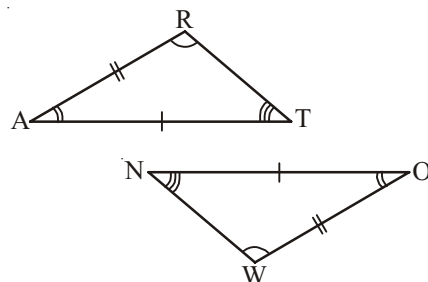


4. In  $\triangle ABC$ ,  $\angle A = 30^\circ$ ,  $\angle B = 40^\circ$  and  $\angle C = 110^\circ$

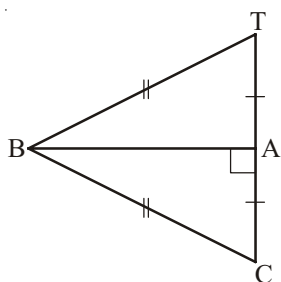
In  $\triangle PQR$ ,  $\angle P = 30^\circ$ ,  $\angle Q = 40^\circ$  and  $\angle R = 110^\circ$

A student says that  $\triangle ABC \cong \triangle PQR$  by AAA congruence criterion. Is he justified? Why or why not?

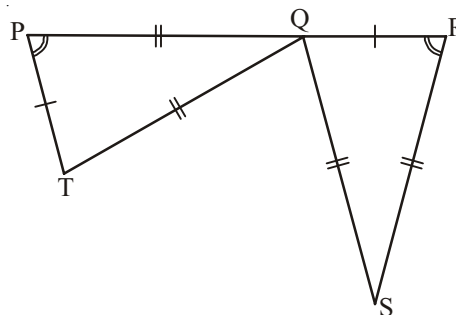
5. In the figure, the two triangles are congruent. The corresponding parts are marked. We can write  $\triangle RAT \cong ?$



6. Complete the congruence statement.



$$\triangle ABC \cong ?$$



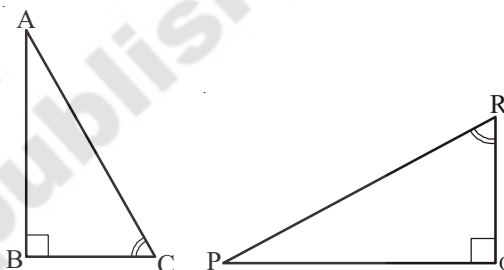
$$\triangle QRS \cong ?$$

7. In a squared sheet, draw two triangles of equal areas such that

- (i) the triangles are congruent.
- (ii) the triangles are not congruent.

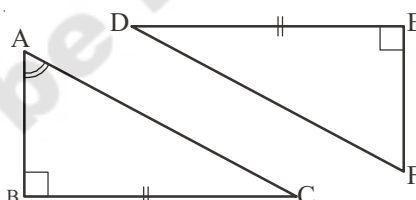
What can you say about their perimeters?

8. If  $\triangle ABC$  and  $\triangle PQR$  are to be congruent, name one additional pair of corresponding parts. What criterion did you use?



9. Explain, why

$$\triangle ABC \cong \triangle FED.$$



### Looking Back

1. Congruent objects are objects having the same shape and size.
2. The method of superimposition examines the congruence of plane figures.
3. Two line segments say, AB and CD are congruent if they have equal lengths. We write this as  $AB \cong CD$ . However, it is common to write it as  $AB = CD$ .
4. If all the parts of one triangle are equal to the corresponding parts of other triangle, then the triangles are congruent.

5. The necessary and sufficient conditions for two triangles to be congruent are as follows:

- (i) Side-Side-Side (SSS) criterion for congruence: If three sides of a triangle are equal to the corresponding three sides of another triangle, then the triangles are congruent.
- (ii) Side-Angle-Side(SAS) criterion for congruence: If two sides and the angle included between the two sides of a triangle are equal to the corresponding two sides and the included angle of another triangle, then the triangles are congruent.
- (iii) Angle-Side-Angle criterion of congruence: If two angles and the included side of a triangle are equal to the corresponding two angles and included side of another triangle then the triangles are congruent.
- (iv) Right-Angle Hypotenuse criterion of congruence: If the hypotenuse and one side of a right-angled triangle are equal to the corresponding hypotenuse and side of the other right-angled triangle, then the triangles are congruent.

