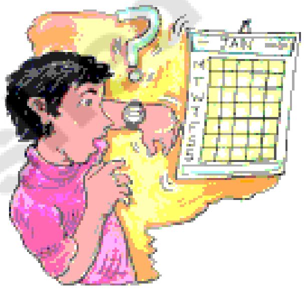
Chapter 15

Playing with Numbers

15.0 Introduction



Imagine … one morning you wake up in a strange world - a world without numbers, how would your day go ?
You will see no calendar to tell you which day of the month it is …
You will not be able to call up your friends to say thanks, if there are no telephone numbers ! And yes! You will get tired of strangers knocking your door, since no house numbers !
These are just few examples ! Think of the other ways in which your life will go for a change in a world without numbers !
You are right . You will get late for your school and miss out yours favourite cartoons/serials, if there would be no clocks. And yes, no cricket, no foot ball, without numbers .
So, it seems that it is not a good idea to be there without numbers. If we wish to find the cost of some article or if want to distribute something equally among your friends, how will you do?
Can you guess which of these are fundamental operations ? All these fundamental operations involve numbers, divisibility rules. The divisibility rules help us to find whether the given number is divisibile by another number or not without doing division. Let us play with numbers using some fundamental operations and divisibility rules.

15.1 Divisibility Rules

Take some numbers and check them which are divisible by 2, which are divisible by 3 and so on till 7.

When a number ‘a’ divides a number ‘b’completely,we say ‘b’ is divisible by ‘a’.

In this chapter we will learn about divisibility of numbers and logic behind them. First recall about place value and factors.

15.1.1 Place value of a digit :

Let us take a number 645 and expand it. $645 = 600 + 40 + 5 = 6 \times 100 + 4 \times 10 + 5 \times 1$

In the given number, the place value of 6 is 600 and the place value of 4 is 40. There are 6 hundreds, 4 tens and 5 ones in 645.

Do this:

Write the place value of numbers underlined?

(i) $\underline{2}9\underline{8}79$ (ii) $10\underline{3}4\underline{4}$ (iii) $9\underline{8}72\underline{5}$

15.1.2 Expanded form of numbers :

We know how to write a number in expanded form .At the same time , we are familiar with how to express a number in expanded form by using powers of ten.

For example

Standard notation				Expanded form
68 =60+8	=	$(10 \times 6) + 8$	=	$(10^1 \times 6) + (10^0 \times 8)$
72=70+2	=	$(10 \times 7) + 2$	=	$(10^1 \times 7) + (10^0 \times 2)$

We know that $10^0 = 1$

Let us consider a two digit number 10a + b having 'a' and 'b' respectively as tens and units digits using the above notations , the number can be written as $(10\times a) + b = (10^1 \times a) + (1 \times b)$. (Where $a \neq 0$)

Let us now consider a number 658 , a three digit number, it can be written as

Standard notation Expanded form

$$658 = 600 + 50 + 8 = 100 \times 6 + 10 \times 5 + 1 \times 8 = 10^2 \times 6 + 10^1 \times 5 + 1 \times 8$$

Similarly $759 = 700 + 50 + 9 = 100\times7 + 10\times5 + 1\times9 = 10^2 \times7 + 10^1\times5 + 1\times9$

In general a three digit number made up of digits a, b, and c is written as $10^2a + 10^1b + c = 100\times a + 10\times b + c = 100a + 10b + c$, (where $a \neq 0$).

We can write a number in such expanded form as

$$3456 = 3000 + 400 + 50 + 6 = 1000 \times 3 + 100 \times 4 + 10 \times 5 + 6$$

$$= 10^3 \times 3 + 10^2 \times 4 + 10^1 \times 5 + 6$$

Similarly a four digit number made up of digits a, b, c and d can be written as

$$1000a + 100b + 10c + d = 1000 \times a + 100 \times b + 10 \times c + d \text{ (where } a \neq 0)$$

$$= 10^3a + 10^2b + 10^1 c + d.$$

Do These :

1. Write the following numbers in expanded form

   (i) 65 (ii) 74 (iii) 153 (iv) 612

2. Write the following in standard notation

   (i) $10 \times 9 + 4$ (ii) $100 \times 7 + 10 \times 4 + 3$

3. Fill in the blanks

   (i) $100 \times 3 + 10 \times$ _____ $+ 7 = 357$

   (ii) $100 \times 4 + 10 \times 5 + 1 =$ _________

   (iii) $100 \times$ _____ $+ 10 \times 3 + 7 = 737$

   (iv) $100 \times$ ____ $+ 10 \times q + r = pqr$

   (v) $100 \times x + 10 \times y + z =$ ____________

15.1.3 Factors and Multiples of numbers:

$36 = 1 \times 36$
$= 2 \times 18$
$= 3 \times 12$
$= 4 \times 9$
$= 6 \times 6$

What are the factors of 36 ?

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.
Which is the biggest factor of 36 ?

We say every factor is less than or equal to the given number .
Greatest factor of a non-zero number is the number it self.

Therefore, every number is a factor of itself. And '1' is a factor of all numbers.

$7 \times 1 = 7$, $9 \times 1 = 9$,

If a natural number other than '1' has no factors except 1 and itself , what do you say about such numbers? Those numbers are **prime numbers.**

Ex : 2, 3, 5, 7, 11, 13,….etc.

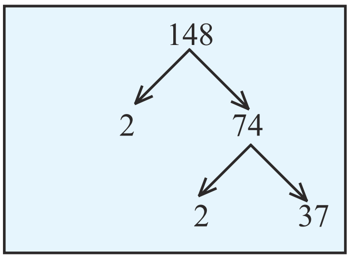
One interesting four sets of numbers 11 , 23 , 4567 , 89 are primes and made with consecutive digits.

Check whether 191, 911, 199, 919, 991 are primes or not?

> The number 8281797877767574737271706968676665646362616059585756555453 52
> 515049484746454443424140393837363534333231302928272625242322212019 18
> 17161514131211109876 54321 is written by starting at 82 and writing backwards to 1 and see that it is a prime number.

Factorize 148 into prime factors.

$148 = 2 \times 74 = 2 \times 2 \times 37 = 2^2 \times 37^1$



Number of factors of 148 is product of (Exponents of factors + 1) of prime factors

i.e. $(2 + 1) \times (1 + 1) = 3\times2 = 6$

Those are 1, 2, 4, 37, 74, 148.

If a number can be written as product of primes i.e. $N = 2^a \times 3^b \times 5^c$ ...

Number of factors of N is $(a + 1)(b + 1)(c + 1)$ ...

What are the first 5 multiples of 6 ?

$6 \times 1 = 6$, $6 \times 2 = 12$, $6 \times 3 = 12$, $6 \times 4 = 24$, $6 \times 5 = 30$

6, 12, 18, 24, 30 are first five multiples of 6.

How many multiples can we write? infinite multiples.

We say number of multiples of a given number is infinite .

Do These :

1. Write all the factors of the following numbers :

   (a) 24 (b) 15 (c) 21 (d) 27

   (e) 12 (f) 20 (g) 18 (h) 23 (i) 36

2. Write first five multiples of given numbers

   (a) 5 (b) 8 (c) 9

3. Factorize the following numbers into prime factors.

   (a) 72 (b) 158 (c) 243

15.1.4 Divisibility by 10 :

Take the multiples of 10 : 10, 20, 30, 40, 50, 60, ...etc

In all these numbers the unit's digit is '0'

Do you say any multiple of 10 will have unit digit as zero? yes,

Therefore if the unit digit of a number is '0',then it is divisible by 10.

Let us see the logic behind this rule .

If we take a three digit number where 'a' is in hundred's place, 'b' is in ten's place and 'c' is in unit's place can be written as $100a + 10b + c = 10(10a + b) + c$

$10(10a + b)$ is multiple of 10. If 'c' is a multiple of 10 then the given number will be divisible by 10. It is possible only if $c = 0$.

Do These :

1. Check whether the following given numbers are divisible by 10 or not ?
   (a) 3860 (b) 234 (c) 1200 (d) 10^3 (e) $10 + 280 + 20$
2. Check whether the given numbers are divisible by 10 or not ?
   (a) 10^{10} (b) 2^{10} (c) $10^3 + 10^1$

Try This :

1. In the division $56Z \div 10$ leaves remainder 6, what might be the value of Z

15.1.5 Divisibility by 5 :

Take the multiples of 5. Those are 5,10,15, 20,25,30,35 ,40,45,50,............etc

In these numbers the unit's digit is '0' or '5'

If the units digit of a number is '0' or '5' then it is divisible by 5.

Let us see the logic behind this rule .

If we take a three digit number $100a + 10b + c$ where 'a' is in hundred's place, b is in ten's place and c is in unit's place, it can be written as $100a + 10b + c = 5(20a + 2b) + c$

$5(20a + 2b)$ is multiple of 5.

The given number is divisible by 5, only if the unit's digit $c = 0$ or 5

Do This :

1. Check whether the given numbers are divisible by 5 or not ?
   (a) 205 (b) 4560 (c) 402 (d) 105 (e) 235785

If 34A is divisible by 5, what might be the value of A?
In the given number the unit digit A is, either 0 or 5 then only it is divisible by 5.
Hence A = 0 or 5.

Try These :

1. If 4B ÷ 5 leaves remainder 1 , what might be the value of B
2. If 76C ÷ 5 leaves remainder 2 , what might be the value of C
3. "If a number is divisible by 10, it is also divisible by 5." Is the statement true? Give reasons.
4. "If a number is divisible by 5, it is also divisible by 10." Is the statement is true or false? Give reasons.

15.1.6 Divisibility by 2:

Take the multiples of 2 : i.e. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...etc

In these numbers the unit's digit ends with 0,2,4,6, 8 .

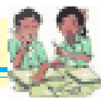
If the unit's digitof the number is 0 or 2 or 4 or 6 or 8 (even number) then it is divisible by 2. Otherwise it will not divisible by 2.

Let us see the logic behind this rule.

If we take a three digit number $100 \times a + 10 \times b + c$ where a in hundred's place, b is in ten's place and c is in unit's place , then it can be written as $100a + 10b + c = 2(50a + 5b) + c$

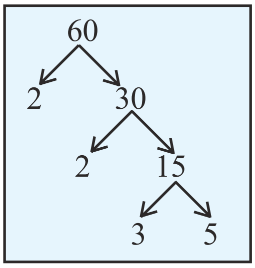
$2(50a + 5b)$ is multiple of 2. If the given number is divisible by 2, it is possible only if the unit's digit $c = 0$ or 2 or 4 or 6 or 8 (even number)

Think, Discuss and Write



1 . Find the digit in the units place of a number if it is divided by 5 and 2 leaves the remainders 3 and 1 respectively.

Example 1: Write the number of factors of 60 and verify by listing the factors



Solution: 60 can be written as product of prime factors as $2^2 \times 3^1 \times 5^1$

$\therefore$ Number of factors are $(2 + 1)(1 + 1)(1 + 1)$

$= 3 \times 2 \times 2 = 12$

The factors are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Exercise - 15.1

1. Using divisibility rules , find which of the following numbers are divisible by 2,5,10 (say yes or no) in the given table . What do you observe ?

Number	Divisible by 2	Divisible by 5	Divisible by 10
524	YES	NO	NO
1200			
535			
836			
780			
3005			
4820			
48630			

2. Using divisibility tests, determine which of following numbers are divisible by 2
   (a) 2144 (b) 1258 (c) 4336 (d) 633 (e) 1352
3. Using divisibility tests , determine which of the following numbers are divisible by 5
   (a) 438750 (b) 179015 (c) 125 (d) 639210 (e) 17852
4. Using divisibility tests, determine which of the following numbers are divisible by 10:
   (a) 54450 (b) 10800 (c) 7138965 (d) 7016930 (e) 10101010
5. Write the number of factors of the following?
   (a) 18 (b) 24 (c) 45 (d) 90 (e) 105
6. Write any 5 numbers which are divisible by 2, 5 and 10.
7. A number 34A is exactly divisible by 2 and leaves a remainder 1, when divided by 5, find A.

15.1.7 Divisibility by 3 and 9 :

Consider the number 378 , it can be written as 378 $= 300 + 70 + 8$

$= 100 \times 3 + 10 \times 7 + 8$

Here '3' can't be taken out as a common factor . $= (99 + 1)\ 3 + (9 + 1)7 + 8$

So let us reorganise the sequence as $378 = 99 \times 3 + 9 \times 7 + (3 + 7 + 8)$

$= 99 \times 3 + 3 \times 3 \times 7 + (3 + 7 + 8)$

$= 3\,(99 + 21) + (3 + 7 + 8)$

$3(99 + 21)$ is a multiple of 3. Therefore the given number is divisible by 3 only when $(3 + 7 + 8)$ sum of digits is a multiple of 3 .

For divisibility of 9:

378 can be written as

$$\begin{aligned} 378 &= 300 + 70 + 8 \\ &= 100 \times 3 + 10 \times 7 + 8 \\ &= (99 + 1)\,3 + (9 + 1)7 + 8 \\ &= 99 \times 3 + 9 \times 7 + (3 + 7 + 8) \\ &= 9\,(11 \times 3 + 1 \times 7) + (3 + 7 + 8) \\ &= 9\,(33 + 7) + (3 + 7 + 8) \end{aligned}$$

$9(33 + 7)$ is multiple of 9. if the given number is divisible by 9, then $(3 + 7 + 8)$,sum of digits is a multiple of 9.

Let us explain this rule :

If we take a three digit number $100a+10b+c$ where 'a' is in hundred's place , 'b' is in ten's place and 'c' is in unit's place.

$$100a + 10b + c = (99 + 1)a + (9 + 1) + c = 99a + 9b + (a + b + c)$$

$$= 9(11a + b) + (a + b + c) \rightarrow \text{sum of given digits}$$

$9(11a + b)$ multiple of 3 and 9 .The given number is divisible by 3 or 9 , only if the sum of the digits $(a + b + c)$ is multiple of 3 or 9 respectively or $(a+b+c)$ is divisibly by 3 or 9.

Is this divisibility rule applicable for the numbers having more than 3-digits? Check by taking 5-digits and 6-digits numbers.

You have noticed that divisibility of a number by 2,5 and 10 is decided by the nature of the digit in unit place, but divisibility by 3 and 9 depends upon other digits also.

Do This:

1. Check whether the given numbers which are divisible by 3 or 9 or by both?

(a) 3663	(b) 186	(c) 342	(d) 18871
(e) 120	(f) 3789	(g) 4542	(h) 5779782

Example 2: 24 P leaves remainder 1 if it is divided by 3 and leaves remainder 2 if it is divided by 5. Find the value of P.

Solution : If 24 P is divided by 5 and leaves remainder 2, then P is either 2 or 7.

If P = 2 the given number when divided by 3 leaves remainder 2. If P = 7, the given number when divided by 3, leaves remainder 1. Hence P = 7.

Exercise -15.2

1. If 345 A 7 is divisible by 3,supply the missing digit in place of 'A'.
2. If 2791 A , is divisible by 9, supply the missing digit in place of 'A'.
3. Write some numbers which are divisible by 2,3,5,9 and 10 also.
4. 2A8 is a number divisible by 2 , what might be the value of A?
5. 50B is a number divisible by 5, what might be the value of B ?
6. 2P is a number which is divisible by 2 and 3, what is the value of P
7. 54Z leaves remainder 2 when divided by 5 , and leaves remainder 1 when divided by 3, what is the value of Z ?
8. 27Q leaves remainder 3 when divided by 5 and leaves remainder 1 when divided by 2, what is the remainder when it is divided by 3 ?

15.1.8 Divisibility by 6 :

Consider a multiple of 6, say 24.

Obviously it is divisible by 6.

Is 24 divisible by the factors of 6 , i.e 2 and 3 ?

Units place of 24 is 4, so it is divisible by 2.

Sum of digits of 24 is 2 + 4 = 6 which is divisible by 3 also.

Now check this with some other multiple of 6.

Now we can conclude that any number divisible by 6 is also divisible by the factors of 6. i.e 2 and 3.

Let us check the converse of the statement.

If a number is divisible by 2 then 2 is its prime factor. If it is divisible by 3 then 3 is its prime factor.

So if the number is divisible by 2 and 3 , then 2 and 3 become its prime factors, then their product $2 \times 3 = 6$ is also a factor of that number.

In other words if a number is divisible by 6, it has to be divisible by 2 and 3.

Do These :

1. Check whether the given numbers are divisible by 6 or not ?

   (a) 1632 (b) 456 (c) 1008 (d) 789 (e) 369 (f) 258

2. Check whether the given numbers are divisible by 6 or not ?

   (a) $458 + 676$ (b) 6^3 (c) $6^2 + 6^3$ (d) $2^2 \times 3^2$

15.1.9 Divisibility by 4 and 8 :

(a) Take a four digit number say $1000a + 100b + 10c + d = 4(250a + 25b) + (10c + d)$. $4(250a + 25b)$ is a multiple of 4.The given number is divisible by 4, only if 10c+d is divisible by 4 .

In a given number if last two digits are divided by 4 or last two digits are '0' then that number is divisible by 4.

Take a number having more than 4 digits and write in expanded form. Can we write the number other than unit digit and ten's digit as multiple of 4?

Check for a number having more than 4 digits, divisibility of 4 depends upon its last two digits or not.

(b) Take a four digit number $1000 \times a + 100 \times b + 10 \times c + d$

$= 1000a + 100b + 10c + d = 8(125a) + (100b + 10c + d)$

$8(125a)$ is always divisible by 8. So the given number is divisible by 8 only when (100b+10c+d) is divisibly by 8.

In a given number if the number formed with its last 3 digits are divisible by 8 or last 3 digits are '0's then that number is divisible by 8.

Take a number having more than 4 digits and write the number in expanded form. Can we write the number other than unit's digit ten's digit and hundred's digit as multiple of 8.

Check for a number having more than 4 digits, divisibility of 8 is depends upon its last three digits or not.

Can you arrange the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 in an order so that the number formed by first two digits is divisible by 2, the number formed by first three digits is divisible by 3, the number formed by first four digits is divisible by 4 and so on upto nine digits?

Solution : The order 123654987 looks promising check and verify.

Example 3: Check whether 6582 is divisible by 4 ?

Solution: The number formed by last two digits is 82, is not divisible by 4. Hence the given number is not divisible by 4.

Example 4: Check whether 28765432 is divisible by 8 ?

Solution : The number formed by last three digits is 432 is divisible by 8, hence it is divisible by 8.

If a number is divisible by 8, then it is divisible by 4 also. Can you say if a number divisible by 4 is it divisible by 8? All multiples of 8 are divisible by 4, but all multiples of 4 may not be divisible by 8.

Do This:

1. Check whether the given numbers are divisible by 4 or 8 or by both 4 and 8?

(a) 464 (b) 782 (c) 3688 (d) 100

(e) 1000 (f) 387856 (g) 4^4 (h) 8^3

Try This :

1. Check whether the given numbers are divisible by 4 or 8 or by both 4 and 8 ?

(a) $4^2 \times 8^2$ (b) 10^3 (c) $10^5 + 10^4 + 10^3$ (d) $4^3 + 4^2 + 4^1 - 2^2$

15.1.10 Divisibility by 7:

Take a three digit number $100 \times a + 10 \times b + c$ can be written as

$$100a + 10b + c = 98a + 7b + (2a + 3b + c)$$

Here 7 is not a common factor , let us re write it in a way that 7 becomes a common factor.

$$= 7(14a + b) + (2a + 3b + c)$$

$7(14a + b)$ is multiple of '7'.The given number is divisible by 7 only when $(2a + 3b + c)$ is divisible by 7.

Example 5: Check whether 364 is divisible by 7 or not ?

Solution : Here $a = 3, b = 6, c = 4, (2a + 3b + c) = 2 \times 3 + 3 \times 6 + 4$

$$= 6 + 18 + 4 = 28 \text{ (is divisible by 7)}$$

Hence, the given number is divisible by '7'

Do This:

1. Check whether the given numbers are divisible by 7?

(a) 322 (b) 588 (c) 952 (d) 553 (e) 448

Try These :

1 Take a four digit general number , make the divisibility rule for '7'

2. Check your rule with the number 3192 which is a mulltiple of 7 .

15.1.11 Divisibility by 11 :

Take a 5 digit number 10000a + 1000b + 100c + 10d + e

Here 11 can't be taken out as a common factor. So let us reorganise the expansion as

$$= (9999 + 1)\,a + (1001 - 1)\,b + (99 + 1)\,c + (11 - 1)\,d + e$$

$$= 9999a + 1001\,b + 99c + 11d + a - b + c - d + e$$

$$= 11\,(909a + 91b + 9c + d) + (a + c + e) - (b + d)$$

11 (909a + 91b + 9c + d) is always divisible by 11.

So the given number is divisible by 11 only if (a + c + e) – (b + d) is divisible by 11.

i.e (a + c + e) – (b + d) is a multiple of 11 or equal to zero.

If the difference between the sum of digits in odd places (a + c + e) and sum of digits in even places (b + d) of a number is a multiple of 11 or equal to zero, then the given number is divisible by 11.

Observe the following table

Number	Sum of the digits at odd places (from the left)	Sum of the digits at even places (from the left)	Difference
308	3 + 8 = 11	0	11 – 0 = 11
1331	1 + 3 = 4	3 + 1 = 4	4 – 4 = 0
61809	6 + 8 + 9 = 23	1 + 0 = 1	23 – 1 = 22

We observe that in each case the difference is either 0 or divisible by 11. Hence all these numbers are divisible by 11.

For the number 5081, the difference of the digits of odd places and even places is $(5+8)-(0+1) = 12$ which is not divisible by 11. Therefore the number 5081 is not divisible by 11.

Do This:

1. Check whether the given numbers are divisible by 11.

   (i) 4867216 (ii) 12221 (iii) 100001

Consider a 3 digit number 123.

Write it two times to make a number as 123123.

Now what is the sum of digits in odd places from left? $1+3+2=6$

What is the sum of digits in even places from the right?

$2+1+3=6$

what is the difference between these sums? Zero. Hence 123123 is divisible by 11.

Take any 3 digits number and make a number by writing it two times. It is exactly divisible by 11.

Try These :

1. Verify whether 789789 is divisible by 11 or not.
2. Verify whether 348348348348 is divisible by 11 or not?
3. Take an even palindrome i.e. 135531 check whether this number is divisible by 11 or not?
4. Verify whether 1234321 is divisible by 11 or not?

Exercise - 15.3

1. Check whether the given numbers are divisible by '6' or not ?

   (a) 273432 (b) 100533 (c) 784076 (d) 24684
2. Check whether the given numbers are divisible by '4' or not ?

   (a) 3024 (b) 1000 (c) 412 (d) 56240
3. Check whether the given numbers are divisible by '8' or not ?

   (a) 4808 (b) 1324 (c) 1000 (d) 76728

4. Check whether the given numbers are divisible by '7' or not ?

(a) 427 (b) 3514 (c) 861 (d) 4676

5. Check whether the given numbers are divisible by '11' or not ?

(a) 786764 (b) 536393 (c) 110011 (d) 1210121

(e) 758043 (f) 8338472 (g) 54678 (i) 13431

(j) 423423 (k) 168861

6. If a number is divisible by '8', then it also divisible by '4' also . Explain ?

7. A 3-digit number 4A3 is added to another 3-digit number 984 to give four digit number 13B7, which is divisible by 11. Find (A + B).

15.2 Some More Divisibility Rules

(a) Let us observe a few more rules about the divisibility of numbers.

Consider a factor of 24 , say 12.

Factors of 12 are 1,2,3,4,6,12

Let us check whether 24 is divisible by 2,3,4,6 we can say that 24 is divisible by all factors of 12.

So, if a number 'a' is divisible by another number 'b', then it is divisible by each of the factors of that number 'b'.

(b) Consider the number 80. It is divisible by 4 and 5. It is also divisible by $4 \times 5 = 20$, where 4 and 5 are co primes to each other. (have no common factors for 4 and 5)

Similarly, 60 is divisible by 3 and 5 which have no common factors each other 60 is also divisible by $3 \times 5 = 15$.

If 'a' and 'b' have no common factors (other than unity), the number divisible by 'a' and 'b' is also divisible by $a \times b$

(Check the property if 'a' and 'b' are not co-primes).

(c) Take two numbers 16 and 20. These numbers are both divisible by 4. The number $16 + 20 = 36$ is also divisible by 4.

Try this for other common divisors of 16 and 20.

Check this for any other pairs of numbers.

If two given numbers are divisible by a number, then their sum is also divisible by that number.

(d) Take two numbers 35 and 20. These numbers are both divisible by 5. Is their difference $35 - 20 = 15$ also divisible by 5? Try this for other pairs of numbers also.

If two given numbers are divisible by a number, then their difference is also divisible by that number.

Do These :

1. Take different pairs of numbers and check the above four rules for given number
2. 144 is divisible by 12. Is it divisible by the factors of 12 ? verify .
3. Check whether $2^3 + 2^4 + 2^5$ is divisible by 2 ? Explain
4. Check whether $3^3 - 3^2$ is divisible by 3 ? Explain

Consider a number, product of three consecutive numbers i.e. $4 \times 5 \times 6 = 120$. This is divisible by 3. Because in these consecutive numbers one number is multiple of 3. Similarly if we take product of any three consecutive numbers among those one number is multiple of 3. Hence product of three consecutive is always divisible by 3.

Try This :

1. Check whether $1576 \times 1577 \times 1578$ is divisible by 3 or not.

Divisibility Rule of 7 for larger numbers

We discussed the divisibility of 7 for 3-digit numbers. If the number of digits of a number are more than 3 we make it simple to find divisibility of 7.

Check a number 7538876849 is divisible by 7 or not.

Step 1 : Make the number into groups of 3-digits each from right to left. If the left most group is less than 3 digits take it as group.

7 | 538 | 876 | 849 |
D C B A

Step 2 : Add the groups in alternate places i.e. A + C and B + D.

$$\begin{array}{r} 849 \\ +\ 538 \\ \hline 1387 \\ \hline \end{array} \qquad \begin{array}{r} 876 \\ +\ \ \ 7 \\ \hline 883 \\ \hline \end{array}$$

Step 3 : Subtract 883 from 1387 and check the divisibility rule of 7 for the resultant 3 digit number as previously learnt

$$\begin{array}{r} 1387 \\ -\ 883 \\ \hline 504 \\ \hline \end{array}$$

By divisibility rule of 7 we know that 504 is divisible by 7.

Hence the given number is divisible by 7.

Try This :

1. Check the above method applicable for the divisibility of 11 by taking 10-digit number.

By using the divisibility rules , we can guess the missing digit in the given number. Suppose a number 84763A9 is divisible by 3, we can guess the value for sum of digits is

8 + 4 + 7 + 6 + 3 + A + 9 = 37 + A. To divisible by 3 , A has values either 2 or 5 or 8.

Exercise - 15.4

1. Check whether 25110 is divisible by 45.
2. Check whether 61479 is divisible by 81 .
3. Check whether 864 is divisible by 36? Verify whether 864 is divisible by all the factors of 36 ?
4. Check whether 756 is divisible by 42? Verify whether 756 is divisible by all the factors of 42 ?
5. Check whether 2156 is divisible by 11 and 7? Verify whether 2156 is divisible by product of 11 and 7 ?
6. Check whether 1435 is divisible by 5 and 7? Verify if 1435 is divisible by the product of 5 and 7 ?

7. Check whether 456 and 618 are divisible by 6? Also check whether 6 divides the sum of 456 and 618 ?

8. Check whether 876 and 345 are divisible by 3? Also check whether 3 divides the difference of 876 and 345 ?

9. Check whether $2^2+2^3+2^4$ is divisible by 2 or 4 or by both 2 and 4 ?

10. Check whether 32^2 is divisible by 4 or 8 or by both 4 and 8 ?

11. If A679B is a 5-dit number is divisible by 72 find 'A' and 'B'?

15.3 Puzzles based on divisibility rules :

Raju and Sudha are playing with numbers . Their conversation is as follows :

Sudha said , let me ask you a question.

Sudha : Choose a 2- digit number

Raju : Ok . I choose. (He choose 75)

Sudha : Reverse the digits (to get a new number)

Raju : Ok .

Sudha : Add this to the number you choosen

Raju : Ok . (I did)

Sudha : Now divide your answer with 11, you will get the remainder zero.

Raju : Yes . but how do you know ?

Can you think why this happens ?

Now let us understand the logic behind the Sudha's trick

Suppose Raju chooses the number 10a + b (such that "a" is a digit in tens place and "b" is a digit in units place and $a \neq 0$) can be written as $10 \times a + b = 10a + b$ and on reversing the digits he gets the number 10b + a. When he adds the two numbers he gets $(10a + b) + (10b + a) = 11a + 11b = 11(a + b)$

The sum is always multiple of 11. Observe that if she divides the sum by 11 , the quotient is (a + b), which is exactly the sum of digits a and b of chosen number.

You may check the same by taking any other two digit number .

Do These :

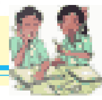
1. Check the result if the numbers chosen were

   (i) 37 (ii) 60 (iii) 18 (iv) 89

2. In a cricket team there are 11 players. The selection board purchased $10x + y$ T-Shirts to players. They again purchased '$10y + x$ T-Shirts and total T-Shirts were distributed to players equally. How many T-Shirts will be left over after they distrubuted equally to 11 players?

   How many each one will get?

Think, Discuss and Write:



Take a two digit number reverse the digits and get another number. Subtract smaller number from bigger number. Is the difference of those two numbers is always divisible by 9?

Do This:

1. In a basket there are '$10a + b$ fruits. ($a \neq 0$ and $a > b$). Among them '$10b + a$ 'fruits are rotten. The remaining fruits distributed to 9 persons equally. How many fruits are left over after equal distribution? How many fruits would each child get ?

15.4 Fun with 3- Digit Numbers:

Sudha : Now think of a 3- digit number.

Raju : Ok . (he chooses 157)

Sudha : Reverse the digits and subtract smaller number from the larger number

Raju : Ok.

Sudha : Divide your answer with 9 or 11. I am sure there will be no remainder .

Raju : Yes. How would you know ?

Right ! How does Sudha know ?

We can derive the logic the way we did for the 3-digit number $100a + 10b + c$.
By reversing the digits she get $100c + 10b + a$.

If (a > c) difference between the numbers is (100a + 10b + c) - (100c + 10b + a)

$= 99a - 99c = 99(a - c) = 9\times 11\times(a - c)$

If (c > a) difference between the numbers is (100c + 10b + a) – (100a + 10b + a)

$= 99c - 99a = 99(c - a) = 9\times 11\times(c - a)$

And if a = c , then the difference is ‘0’

In each case the result is a multiple of 99. Therefore, it is divisible by both 9 and 11, and the quotient is (a – c) or (c – a).

Do This:

1. Check in the above activity with the following numbers ?

(i) 657 (ii) 473 (iii) 167 (iv) 135

Try This:

Take a three digit number and make the new numbers by replacing its digits as (ABC, BCA, CAB). Now add these three numbers. For what numbers the sum of these three numbers is divisible?

15.5 Puzzles with missing digits

We can also have some puzzles in which we have alphabet in place of digits in an arithmetic sum and the task is to find out which alphabet represents which digit . let us do some problems of addition and multiplication.

The three conditions for the puzzles.

1. Each letter of the puzzle must stand for just one digit . Each digit must be represented by just one letter.
2. The digit with highest place value of the number can not be zero.
3. The puzzle must have only one answer.

Example 6: Find A in the addition

$$\begin{array}{r} 17A \\ +\ 2A4 \\ \hline 407 \\ \hline \end{array}$$

Solution : By observation $A + 4 = 7$. Hence $A = 3$

$173 + 234 = 407$

or

$$\begin{array}{r} 100 + 70 \; + A \\ 200 + 10A + 4 \\ \hline 300 + 70 + 11A + 4 = 407 \\ 11A = 33 \\ A = 3 \end{array}$$

Example 7 : Find M and Y in the addition $Y + Y + Y = MY$

Solution : $Y + Y + Y = MY$

$3Y = 10M + Y$

$2Y = 10M$

$M = \dfrac{Y}{5}$ (i.e. Y is divisible by 5. Hence $Y = 0$ or 5)

From above, if $Y = 0$, $Y + Y + Y = 0 + 0 + 0 = 0$, $M = 0$

if $Y = 5$, $Y + Y + Y = 5 + 5 + 5 = 15$, $MY = 15$ Hence $M = 1$, $Y = 5$

Example 8: $A2 - 15 = 5A$,

Solution : $2 - 5 = a$ is possible when $12 - 5 = 7$, There fore $A = 7$

or

$(10A + 2) - (10 + 5) = 50 + A$

$10A - 13 = 50 + A$

$9A = 63$

$A = 7$

Example 9: $5A1 - 23A = 325$

Solution : $1 - A = 5$? i.e. $11 - A = 5$, There fore $A = 6$

or

$(500 + 10A + 1) - (200 + 30 + A) = 325$

$501 - 230 + 10A - A = 325$

$271 + 9A = 325$

$271 + 9A = 325$

$271 - 271 + 9A = 325 - 271$

$9A = 54$

$A = 6$

Example 10: $1A \times A = 9A$

Solution : For $A \times A = A$ or $(10 + A)\,A = (90 + A)$

From square tables 1, 5, 6

$1\times1 = 1,$

$5\times5 = 25,$

$6\times6 = 36,$

if $A = 6,$

$16\times6 = 96$

$$(10 + A)\,A = (90 + A)$$
$$10A + A^2 = 90 + A$$
$$A^2 + 9A - 90 = 0$$
$$A^2 + 2.A\left(\frac{9}{2}\right) + \left(\frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 - 90 = 0$$
$$\left(A + \frac{9}{2}\right)^2 - \frac{81}{4} - 90 = 0$$
$$\left(A + \frac{9}{2}\right)^2 = \frac{441}{4}$$
$$A + \frac{9}{2} = \frac{21}{2}$$
$$A = \frac{12}{2} = 6$$

Example 11 : $BA \times B3 = 57A.$

Solution : In this example we estimate the value of digits from multiplication tables by trial and error method. In one's place $A \times 3 = A$. For $A = 0$ or 5, the unit digit of product becomes same digit. Hence A is either 0 or 5. If we take 1 at tens place then, to the utmost value of two digit number is 19. The product could be $19\times19 = 361$. Which is less than 500. Further if we take 3 at tens place then the atleast value of both two digit number will be $30\times30 = 900$ which is greater than 500. So, it will be 2 at tens place. Then $20\times23 = 460$ or $25\times23 = 575$.

Hence, the required answer is $25\times23 = 575$.

Do These :

1. If 21358AB is divisible by 99 , find the values of A and B
2. Find the value of A and B of the number 4AB8 (A , B are digits) which is divisible by 2,3,4,6,8 and 9.

Example 12: Find the value of the letters in the given multiplication

$$\begin{array}{r} AB \\ \times\ 5 \\ \hline CAB \end{array}$$

Solution : If we take $B = 0$ or 1 or 5 then $0 \times 5 = 0$, $1 \times 5 = 5$, $5\times 5 = 25$

If $B = 0$, then $A0 \times 5 = CA0$

then if we take A = 5 , then $50 \times 5 = 250$

$\therefore$ CAB = 250 .

Try These :

1. If YE $\times$ ME = TTT find the numerical value of Y + E + M + T

   [Hint : TTT = 100T + 10T + T = T(111) = T(37 $\times$ 3)]

2. If cost of 88 articles is A733B . find the value of A and B

Exercise -15. 5

1. Find the missing digits in the following additions.

(a)
$$\begin{array}{r} 111 \\ +\quad A \\ +\ 77 \\ \hline 197 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 222 \\ +\quad 8 \\ +\ BB \\ \hline 285 \\ \hline \end{array}$$

(c)
$$\begin{array}{r} AAA \\ +\quad 7 \\ +\ AA \\ \hline 373 \\ \hline \end{array}$$

(d)
$$\begin{array}{r} 2222 \\ +\ 99 \\ +\quad 9 \\ AAA \\ \hline 299A \\ \hline \end{array}$$

(e)
$$\begin{array}{r} BB \\ +\quad 6 \\ AAA \\ \hline 461 \\ \hline \end{array}$$

2. Find the value of A in the following

   (a) 7A – 16 = A9 (b) 107 – A9 = 1A (c) A36 – 1A4 = 742

3. Find the numerical value of the letters given below-

(a)
$$\begin{array}{r} \boxed{D}\,\boxed{E} \\ \times\ 3 \\ \hline \boxed{F}\,\boxed{D}\,\boxed{E} \\ \hline \end{array}$$

(b)
$$\begin{array}{r} \boxed{G}\,\boxed{H} \\ \times\ 6 \\ \hline \boxed{C}\,\boxed{G}\,\boxed{H} \\ \hline \end{array}$$

4. Replace the letters with appropriate digits

   (a) 73K $\div$ 8 = 9L (b) 1MN $\div$ 3 = MN

5. If ABB $\times$ 999 = ABC123 (where A, B, C are digits) find the values of A, B, C.

15.6 Finding of divisibility by taking remainders of place values

In this method we take remainders by dividing the place values, with given number.

If we divide the place values of a number by 7, we get the remainders as

1000 $\div$ 7 (Remainder 6. This can be taken as 6 – 7 = –1)

100 $\div$ 7 (Remainder 2)

10 $\div$ 7 (Remainder 3)

1 $\div$ 7 (Remainder 1)

Place value	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
Remainders divide by 7	3	2	1	–2	–3	–1	2	3	1

Suppose to check whether 562499 is divisible by 7 or not.

Digits	5	6	2	4	9	9
Place values	5×10^5	6×10^4	2×10^3	4×10^2	9×10^1	9×10^0
Remainders divided by 7	$5\times(-2)$	$6\times(-3)$	$2\times(-1)$	4×2	9×3	9×1

Sum of product of face values and remainders of place values is

$-10 - 18 - 2 + 8 + 27 + 9 = -30 + 44 = 14$(divisible by 7)

Hence 562499 is divisible by 7.

Do These :

1. By using the above method check whether 7810364 is divisible by 4 or not.
2. By using the above method check whether 963451 is divisible by 6 or not.

15.7 Some more puzzles on divisibility rules :

Example 13: Is every even number of palindrome is divisible by '11' ?

Solution: Let us take an even number of palindrome i.e. 12344321. The sum of digits in odd places is 1 + 3 + 4 + 2. Sum of digits in even places 2 + 4 + 3 + 1. Their difference is 0. Hence, it is divisible by 11.

Example 14: Is 10^{1000}-1 divisible by both 9 and 11?

Solution: Let us write 10^{1000}-1 as 999 .... 999 (1000 times). The digits in all places are 9. Hence, it is divisible by 9. And there are 1000 digits. Sum of digits in odd places and sum digits even places are same. Their difference is 0. Hence, it is divisible by 11.

Think, Discuss and Write:

1. Can we conclude 10^{2n}-1 is divisible by both 9 and 11? Explain.
2. Is 10^{2n+1}-1 is divisible by 11 or not. Explain.

Example 15: Take any two digit number three times to make a 6-digit number. Is it divisible by 3 ?

Solution: Let us take a 2-digit number 47. Write three times to make 6-digit number i.e. 474747.

474747 can be written as 47(10101). 10101 is divisible by 3. Because sum of its digit is 1 + 1 + 1 = 3. Hence 474747 is divisible by 3.

Example 16: Take any three digit number and write it two times to make a 6-digit number. Verify whether it is divisible by both 7 and 11.

Solution: Let us take a 3-digit number 345. Write it two times to get 6-digit number i.e. 345345.

345345 can be written as $345345 = 345000 + 345 = 345\ (1000 + 1)$

$= 345\ (1001)$

$= 345\ (7 \times 11 \times 13)$

Hence 345345 is divisible by 7, 11 and 13 also.

Try This :

1. Check whether 456456456456 is divisible by 7, 11 and 13?

Example 17: Take a three digit number in which all digits are same. Divide the number with reduced number. What do you notice?

Solution: Consider 444. Reduced number of 444 is 4 + 4 + 4 = 12

Now divide 444 by 12, $444 \div 12 = 37$. Do the process with 333 , 666,etc.

You will be supposed the quotient is 37 for all the numbers .

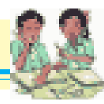
Example 18: Is $2^3 + 3^3$ is divisible by (2 + 3) or not?

Solution: We know that $a^3 + b^3 = (a + b)\ (a^2 - ab + b^2)$.

So $2^3 + 3^3 = (2 + 3)\ (2^2 - 2 \times 3 + 3^2)$. It is multiple of (2 + 3).

Hence $2^3 + 3^3$ is divisible by (2 + 3).

Think, Discuss and Write:



1. Verify $a^5 + b^5$ is divisible by (a + b) by taking different natural numbers for ‘a’ and ‘b’?
2. Can we conclude $(a^{2n+1} + b^{2n+1})$ is divisible by (a + b)?

15.8 Finding Sum of Consecutive numbers :

We can find the sum of consecutive numbers from 1 to 100 without adding.

$1 + 2 + 3 + \ldots + 50 + 51 + \ldots + 98 + 99 + 100$

$= (1 + 100) + (2 + 99) + (3 + 98) \ldots\ldots (50 + 51)$

$= 101 + 101 + 101 + \ldots\ldots\ldots$ 50 pairs are there. $= 50 \times 101 = 5050$

This can be written as $\frac{100\times 101}{2} = 5050$.

What is the sum of first 48 natural numbers? What do you observe ?

What is the sum of first 'n' natural numbers? It is $\frac{n(n+1)}{2}$ (verify)

Example 19: Find the sum of integers which are divisible by 5 from 50 to 85.

Solution: Sum of integers which are divisible by 5 from 50 to 85 = (Sum of integers which are divisble by 5 from 1 to 85) – (Sum of integers which are divisble by 5 from 1 to 49)

$= (5 + 10 + \ldots + 85) - (5 + 10 + \ldots + 45)$

$= 5(1 + 2 + \ldots + 17) - 5(1 + 2 + \ldots + 9)$

$= 5\times\left(\frac{17\times \cancel{18}^{9}}{\cancel{2}}\right) - 5\times\left(\frac{9\times \cancel{10}^{5}}{\cancel{2}}\right)$

$= 5\times 9\times 17 - 5\times 9\times 5$

$= 5\times 9\times (17 - 5)$

$= 5\times 9\times 12 = 540$

Example 20: Find the sum of integers from 1 to 100 which are divisible by 2 or 3.

Solution: The numbers which are divisible by 2 from 1 to 100 are 2, 4, ... 98, 100.

The numbers which are divisible by 3 from 1 to 100 are 3, 6, ... 96, 99.

In the above series some numbers are repeated twice. Those are multiple of 6 i.e. LCM of 2 and 3.

Sum of integers which are divisible by 2 or 3 from 1 to 100 = (Sum of integers which are divisble by 2 from 1 to 100) + (Sum of integers which are divisble by 3 from 1 to 100) – (Sum of integers which are divisble by 6 from 1 to 100)

$= (2 + 4 + \ldots + 100) + (3 + 6 + \ldots + 99) - (6 + 12 + \ldots 96)$

$= 2(1 + 2 + \ldots + 50) + 3(1 + 2 + \ldots + 33) - 6(1 + 2 + \ldots + 16)$

$= 2\times\left(\frac{50\times(50+1)}{2}\right) + 3\times\left(\frac{33\times(33+1)}{2}\right) - 6\times\left(\frac{16\times(16+1)}{2}\right)$

$$= \cancel{2} \times \left(\frac{50 \times 51}{\cancel{2}}\right) + 3 \times \left(\frac{33 \times \cancel{34}^{17}}{\cancel{2}}\right) - 6 \times \left(\frac{{}^{8}\cancel{16} \times 17}{\cancel{2}}\right)$$

$$= 2550 + 1683 - 816$$

$$= 4233 - 816 = 3417$$

Exercise – 15.6

1. Find the sum of integers which are divisible by 5 from 1 to 100.
2. Find the sum of integers which are divisible by 2 from 11 to 50.
3. Find the sum of integers which are divisible by 2 and 3 from 1 to 50.
4. $(n^3 - n)$ is divisible by 3. Explain the reason.
5. Sum of 'n' odd number of consecutive numbers is divisible by 'n'. Explain the reason.
6. Is $1^{11} + 2^{11} + 3^{11} + 4^{11}$ divisible by 5? Explain.
7. 

   Find the number of rectangles of the given figure ?
8. Rahul's father wants to deposit some amount of money every year on the day of Rahul's birthday. On his 1st birth day Rs.100, on his 2nd birth day Rs.300, on his 3rd birth day Rs.600, on his 4th birthday Rs.1000 and so on. What is the amount deposited by his father on Rahul's 15th birthday.
9. Find the sum of integers from 1 to 100 which are divisible by 2 or 5.
10. Find the sum of integers from 11 to 1000 which are divisible by 3.

What we have discussed

1. Writing and understanding a 3-digit number in expanded form $100a + 10b + c$. Where a, b, c digits $a \neq 0$, b, c is from 0 to 9
2. Deducing the divisibility test rules of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 for two or three digit number expressed in the general form.
3. Logic behind the divisibility laws of 2, 3, 4, 5, 6, 7,8, 9, 10, 11.
4. Number puzzles and games.