Chapter **2**

Linear Equations in One Variable

2.0 Introduction

Sagar and Latha are playing with numbers. Sagar tells Latha "I think of a number. If I double it and take 7 away I get 35. Can you tell the number that I thought of" ?

Latha thinks for a while and tells the answer. Can you too tell the answer?

Let us see how Latha told the answer.

Let the number be 'x'. By doubling it we get '$2x$'

Next 7 was taken away i.e., 7 was subtracted from '$2x$'. After subtraction the resulting number is $2x-7$

But according to Sagar it is equal to 35

$\Rightarrow \quad 2x - 7 = 35$

$\therefore \quad 2x = 35 + 7$ (Transposing 7 to RHS)

$2x = 42$

$\therefore \quad x = \dfrac{42}{2}$ (Transposing 2 to RHS)

$\therefore \quad x = 21$

$\therefore$ The number that Sagar thought of is 21.

Trick
Take the final result. Add 7 to it and then halve the result.

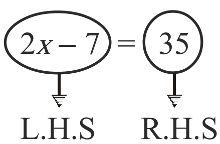
Note
When we transpose terms
'+' quantity becomes '–' quantity
'–' quantity becomes '+' quantity
'×' quantity becomes '÷' quantity
'÷' quantity becomes '×' quantity

We learnt in earlier classes that $2x - 7 = 35$ is an example of an equation. By solving this equation in the above method, Latha was able to find the number that Sagar thought of.

In this chapter we will discuss about linear equations in one variable or simple equations, technique of solving such equations and its application in daily life problems.

Let us briefly revise what we know about equations:

(i) An algebraic equation is equality of algebraic expressions involving variables and constants



(ii) It has an equality sign

(iii) The expression on the left of the equality sign is called the L.H.S (Left Hand Side) of the equation

(iv) The expression on the right of the equality sign is called R.H.S (Right Hand Side) of the equation

(v) In an equation, the values of LHS and RHS are equal. This happens to be true only for certain value of the Variable. This value is called the solution of the equation.

$2x - 7 = 35$ is true for $x = 21$ only

i.e., if $x = 21$

$\text{LHS} = 2x - 7$

$= 2 \times 21 - 7$

$= 35$

$= \text{RHS}$

$\therefore$ Solution is $x = 21$

2.1 Linear Equations

Consider the following equations:

(1) $2x - 7 = 35$ (2) $2x + 2y = 48$ (3) $4x - 1 = 2x + 5$ (4) $x^2 + y = z$

Degree of each equation (1), (2) and (3) is one. So they are called linear equations. While degree of equation (4) is not one. So it is not a linear equation.

So equations (1), (2) and (3) are examples of linear equations. Since the degree of the fourth equation is not one, it is not a linear equation.

Do This:

Which of the following are linear equations:

(i) $4x + 6 = 8$ (ii) $4x - 5y = 9$ (iii) $5x^2 + 6xy - 4y^2 = 16$

(iv) $xy + yz + zx = 11$ (v) $3x + 2y - 6 = 0$ (vi) $3 = 2x + y$

(vii) $7p + 6q + 13s = 11$

2.2 Simple equations or Linear equations in one variable:

Consider the following equations:

(i) $2x - 7 = 35$ (ii) $4x - 1 = 2x + 5$ (iii) $2x + 2y = 48$

We have just learnt that these are examples of linear equations. Observe the number of variable in each equation.

(i) and (ii) are examples of linear equations in one variable. But the (iii) equation has two variables 'x' and 'y'. So this is called linear equation in two variables.

Thus an equation of the form $ax + b = 0$ or $ax = b$ where a, b are constants and $a \neq 0$ is called linear equation in one variable or simple equation.

Do This:

Which of the following are simple equations?

(i) $3x + 5 = 14$ (ii) $3x - 6 = x + 2$ (iii) $3 = 2x + y$

(iv) $\frac{x}{3} + 5 = 0$ (v) $x^2 + 5x + 3 = 0$ (vi) $5m - 6n = 0$

(vii) $7p + 6q + 13s = 11$ (viii) $13t - 26 = 39$

2.3 Solving Simple equation having the variable on one side

Let us recall the technique of solving simple equations (having the variable on one side). Using the same technique Latha was able to solve the puzzle and tell the number that Sagar thought of.

Example 1: Solve the equation $3y + 39 = 8$

Solution: Given equation : $3y + 39 = 8$

$3y = 8 - 39$ (Transposing 39 to RHS)

$3y = -31$

$y = \frac{-31}{3}$ (Transposing 3 to RHS)

$\therefore$ The solution of $3y + 39 = 8$ is $y = \frac{-31}{3}$

Do you notice that the solution ($\frac{-31}{3}$) is a rational number?

Say True or false? Justify your answer?

While solving an equation Kavya does the following:

$3x + x + 5x = 72$

$9x = 72$ $x = 72 \times 9 = 648$

Where has she gone wrong? Find the correct answer?

Check: LHS $= 3y + 39 = 3\left(\frac{-31}{3}\right) + 39 = -31 + 39 = 8$ RHS

Example 2: Solve $\frac{7}{4} - p = 11$

Solution: $\frac{7}{4} - p = 11$

$$-p = 11 - \frac{7}{4} \qquad \text{(Transposing } \frac{7}{4} \text{ to RHS)}$$

$$-p = \frac{44-7}{4}$$

$$-p = \frac{37}{4}$$

$$\therefore p = -\frac{37}{4} \qquad \text{(Multiplying both sides by } -1\text{)}$$

Transpose p from LHS to RHS and find the value of p.

Is there any change in the value of p?

Check: LHS = $\frac{7}{4} - p = \frac{7}{4} - (-\frac{37}{4}) = \frac{7}{4} + \frac{37}{4} = \frac{7+37}{4} = \frac{44}{4} = 11 = \text{RHS}$

Exercise - 2.1

Solve the following Simple Equations:

(i) $6m = 12$ (ii) $14p = -42$

(iii) $-5y = 30$ (iv) $-2x = -12$

(v) $34x = -51$ (vi) $\frac{n}{7} = -3$

(vii) $\frac{2x}{3} = 18$ (viii) $3x + 1 = 16$

(ix) $3p - 7 = 0$ (x) $13 - 6n = 7$

(xi) $200y - 51 = 49$ (xii) $11n + 1 = 1$

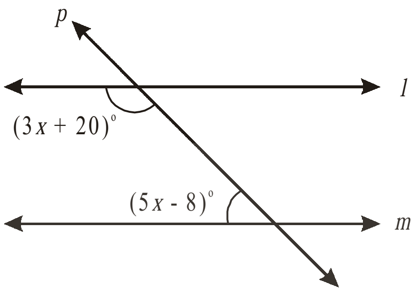
(xiii) $7x - 9 = 16$ (xiv) $8x + \frac{5}{2} = 13$

(xv) $4x - \frac{5}{3} = 9$ (xvi) $x + \frac{4}{3} = 3\frac{1}{2}$

2.3.1 Some Applications:

Consider the following examples:

Example 3: If $l \parallel m$, find the value of 'x'?



Solution: Here $l \parallel m$ and p is transversal.

Therefore $(3x+20)^o + (5x-8)^o = 180^o$ (sum of the interior angles on the same side of a transversal)

$3x + 20^o + 5x - 8^o = 180^o$

$8x + 12^o = 180^o$

$8x = 180^o - 12^o$

$8x = 168^o$

$x = \frac{168^o}{8} = 21^o$

Example 4: Sum of two numbers is 29 and one number exceeds another by 5. Find the numbers.

Solution: We have a puzzle here. We don't know the numbers. We have to find them.

Let the smaller number be 'x', then the bigger number will be '$x + 5$'.

But it is given that sum of these two numbers is 29

$\Rightarrow x + x + 5 = 29$

$\Rightarrow 2x + 5 = 29$

$\therefore 2x = 29 - 5$

$\therefore 2x = 24$

$x = \frac{24}{2}$ (Transposing '2' to RHS)

$x = 12.$

Therefore smaller number = x = 12 and

Bigger number = $x + 5 = 12 + 5 = 17$.

Check: 17 exceeds 12 by 5 and their sum = 12 + 17 = 29.

Example 5: Four times a number reduced by 5 equals 19. Find the number.

Solutions: If the number is taken to be 'x'

Then four times of the number is '$4x$'

When it is reduced by 5 it equals to 19

$$\Rightarrow \quad 4x - 5 = 19$$

$$4x = 19 + 5 \qquad \text{(Transposing -5 to RHS)}$$

$$4x = 24$$

$$\therefore \; x = \frac{24}{4} \qquad \text{(Transposing 4 to RHS)}$$

$$\Rightarrow \; x = 6$$

Hence the required number is 6

Check: 4 times of 6 is 24 and 24 – 5 = 19.

Example 6: The length of a rectangle shaped park exceeds its breadth by 17 meters. If the perimeter of the park is 178 meters find the dimensions of the park?

Solution: Let the breadth of the park be = x meters

Then the length of the park = $x + 17$ meters

$\therefore$ perimeter of the park = 2 (length + breadth)

= $2\,(x + 17 + x)$ meters

= $2\,(2x + 17)$ meters

But it is given that the perimeter of the rectangle is 178 meters

$$\therefore \quad 2\,(2x + 17) = 178$$

$$4x + 34 = 178$$

$$4x = 178 - 34$$

$$4x = 144$$

$$x = \frac{144}{4} = 36$$

Therefore, breadth of the park = 36 meters

length of the park = 36 + 17 = 53 meters.

Try and Check it on your **own**.

Example 7: Two supplementary angles differ by 34. Find the angles?

Solution: Let the smaller angle be x^o

Since the two angles differ by 34^o, the bigger angle = $x + 34^o$

Since the sum of the supplementary angles is 180^o

$$\text{We have } x + (x + 34) = 180^o$$

$$2x + 34 = 180^o$$

$$2x = 180 - 34 = 146^o$$

$$x = \frac{146^o}{2} = 73^o$$

Therefore smaller angle $= x = 73^o$

Bigger angle $= x + 34 = 73 + 34 = 107^o$

Example 8: The present age of Vijaya's mother is four times the present age of Vijaya. After 6 years the sum of their ages will be 62 years. Find their present ages.

Solution: Let Vijaya's present age be 'x' years

Then we can make the following table

	Vijaya	Vijaya's mother
Present age	x	$4x$
Age after 6 years	$x + 6$	$4x + 6$

$\therefore$ Sum of their ages after 6 years $= (x + 6) + (4x + 6)$

$= x + 6 + 4x + 6$

$= 5x + 12$

But it is given that sum of their ages after 6 years is 62

$$\Rightarrow 5x + 12 = 62$$

$$5x = 62 - 12$$

$$5x = 50$$

$$x = \frac{50}{5} = 10$$

Therefore, Present age of Vijaya = x = 10 years

Present age of Vijaya's mother = $4x = 4 \times 10 = 40$ years

Example 9 : There are 90 multiple choice questions in a test. Two marks are awarded for every correct answer and one mark is deducted for every wrong answer . If Sahana got 60 marks in the test while she answered all the questions, then how many questions did she answer correctly?

Solution: Suppose the number of correctly answered questions be 'x', then number of wrongly answer questions = $90 - x$.

It is given that for every correct answer 2 marks are awarded.

$\therefore$ Number of marks scored for correct answers $= 2x$

And it is given that for every wrongly answered questions '1' mark is deducted

$\therefore$ Number of marks to be deducted from the score

$= (90 - x) \times 1 = 90 - x$

Total score = $2x - (90 - x) = 2x - 90 + x = 3x - 90$

But it is given that total score is 60

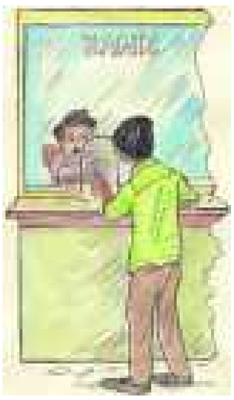
$\Rightarrow \quad 3x - 90 = 60$

$3x = 60 + 90$

$3x = 150$

$$x = \frac{150}{3} = 50$$

Number of questions answered correctly = x = 50



Example 10: Ravi works as a cashier in a bank. He has currency of denominations ₹ 100, ₹ 50, ₹ 10 respectively. The ratio of number of these notes is 2 : 3 : 5. The total cash with Ravi is ₹ 4,00,000.

How many notes of cash of each denomination does he have?

Solution: Let the number of ₹ 100 notes = $2x$

Number of ₹ 50 notes $= 3x$

and Number of ₹ 10 notes $= 5x$

Note that $2x : 3x : 5x$ is same as 2 : 3 : 5

$\therefore$ Total Money = $(2x \times 100) + (3x \times 50) + (5x \times 10)$

$200x + 150x + 50x = 400x$

But according to the problem the total money is Rs.4, 00,000.

$\Rightarrow \qquad 400x = 4,00,000$

$$x = \frac{400000}{400} = 1000$$

Therefore number of ₹100 notes $= 2x = 2 \times 1000 = 2000$

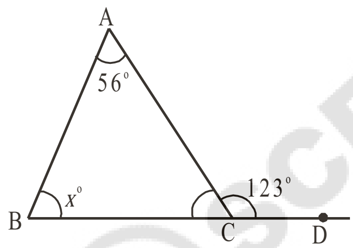
Number of ₹ 50 notes $= 3x = 3 \times 1000 = 3000$

Number of ₹10 notes $= 5x = 5 \times 1000 = 5000$

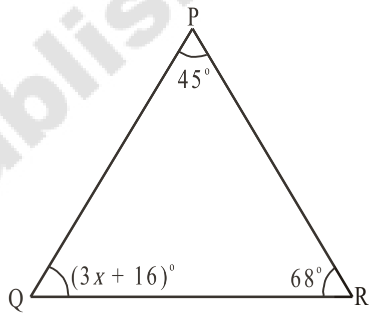
Exercise - 2.2

1. Find 'x' in the following figures?

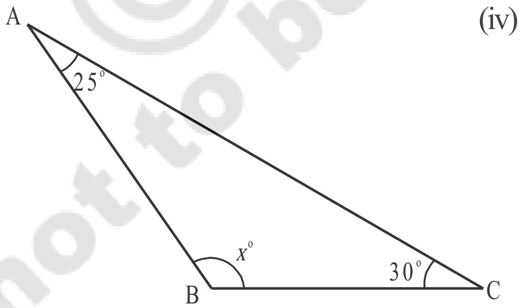
(i)



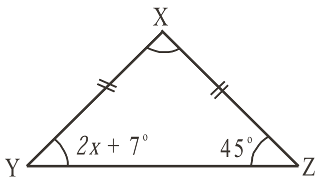
(ii)



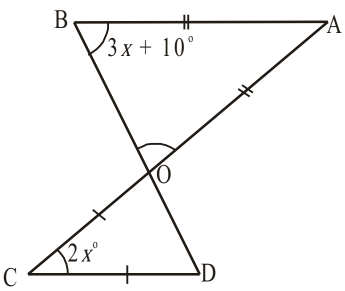
(iii)



(iv)



(v)



2. The difference between two numbers is 8. If 2 is added to the bigger number the result will be three times the smaller number . Find the numbers.

3. What are those two numbers whose sum is 58 and difference is 28?

4. The sum of two consecutive odd numbers is 56. Find the numbers.

5. The sum of three consecutive multiples of 7 is 777. Find these multiples.

   (Hint: Three consecutive multiples of 7 are 'x', '$x + 7$', '$x + 14$')

6. A man walks 10 km, then travels a certain distance by train and then by bus as far as twice by the train. If the whole journey is of 70km, how far did he travel by train?

7. Vinay bought a pizza and cut it into three pieces. When he weighed the first piece he found that it was 7g lighter than the second piece and 4g.heavier than the third piece. If the whole pizza weighed 300g. How much did each of the three pieces weigh?

   (Hint: weight of normal piece be 'x' then weight of largest piece is '$x + 7$', weight of the smallest piece is '$x - 4$')

8. The distance around a rectangular field is 400 meters.The length of the field is 26 meters more than the breadth. Calculate the length and breadth of the field?

9. The length of a rectangular field is 8 meters less than twice its breadth. If the perimeter of the rectangular field is 56 meters, find its length and breadth?

10. Two equal sides of a triangle are each 5 meters less than twice the third side. If the perimeter of the triangle is 55 meters, find the length of its sides?

11. Two complementary angles differ by 12^{o}, find the angles?

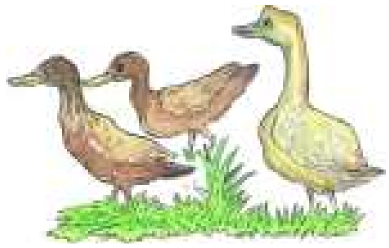
12. The ages of Rahul and Laxmi are in the ratio 5:7. Four years later, the sum of their ages will be 56 years. What are their present ages?

13. There are 180 multiple choice questions in a test. A candidate gets 4 marks for every correct answer, and for every un-attempted or wrongly answered questions one mark is deducted from the total score of correct answers. If a candidate scored 450 marks in the test how many questions did he answer correctly?

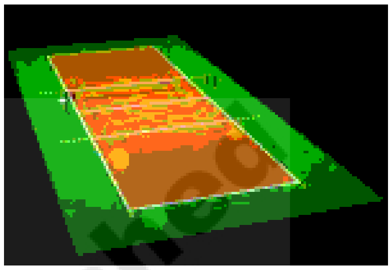
14. A sum of ₹ 500 is in the form of denominations of ₹ 5 and ₹ 10. If the total number of notes is 90 find the number of notes of each denomination.

   (Hint: let the number of 5 rupee notes be 'x', then number of 10 rupee notes = $90 – x$)

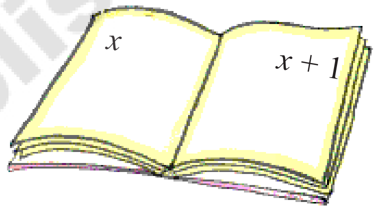
15. A person spent ₹ 564 in buying geese and ducks,if each goose cost ₹ 7 and each duck ₹ 3 and if the total number of birds bought was 108, how many of each type did he buy?



16. The perimeter of a school volleyball court is 177 ft and the length is twice the width. What are the dimensions of the volleyball court?



17. The sum of the page numbers on the facing pages of a book is 373. What are the page numbers?
(Hint :Let the page numbers of open pages are x and $x + 1$)



2.4 Solving equation that has variables on both the sides:

We know that an equation is the equality of the values of two expressions. In the equation $2x - 7 = 35$, the two expressions are 2x-7 and 35. In most examples that we have come across so far the RHS is just a number. But it need not be always. So, both sides could have expressions with variables. Let us see how this happens.

Consider the following example

Example 11: The present ages of Rafi and Fathima are in the ratio 7 : 5. Ten years later the ratio of their ages will be 9 : 7. Find their present ages?

Solution: Since the presen ratios of ages of Rafi and Fathima is 7:5,

We may take, Rafi's age to be $7x$ and the Fathima's age to be $5x$

(Note that ratio of $7x$ and $5x$ is $7x : 5x$ and which is same as 7:5)

After 10 years Rafi's age = $7x + 10$

and Fathima's age = $5x + 10$

After 10 years, the ratio of Rafi's age and Fathima's age is $7x + 10 : 5x + 10$

But according to the given data this ratio is equal to 9 : 7

$\Rightarrow \quad 7x + 10 : 5x + 10 = 9 : 7$

i.e., $7 (7x + 10) = 9 (5x + 10)$

$\Rightarrow \qquad 49\,x + 70 = 45\,x + 90.$

Did you notice that in the above equation we have algebraic expressions on both sides.

Now let us learn how to solve such equations.

The above equation is $49\,x + 70 = 45x + 90$

$\Rightarrow \qquad 49x - 45x = 90 - 70$ (Transposing 70 to RHS and $45x$ to LHS)

$\therefore \qquad 4x = 20$

$$\therefore \qquad x = \frac{20}{4} = 5$$

Therefore Rafi's age = $7x = 7 \times 5 = 35$ years

And Fathima's age $= 5x = 5 \times 5 = 25$ years

Example 12: Solve $5\,(x + 2) - 2\,(3 - 4x) = 3\,(x + 5) - 4\,(4 - x)$

Solution : $5x + 10 - 6 + 8x = 3x + 15 - 16 + 4x$ (removing brackets)

$13x + 4 = 7x - 1$ (adding like terms)

$13x - 7x = -1 - 4$ (transposing 4 to RHS, $7x$ to LHS)

$6x = -5$

$$x = \frac{-5}{6}$$ (transposing 6 to RHS)

Exercise - 2.3

Solve the following equations:

1. $7x - 5 = 2x$
2. $5x - 12 = 2x - 6$
3. $7p - 3 = 3p + 8$
4. $8m + 9 = 7m + 8$
5. $7z + 13 = 2z + 4$
6. $9y + 5 = 15y - 1$
7. $3x + 4 = 5\,(x - 2)$
8. $3\,(t - 3) = 5\,(2t - 1)$

9. $5(p-3) = 3(p-2)$

10. $5(z+3) = 4(2z+1)$

11. $15(x-1) + 4(x+3) = 2(7+x)$

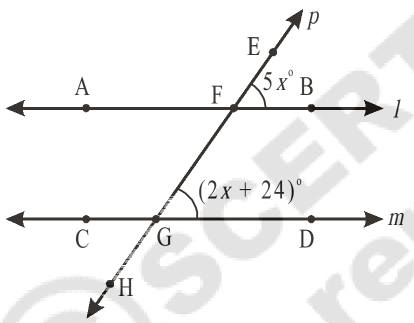
12. $3(5z-7) + 2(9z-11) = 4(8z-7) - 111$

13. $8(x-3) - (6-2x) = 2(x+2) - 5(5-x)$

14. $3(n-4) + 2(4n-5) = 5(n+2) + 16$

2.4.1 Some more applications

Example 13: In the figure $l \parallel m$, and p a transversal find the value of 'x'?



Solution: It is given that $l \parallel m$ and p is a transversal.

Therefore $\angle EFB = \angle FGD$ (corresponding angles)

Therefore $5x^o = (2x+24)^o$

$5x - 2x = 24$

$3x = 24$

$$x = \frac{24}{3} = 8^o$$

Example 14: Hema is 24 years older than her daughter Dhamini. 6 years ago, Hema was thrice as old as Dhamini. Find their present ages.

Solution: Let the Present age of Dhamini be 'x' years, then we can make the following table.

	Dhamini	Hema
Present age	x	$x + 24$
6 years ago	$x - 6$	$(x + 24) - 6 = x + 24 - 6 = x + 18$

But as given that 6 years ago Hema was thrice as old as Dhamini

$\therefore \quad x + 18 = 3(x - 6)$

$x + 18 = 3x - 18$

$x - 3x = -18 - 18$

$-2x = -36$

$x = 18.$

Therefore present age of Dhamini = x = 18 years

Present age of Hema $= x + 24 = 18 + 24 = 42$ years

Example 15: In a two digit number the sum of the two digits is 8. If 18 is added to the number its digits are reversed. Find the number.

Solution: Let the digit at ones place be 'x'

Then the digit at tens place = $8 - x$ (sum of the two digits is 8)

Therefore number $10(8 - x) + x = 80 - 10x + x = 80 - 9x$ —— (1)

Now, number obtained by reversing the digits = $10 \times (x) + (8 - x)$

$= 10x + 8 - x = 9x + 8$

It is given that if 18 is added to the number its digits are reversed

$\therefore$ number + 18 = Number obtained by reversing the digits

$\Rightarrow \quad (80 - 9x) + 18 = 9x + 8$

$98 - 9x = 9x + 8$

$98 - 8 = 9x + 9x$

$90 = 18x$

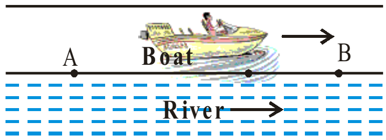
$$x = \frac{90}{18} = 5$$

By substituting the value of x in equation (1) we get the number

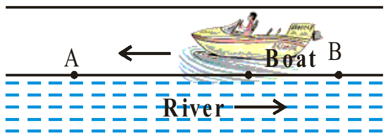
$\therefore$ Number $= 80 - 9 \times 5 = 80 - 45 = 35.$

Example 16: A motorboat goes down stream in a river and covers the distance between two coastal towns in five hours. It covers this distance upstream in six hours. If the speed of the stream is 2 km/hour find the speed of the boat in still water.

Solution: Since we have to find the speed of the boat in still water, let us suppose that it is x km/h.



This means that while going downstream the speed of the boat will be $(x + 2)$ kmph because the water current is pushing the boat at 2 kmph in addition to its own speed 'x' kmph.



Now the speed of the boat down stream $= (x + 2)$ kmph

$\Rightarrow$ distance covered in 1 hour $= x + 2$ km.

$\therefore$ distance covered in 5 hours $= 5(x + 2)$ km

Hence the distance between A and B is $5(x + 2)$ km

But while going upstream the boat has to work against the water current.

Therefore its speed upstream will be $(x - 2)$ kmph.

$\Rightarrow$ Distance covered in 1 hour $= (x - 2)$ km

Distance covered in 6 hours $= 6(x - 2)$ km

$\therefore$ distance between A and B is $6(x - 2)$ km

But the distance between A and B is fixed

$\therefore$ $5(x + 2) = 6(x - 2)$

$\Rightarrow$ $5x + 10 = 6x - 12$

$\Rightarrow$ $5x - 6x = -12 - 10$

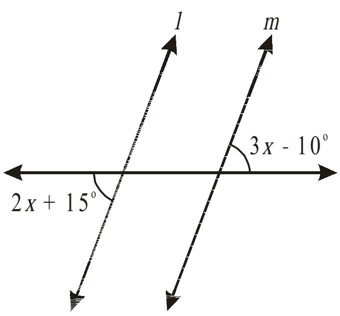
$\therefore$ $-x = -22$

$x = 22.$

Therefore speed of the boat in still water is 22 kmph.

Exercise- 2.4

1. Find the value of 'x' so that $l \parallel m$.



2. Eight times of a number reduced by 10 is equal to the sum of six times the number and 4. Find the number.
3. A number consists of two digits whose sum is 9. If 27 is subtracted from the number its digits are reversed. Find the number.
4. A number is divided into two parts such that one part is 10 more than the other. If the two parts are in the ratio 5:3, find the number and the two parts.
5. When I triple a certain number and add 2, I get the same answer as I do when I subtract the number from 50. Find the number.
6. Mary is twice older than her sister. In 5 years time, she will be 2 years older than her sister. Find how old are they both now.
7. In 5 years time, Reshma will be three times old as she was 9 years ago. How old is she now?
8. A town's population increased by 1200 people, and then this new population decreased 11%. The town now had 32 less people than it did before the 1200 increase. Find the original population.
9. A man on his way to dinner shortly after 6.00 p.m. observes that the hands of his watch form an angle of 110^0. Returning before 7.00 p.m. he notices that again the hands of his watch form an angle of 110^0. Find the number of minutes that he has been away.

2.5 Reducing Equations to Simpler Form - Equations Reducible to Linear Form:

Example 17: Solve $\frac{x}{2}-\frac{1}{4}=\frac{x}{3}+\frac{1}{2}$

Solution: $\frac{x}{2}-\frac{1}{4}=\frac{x}{3}+\frac{1}{2}$

$\frac{x}{2}-\frac{x}{3}=\frac{1}{2}+\frac{1}{4}$ (Transposing $\frac{x}{3}$ to L.H.S. and $\frac{1}{4}$ to R.H.S.)

$\frac{3x-2x}{6}=\frac{2+1}{4}$ (LCM of 2 and 3 is 6 ; 2 and 4 is 4)

$\frac{x}{6}=\frac{3}{4}$

$\therefore x=\frac{3}{4}\times 6$ (Transposing 6 to R.H.S.)

$\therefore x=\frac{9}{2}$

$\therefore x=\frac{9}{2}$ is the solution of the given equation.

Example 18: Solve $\frac{x-4}{7}-\frac{x+4}{5}=\frac{x+3}{7}$

Solution :

$$\frac{x-4}{7}-\frac{x+4}{5}=\frac{x+3}{7}$$

$$\frac{5(x-4)-7(x+4)}{35}=\frac{x+3}{7}$$

$$\frac{5x-20-7x-28}{35}=\frac{x+3}{7}$$

$$\frac{-2x-48}{35}=\frac{x+3}{7}$$

$$-2x-48=\frac{(x+3)}{7}\times 35$$

$\Rightarrow$ $-2x-48=(x+3)\times 5$

$\Rightarrow$ $-2x-48=5x+15$

$\Rightarrow$ $-2x-5x=15+48$

$$-7x=63$$

$$x=\frac{63}{-7}=-9.$$

Example 19: Solve the equation $\frac{5x+2}{2x+3}=\frac{12}{7}$ ————(1)

Solution: Let us multiply both sides of the given equation by $2x+3$. This gives

$$\frac{5x+2}{2x+3}\times(2x+3)=\frac{12}{7}\times(2x+3)$$

$$5x+2=\frac{12}{7}\times(2x+3)$$

Again multiply both sides of the equation by 7. This gives

$$7\times(5x+2)=7\times\frac{12}{7}\times(2x+3)$$

$\Rightarrow 7\times(5x+2)=12\times(2x+3)$ ——————(2)

$35x+14=24x+36$

$35x-24x=36-14$

$11x=22$

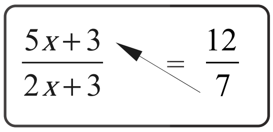
$\therefore x=\frac{22}{11}=2$

Now look at the given equation i.e., (1) and equation (2) carefully.

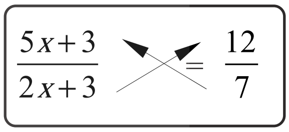
Given equation	Simplified form of the given equation
$\frac{5x+2}{2x+3}=\frac{12}{7}$	$7\times(5x+2)=12\times(2x+3)$

What did you notice? All we have done is :

1. Multiply the numerator of the LHS by the denominator of the RHS



2. Multiply the numerator of the RHS by the denominator of the LHS.

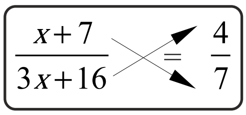


3. Equate the expressions obtained in (1) and (2)
   $7\times(5x+2)=12\times(2x+3)$

For obvious reasons, we call this method of solution as the "method of cross multiplication". Let us now illustrate the method of cross multiplication by examples

Example 20: Solve the equation $\frac{x+7}{3x+16}=\frac{4}{7}$

Solution: By cross multiplication, we get



$7 \times (x+7) = 4 \times (3x+16)$

$7x+49 = 12x+64$

$7x-12x = 64-49$

$-5x = 15$

$x = -3$

Example 21: Rehana got 24% discount on her frock. She paid ₹ 380 after discount. Find the marked price of the frock.

Solution: Let the marked price of the frock be ₹ x

Then the discount is 24% of x

She paid $x - 24\%$ of x i.e. 380

$x - 24\%$ of $x = 380$

$$\Rightarrow \quad x - \frac{24}{100} \times x = 380$$

$$\Rightarrow \quad \frac{100x - 24x}{100} = 380$$

$$\Rightarrow \quad \frac{76x}{100} = 380$$

$$x = \frac{380 \times 100}{76}$$

$\therefore \quad x = 500$

$\therefore$ Marked price = ₹ 500

Example 22: Four fifths of a number is greater than three fourths of the number by 4. Find the number.

Solution: Let the required number be 'x',

then four fifths of the number $= \frac{4}{5}x$

And three fourths of the number $= \frac{3}{4}x$

It is given that $\frac{4}{5}x$ is greater than $\frac{3}{4}x$ by 4

$$\Rightarrow \frac{4}{5}x - \frac{3}{4}x = 4$$

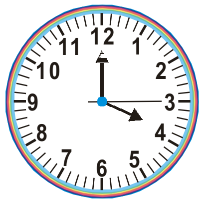
$$\frac{16x - 15x}{20} = 4$$

$$\Rightarrow \frac{x}{20} = 4 \quad \Rightarrow x = 80$$

Hence the required number is 80.

Example 23: John sold his watch for ₹ 301 and lost 14% on it. Find the cost price of the watch.

Solution: Let the cost price of the watch = ₹ x



The loss on it = 14% of 'x' = $\frac{14}{100} \times x = \frac{14x}{100}$

Selling price of the watch = Cost price – Loss

$$\Rightarrow 301 = x - \frac{14x}{100}$$

$$301 = \frac{100x - 14x}{100}$$

$$301 = \frac{86x}{100}$$

$$\frac{301 \times 100}{86} = x$$

$$350 = x$$

Therefore the cost price of the watch = ₹ 350.

Example 24: A man had to walk a certain distance. He covered two thirds of it at 4kmph and the remaining at 5 kmph. If the total time taken is 42 minutes, find the total distance.

Solution: Let the distance be 'x' km.

	First part	Second part
Distance covered	$\frac{2}{3}$ of 'x' = $\frac{2x}{3}$	Remaining distance = $x - \frac{2x}{3} = \frac{x}{3}$
Speed	4 kmph	5 kmph
Time taken	$\frac{\frac{2}{3}x}{4} = \frac{2x}{12}$ hr.	$\frac{\frac{x}{3}}{5} = \frac{x}{15}$ hr.

Therefore total time taken = $\frac{2x}{12} + \frac{x}{15}$ hr.

$$\Rightarrow \quad (\frac{2x}{12}+\frac{x}{15})\text{ hr} = 42\text{ min}.$$

$$\Rightarrow \quad (\frac{2x}{12}+\frac{x}{15})\text{ hr} = \frac{42}{60}\text{hr}.$$

$$\frac{2x}{12}+\frac{x}{15}=\frac{42}{60}$$

$$\frac{10x+4x}{60}=\frac{42}{60}$$

$$\Rightarrow \quad 14x = 42$$

$$\Rightarrow \quad x = 3$$

Total distance $x = 3$ km.

Example 25: The numerator of a fraction is 6 less than the denominator. If 3 is added to the numerator ,the fraction is equal to $\frac{2}{3}$, find the original fraction

Solution: Let the denominator of the fraction be 'x' then

Numerator of the fraction = $x-6$

Therefore the fraction = $\frac{x-6}{x}$

If 3 is added to the numerator , it becomes $\frac{2}{3}$

$$\Rightarrow \frac{x-6+3}{x}=\frac{2}{3}$$

$$\frac{x-3}{x}=\frac{2}{3}$$

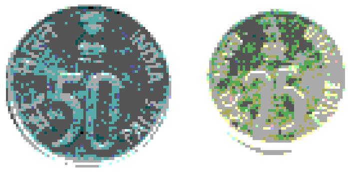
$$\Rightarrow 3x-9=2x$$

$$x=9$$

$$\therefore \text{ Fraction} = \frac{x-6}{x} = \frac{9-6}{9} = \frac{3}{9}$$

Therefore original fraction is $\frac{3}{9}$.

Example 26: Sirisha has ₹9 in fifty-paisa and twenty five paise coins. She has twice as many twenty five paise coins as she has fifty paise coins. How many coins of each kind does she have?



Solution: Let the number of fifty paise coins = x

Therefore the number of twenty five paise coins = $2x$

Value of fifty paise coins = $x \times 50$ paisa = ₹ $\frac{50x}{100}$ = ₹ $\frac{x}{2}$

Value of twenty five paise coins = $2x \times 25$ paisa = $2x \times \frac{25}{100}$

$= 2x \times \frac{1}{4}$ = ₹ $\frac{x}{2}$

Total value of all coins = ₹ $\left(\frac{x}{2} + \frac{x}{2}\right)$

But the total value of money is ₹9

$\Rightarrow \frac{x}{2} + \frac{x}{2} = 9$

$\frac{2x}{2} = 9$

$\therefore \quad x = 9$

Therefore number of fifty paise coins = $x = 9$

Number of twenty paise coins = $2x = 2 \times 9 = 18$.

Example 27: A man driving his moped at 24 kmph reaches his destination 5 minutes late to an appointment. If he had driven at 30 kmph he would have reached his destination 4 minutes before time. How far is his destination?

Solution: Let the distance be 'x' km.

Therefore time taken to cover 'x' km. at 24 kmph = $\frac{x}{24}$ hr.

Time taken to cover 'x' km. at 30 kmph = $\frac{x}{30}$ hr.

But it is given that the difference between two timings = 9 min = $\frac{9}{60}$ hr.

$$\therefore \quad \frac{x}{24} - \frac{x}{30} = \frac{9}{60}$$

$$\therefore \quad \frac{5x - 4x}{120} = \frac{9}{60}$$

$$\Rightarrow \quad \frac{x}{120} = \frac{9}{60}$$

$$\Rightarrow \quad x = \frac{9}{60} \times 120 = 18$$

Therefore the distance is 18 km.

Exercise - 2.5

1. Solve the following equations.

(i) $\frac{n}{5} - \frac{5}{7} = \frac{2}{3}$ (ii) $\frac{x}{3} - \frac{x}{4} = 14$

(ii) $\frac{z}{2} + \frac{z}{3} - \frac{z}{6} = 8$ (iv) $\frac{2p}{3} - \frac{p}{5} = 11\frac{2}{3}$

(v) $9\frac{1}{4} = y - 1\frac{1}{3}$ (vi) $\frac{x}{2} - \frac{4}{5} + \frac{x}{5} + \frac{3x}{10} = \frac{1}{5}$

(vii) $\frac{x}{2} - \frac{1}{4} = \frac{x}{3} + \frac{1}{2}$ (viii) $\frac{2x-3}{3x+2} = \frac{-2}{3}$

(ix) $\frac{8p-5}{7p+1} = \frac{-2}{4}$ (x) $\frac{7y+2}{5} = \frac{6y-5}{11}$

(xi) $\frac{x+5}{6} - \frac{x+1}{9} = \frac{x+3}{4}$ (xii) $\frac{3t+1}{16} - \frac{2t-3}{7} = \frac{t+3}{8} + \frac{3t-1}{14}$

2. What number is that of which the third part exceeds the fifth part by 4?

3. The difference between two positive integers is 36. The quotient when one integer is divided by other is 4. Find the integers.

   (Hint: If one number is 'x', then the other number is '$x-36$')

4. The numerator of a fraction is 4 less than the denominator. If 1 is added to both its numerator and denominator , it becomes $\frac{1}{2}$. Find the fraction.

5. Find three consecutive numbers such that if they are divided by 10, 17, and 26 respectively, the sum of their quotients will be 10.

   (Hint: Let the consecutive numbers = x, $x+1$, $x+2$, then $\frac{x}{10}+\frac{x+1}{17}+\frac{x+2}{26}=10$)

6. In class of 40 pupils the number of girls is three-fifths of the number of boys. Find the number of boys in the class.

7. After 15 years , Mary's age will be four times of her present age. Find her present age.

8. Aravind has a kiddy bank. It is full of one-rupee and fifty paise coins. It contains 3 times as many fifty paise coins as one rupee coins. The total amount of the money in the bank is ₹ 35. How many coins of each kind are there in the bank?

9. A and B together can finish a piece of work in 12 days. If 'A' alone can finish the same work in 20days , in how many days B alone can finish it?

10. If a train runs at 40 kmph it reaches its destination late by 11 minutes . But if it runs at 50 kmph it is late by 5 minutes only. Find the distance to be covered by the train.

11. One fourth of a herd of deer has gone to the forest. One third of the total number is grazing in a field and remaining 15 are drinking water on the bank of a river. Find the total number of deer.

12. By selling a radio for ₹903, a shop keeper gains 5%. Find the cost price of the radio.

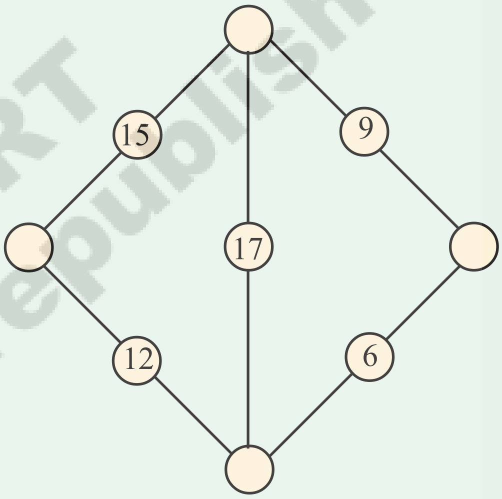
13. Sekhar gives a quarter of his sweets to Renu and then gives 5 sweets to Raji. He has 7 sweets left. How many did he have to start with?

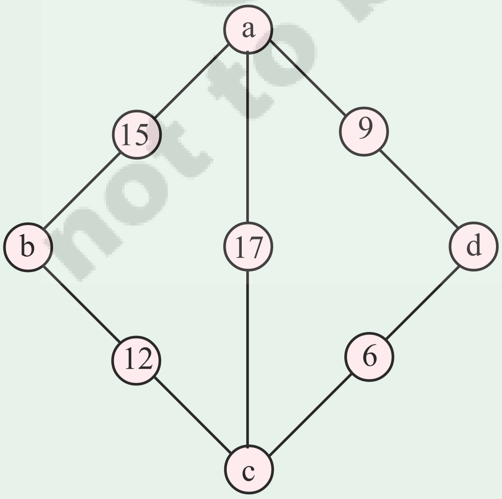
What we have discussed

1. If the degree of an equation is one then it is called a linear equation.
2. If a linear equation has only one variable then it is called a linear equation in one variable or simple equation.
3. The value which when subsitututed for the variable in the given equation makes L.H.S. = R.H.S. is called a solution or root of the given equation.
4. Just as numbers, variables can also be transposed from one side of the equation to the other side.

A magic Diamond

Find numbers to put in the circles so that the total along each line of the dimond is the same.





Hint : The number will be of the form

$a = x$, $b = 5 + x$, $c = 3 + x$, $d = 11 + x$

where x is any number and the total along each line will be $20 + 2x$

for example if $x = 1$, then $a = 1$, $b = 6$, $c = 4$, $d = 12$ and each line total will be 22.