

Exponents and Powers

4.0 Introduction

We know $3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$ and

$$3^m = 3 \times 3 \times 3 \times 3 \times 3 \times \dots \dots \dots \text{(m times)}$$

Do you know?

The estimated diameter of the sun is 1,40,00,00,000 m and

Mass of the sun is 1, 989, 100, 000, 000, 000, 000, 000, 000, 000 kg

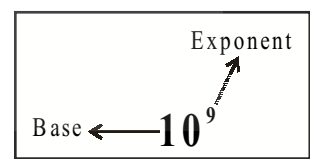
The distance from the Sun to Earth is 149, 600, 000, 000 m. The universe is estimated to be about 12,000,000,000 years old. The earth has approximately 1,353,000,000 cubic km of sea water.

Each square of a chess board is filled with grain. First box is filled with one grain and remaining boxes are filled in such a way that number of grains in a box is double of the previous box. Do you know how many number of grains required to fill all 64 boxes? It is 18,446,744,073,709,551,615.

Do we not find it difficult to read, write and understand such large numbers? Try to recall how we have written these kinds of numbers using exponents

$$1,40,00,00,000 \text{ m} = 1.4 \times 10^9 \text{ m.}$$

We read 10^9 as 10 raised to the power of 9



Do This

1. Simplify the following-

$$(i) 3^7 \times 3^3 \quad (ii) 4 \times 4 \times 4 \times 4 \times 4 \quad (iii) 3^4 \times 4^3$$

2. The distance between Hyderabad and Delhi is 1674.9 km by rail. How would you express this in centimeters? Also express this in the scientific form.

4.1 Powers with Negative Exponents

Usually we write

$$\text{Diameter of the sun} = 1400000000 \text{ m} = 1.4 \times 10^9 \text{ m}$$

$$\text{Avagadro number} = 6.23 \times 10^{23}$$

These numbers are large numbers and conveniently represented in short form.

But what if we need to represent very small numbers even less than unit, for example

$$\text{Thickness of hair} = 0.000005 \text{ m}$$

$$\text{Thickness of micro film} = 0.000015 \text{ m}$$

Let us find how we can represent these numbers that are less than a unit.

Let us recall the following patterns from earlier class

$$10^3 = 10 \times 10 \times 10 = 1000$$

$$10^2 = 10 \times 10 = 100 = 1000/10$$

$$10^1 = 10 = 100/10$$

$$10^0 = 1 = 10/10$$

$$10^{-1} = ?$$

As the exponent decreases by 1, the value becomes one-tenth of the previous value.

Continuing the above pattern we say that $10^{-1} = \frac{1}{10}$

$$\text{Similarly } 10^{-2} = \frac{1}{10} \div 10 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = \frac{1}{10^2}$$

$$10^{-3} = \frac{1}{100} \div 10 = \frac{1}{100} \times \frac{1}{10} = \frac{1}{1000} = \frac{1}{10^3}$$

From the above illustrations we can write $\frac{1}{10^n} = 10^{-n}$ or $\frac{1}{10^{-n}} = 10^n$

Observe the following table:

1 kilometre	1 hectometre	1 decametre	1 metre	1 decimeter	1 centimetre	1 millimetre
1000m	100m	10m	1 m	$\frac{1}{10}$ m	$\frac{1}{100}$ m	$\frac{1}{1000}$ m
10^3 m	10^2 m	10^1 m	10^0 m	10^{-1} m	10^{-2} m	10^{-3} m

**Do This**

What is 10^{-10} equal to?

Observe the pattern-

$$(i) \quad 8 = 2 \times 2 \times 2 = 2^3$$

$$(ii) \quad \frac{8}{2} = 4 = 2 \times 2 = 2^2$$

$$(iii) \quad \frac{4}{2} = 2 = 2^1$$

$$(iv) \quad \frac{2}{2} = 1 = 2^0$$

$$(v) \quad \frac{1}{2} = 2^{-1}$$

$$(vi) \quad \frac{1}{2^2} = 2^{-2}$$

In general we could say that for any non zero integer 'a', $a^{-m} = \frac{1}{a^m}$, which is multiplicative inverse of a^m . (How ?)

$$\text{That is } a^m \times a^{-m} = a^{m+(-m)} = a^0 = 1$$

**Do This**

Find the multiplicative inverse of the following

$$(i) \quad 3^{-5} \quad (ii) \quad 4^{-3} \quad (iii) \quad 7^{-4} \quad (iv) \quad 7^{-3}$$

$$(v) \quad x^{-n} \quad (vi) \quad \frac{1}{4^3} \quad (vii) \quad \frac{1}{10^3}$$

Look at this!

We know that $\text{speed} = \frac{\text{distance}}{\text{time}}$

Writing this symbolically, $s = \frac{d}{t}$. When distance is expressed in meters (m) and time in seconds(s), the unit for speed is written as $\text{m} \times \text{s}^{-1}$. Similarly the unit for acceleration is

$\frac{m}{s^2}$. This is also expressed as $\text{m} \times \text{s}^{-2}$

We can express the numbers like 3456 in the expanded form as follows :

$$3456 = (3 \times 1000) + (4 \times 100) + (5 \times 10) + (6 \times 1)$$

$$3456 = (3 \times 10^3) + (4 \times 10^2) + (5 \times 10) + (6 \times 10^0)$$

Similarly $7405 = (7 \times 10^3) + (4 \times 10^2) + (0 \times 10) + (5 \times 10^0)$

Let us now see how we can express the decimal numbers like 326.57 in the expanded form by using exponentials.

$$326.57 = (3 \times 10^2) + (2 \times 10) + (6 \times 10^0) + \left(\frac{5}{10}\right) + \left(\frac{7}{10^2}\right)$$

$$= (3 \times 10^2) + (2 \times 10) + (6 \times 10^0) + (5 \times 10^{-1}) + (7 \times 10^{-2})$$

(We have

$$\frac{1}{10} = 10^{-1} \text{ \& } \frac{1}{10^2} = 10^{-2})$$

$$\text{Also } 734.684 = (7 \times 10^2) + (3 \times 10) + (4 \times 10^0) + \left(\frac{6}{10}\right) + \left(\frac{8}{10^2}\right) + \left(\frac{4}{10^3}\right)$$

$$= (7 \times 10^2) + (3 \times 10) + (4 \times 10^0) + (6 \times 10^{-1}) + (8 \times 10^{-2}) + (4 \times 10^{-3})$$



Do This

Expand the following numbers using exponents

- (i) 543.67 (ii) 7054.243 (iii) 6540.305 (iv) 6523.450

4.2 Laws of Exponents

We have learnt that for any non-zero integer 'a', $a^m \times a^n = a^{m+n}$; where 'm' and 'n' are natural numbers.

Does this law also hold good for negative exponents?

Let us verify

- (i) Consider $3^2 \times 3^{-4}$

$$\text{We know that } 3^{-4} = \frac{1}{3^4}$$

$$\text{Therefore } 3^2 \times 3^{-4} = 3^2 \times \frac{1}{3^4} = \frac{3^2}{3^4}$$

$$= 3^{2-4} = 3^{-2}$$

$$\text{i.e., } 3^2 \times 3^{-4} = 3^{-2}$$

- (ii) Take $(-2)^{-3} \times (-2)^{-4}$

$$(-2)^{-3} \times (-2)^{-4} = \frac{1}{(-2)^3} \times \frac{1}{(-2)^4} = \frac{1}{(-2)^{3+4}} \quad (\because a^m \times a^n = a^{m+n})$$

$$a^{-m} = \frac{1}{a^m} \text{ for any non zero integer 'a',}$$

$$\left(\text{We know } \frac{a^m}{a^n} = a^{m-n} \right)$$

$$= \frac{1}{(-2)^7} = (-2)^{-7} \quad (\because \frac{1}{a^m} = a^{-m})$$

Therefore $(-2)^{-3} \times (-2)^{-4} = (-2)^{-7}$

(iii) Let us take $(-5)^2 \times (-5)^{-5}$

$$\begin{aligned} (-5)^2 \times (-5)^{-5} &= (-5)^2 \times \frac{1}{(-5)^5} \\ &= \frac{1}{(-5)^{5-2}} \quad \left(\because \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \right) \\ &= (-5)^{-3} \\ &= \frac{1}{(-5)^{5-2}} \\ &= \frac{1}{(-5)^3} = (-5)^{-3} \end{aligned}$$

Therefore $(-5)^2 \times (-5)^{-5} = (-5)^{-3}$ (We know $2+(-5) = -3$)

In general we could infer that for any non-zero integer 'a', $a^m \times a^n = a^{m+n}$; where 'm' and 'n' are integers.



Do This

Simplify and express the following as single exponent-

- (i) $2^{-3} \times 2^{-2}$ (ii) $7^{-2} \times 7^5$ (iii) $3^4 \times 3^{-5}$ (iv) $7^5 \times 7^{-4} \times 7^{-6}$
 (v) $m^5 \times m^{-10}$ (vi) $(-5)^{-3} \times (-5)^{-4}$

Similarly, we can also verify the following laws of exponents where 'a' and 'b' are non zero integers and 'm' and 'n' are any integers.

1. $\frac{a^m}{a^n} = a^{m-n}$
2. $(a^m)^n = a^{mn}$
3. $(a^m \times b^m) = (ab)^m$

You have studied these laws in lower classes only for positive exponents

$$4. \quad \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

$$5. \quad a^0 = 1$$

Do you find any relation between 'm' and 'n' if $a^m = a^n$ where 'a' is a non zero integer and $a \neq 1, a \neq -1$. Let us see:

$$\text{Let } a^m = a^n \quad \text{then } \frac{a^m}{a^n} = 1 \quad (\text{Dividing both sides by } a^n)$$

$$\begin{aligned} \text{That is } a^{m-n} &= 1. & a^{m-n} &= a^0 \\ & & \therefore m-n &= 0 \\ & & \therefore m &= n \end{aligned}$$

Why $a \neq 1$?

If $a = 1, m = 7$ and $n = 6$
then $1^7 = 1^6$

$$\Rightarrow 7 = 6$$

is it true?

so $a \neq 1$

if $a = -1$ what happens.

Thus we can conclude that if $a^m = a^n$ then $m = n$.

Example 1: Find the value of (i) 5^{-2} (ii) $\frac{1}{2^{-5}}$ (iii) $(-5)^2$

$$\text{Solution:} \quad (i) \quad 5^{-2} = \frac{1}{(5)^2} = \frac{1}{5 \times 5} = \frac{1}{25} \quad (\because a^{-m} = \frac{1}{a^m})$$

$$(ii) \quad \frac{1}{2^{-5}} = 2^5 = 2 \times 2 \times 2 \times 2 \times 2 \quad (\because \frac{1}{a^{-m}} = a^m)$$

$$2^5 = 32$$

$$(iii) \quad (-5)^2 = (-5)(-5) = 25$$

Example 2: Simplify the following

$$(i) \quad (-5)^4 \times (-5)^{-6} \quad (ii) \quad \frac{4^7}{4^4} \quad (iii) \quad \left(\frac{3^5}{3^3}\right)^5 \times 3^{-6}$$

$$\text{Solution:} \quad (i) \quad (-5)^4 \times (-5)^{-6} \quad (\because a^m \times a^n = a^{m+n})$$

$$= (-5)^{4+(-6)} = (-5)^{-2}$$

$$= \frac{1}{(-5)^2} = \frac{1}{(-5) \times (-5)} = \frac{1}{25} \quad (\because a^{-m} = \frac{1}{a^m})$$

$$(ii) \quad \frac{4^7}{4^4} \quad (\because \frac{a^m}{a^n} = a^{m-n})$$

$$= 4^{7-4} = 4^3 = 64$$

$$\begin{aligned}
 \text{(iii)} \quad & \left(\frac{3^5}{3^3}\right)^5 \times 3^{-6} \\
 & = (3^{5-3})^5 \times 3^{-6} && (\because \frac{a^m}{a^n} = a^{m-n}) \\
 & = (3^2)^5 \times 3^{-6} && (\because (a^m)^n = a^{mn}) \\
 & = 3^{10} \times 3^{-6} = 3^4 = 81
 \end{aligned}$$

Example 3: Express each of the following with positive exponents.

$$\text{(i)} \ 4^{-7} \quad \text{(ii)} \ \frac{1}{(5)^{-4}} \quad \text{(iii)} \ \left(\frac{4}{7}\right)^{-3} \quad \text{(iv)} \ \frac{7^{-4}}{7^{-6}}$$

Solution : (i) 4^{-7} (We know $a^{-m} = \frac{1}{a^m}$)

$$= \frac{1}{(4)^7}$$

$$\text{(ii)} \ \frac{1}{(5)^{-4}} \\
 = 5^4$$

$$(\because \frac{1}{a^{-m}} = a^m)$$

$$\text{(iii)} \ \left(\frac{4}{7}\right)^{-3} = \frac{4^{-3}}{7^{-3}} \\
 = \frac{7^3}{4^3} = \left(\frac{7}{4}\right)^3$$

$$\left(a^{-m} = \frac{1}{a^m} \text{ and } a^m = \frac{1}{a^{-m}}\right)$$

$$\text{(iv)} \ \frac{7^{-4}}{7^{-6}} \\
 = 7^{-4 - (-6)} \\
 = 7^{-4+6} = 7^2$$

$$\therefore \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$



Example 4 : Express 27^{-4} as a power with base 3

Solution : 27 can be written as $3 \times 3 \times 3 = 3^3$

$$\text{Therefore } 27^{-4} = (3^3)^{-4}$$

$$= 3^{-12}$$

$$(\because (a^m)^n = a^{mn})$$

Example 5: Simplify

$$(i) \left(\frac{1}{27}\right) \times 2^{-3} \quad (ii) 4^4 \times 16^{-2} \times 4^0$$

Solution: (i) $\left(\frac{1}{27}\right) \times 2^{-3}$

27 can be expressed as $3 \times 3 \times 3 = 3^3$

$$\text{So, } \left(\frac{1}{27}\right) \times 2^{-3} = \frac{1}{3^3} \times 2^{-3}$$

$$= \frac{1}{3^3} \times \frac{1}{2^3} \quad (\because \frac{1}{a^m} = a^{-m})$$

$$= \frac{1}{(3 \times 2)^3} \quad (\because a^m \times b^m = (ab)^m)$$

$$= \frac{1}{6^3} = \frac{1}{216}$$

$$(ii) 4^4 \times 16^{-2} \times 4^0$$

$$= 4^4 \times (4^2)^{-2} \times 4^0 \quad (\because (a^m)^n = a^{mn})$$

$$= 4^4 \times 4^{-4} \times 4^0 \quad (\because a^m \times a^n = a^{m+n})$$

$$= 4^{4+0} = 4^0 \quad (\because a^0 = 1)$$

$$= 1$$

Example 6: Can you guess the value of 'x' when

$$2^x = 1$$

Solution: as we discussed before $a^0 = 1$

$$\text{Obviously } 2^x = 1$$

$$2^x = 2^0$$

$$\Rightarrow x = 0$$

Example 7: Find the value of 'x' such that

$$(i) 25 \times 5^x = 5^8$$

$$(ii) \frac{1}{49} \times 7^{2x} = 7^8$$

$$(iii) (3^6)^4 = 3^{12x}$$

$$(iv) (-2)^{x+1} \times (-2)^7 = (-2)^{12}$$

Solution : $25 \times 5^x = 5^8$ as $25 = 5 \times 5 = 5^2$
 $5^2 \times 5^x = 5^8$ But $a^m \times a^n = a^{m+n}$
 $5^{2+x} = 5^8$ If $a^m = a^n \Rightarrow m = n$
 $2 + x = 8$
 $\therefore x = 6$

(ii) $\frac{1}{49} \times 7^{2x} = 7^8$

$\frac{1}{7^2} \times 7^{2x} = 7^8$ ($\because \frac{1}{a^m} = a^{-m}$)

$7^{-2} \times 7^{2x} = 7^8$

$7^{2x-2} = 7^8$

As bases are equal, Hence

$2x - 2 = 8$

$2x = 8 + 2$

$2x = 10$

$x = \frac{10}{2} = 5$

$\therefore x = 5$

(iii) $(3^6)^4 = 3^{12x}$ [$\because (a^m)^n = a^{mn}$]
 $3^{24} = 3^{12x}$

As bases are equal, Hence

$24 = 12x$

$\therefore x = \frac{24}{12} = 2$

(iv) $(-2)^{x+1} \times (-2)^7 = (-2)^{12}$

$(-2)^{x+1+7} = (-2)^{12}$

$(-2)^{x+8} = (-2)^{12}$

As bases are equal, Hence

$x + 8 = 12$

$\therefore x = 12 - 8 = 4$

Example 8 : Simplify $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{25}{4}\right)^{-2}$

$$\frac{25}{4} = \frac{5 \times 5}{2 \times 2} = \frac{5^2}{2^2}$$

$$\left(\frac{2}{5}\right)^{-3} \times \left(\frac{25}{4}\right)^{-2} = \left(\frac{2}{5}\right)^{-3} \times \left(\frac{5^2}{2^2}\right)^{-2} \quad (\because (a^m)^n = a^{mn})$$

$$= \frac{5^3}{2^3} \times \frac{2^4}{5^4} = 5^{3-4} \times 2^{4-3} \quad \text{As } \frac{1}{a^m} = a^{-m} \text{ and } \frac{1}{a^{-m}} = a^m$$

$$= 5^{-1} \times 2^1 = \frac{2}{5}$$

Example 9 : Simplify $\left\{ \left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3} \div \left(\frac{1}{5}\right)^{-2} \right\}$

Solution: $\left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3} \div \left(\frac{1}{5}\right)^{-2} \right] \quad (\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m})$

$$= \left[\left(\frac{1^{-3}}{3^{-3}} - \frac{1^{-3}}{2^{-3}}\right) \div \frac{1^{-2}}{5^{-2}} \right] \quad (\because a^{-m} = \frac{1}{a^m} \text{ and } a^m = \frac{1}{a^{-m}})$$

$$= \left[\left(\frac{3^3}{1^3} - \frac{2^3}{1^3}\right) \div \frac{5^2}{1^2} \right] = \left(\frac{27}{1} - \frac{8}{1}\right) \div 25$$

$$= (27 - 8) \div 25 = \frac{19}{25}$$

Example 10 : If $x = \left(\frac{3}{2}\right)^2 \times \left(\frac{2}{3}\right)^{-4}$ find the value of x^{-2}

Solution: $x = \left(\frac{3}{2}\right)^2 \times \left(\frac{2}{3}\right)^{-4}$

$$x = \left(\frac{3}{2}\right)^2 \times \frac{2^{-4}}{3^4} \quad (\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m})$$

$$x = \frac{3^2}{2^2} \times \frac{3^4}{2^4} = \frac{3^{2+4}}{2^{2+4}} = \frac{3^6}{2^6} = \left(\frac{3}{2}\right)^6$$

$$x = \left(\frac{3}{2}\right)^6$$

$$x^{-2} = \left[\left(\frac{3}{2}\right)^6\right]^{-2} = \left(\frac{3}{2}\right)^{-12} = \frac{3^{-12}}{2^{-12}} = \frac{2^{12}}{3^{12}} = \left(\frac{2}{3}\right)^{12}$$



Exercise - 4.1

1. Simplify and give reasons

(i) 4^{-3} (ii) $(-2)^7$ (iii) $\left(\frac{3}{4}\right)^{-3}$ (iv) $(-3)^{-4}$

2. Simplify the following :

(i) $\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^6$ (ii) $(-2)^7 \times (-2)^3 \times (-2)^4$

(iii) $4^4 \times \left(\frac{5}{4}\right)^4$ (iv) $\left(\frac{5^{-4}}{5^{-6}}\right) \times 5^3$ (v) $(-3)^4 \times 7^4$

3. Simplify (i) $2^2 \times \frac{3^2}{2^{-2}} \times 3^{-1}$ (ii) $(4^{-1} \times 3^{-1}) \div 6^{-1}$

4. Simplify and give reasons

(i) $(4^0 + 5^{-1}) \times 5^2 \times \frac{1}{3}$ (ii) $\left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{4}\right)^{-3} \times \left(\frac{1}{5}\right)^{-3}$

(iii) $(2^{-1} + 3^{-1} + 4^{-1}) \times \frac{3}{4}$ (iv) $\frac{3^{-2}}{3} \times (3^0 - 3^{-1})$

(v) $1 + 2^{-1} + 3^{-1} + 4^0$ (vi) $\left[\left(\frac{3}{2}\right)^{-2}\right]^2$

5. Simplify and give reasons (i) $\left[(3^2 - 2^2) \div \frac{1}{5}\right]^2$ (ii) $((5^2)^3 \times 5^4) \div 5^6$
6. Find the value of 'n' in each of the following :
- (i) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^5 = \left(\frac{2}{3}\right)^{n-2}$
- (ii) $(-3)^{n+1} \times (-3)^5 = (-3)^{-4}$
- (iii) $7^{2n+1} \div 49 = 7^3$
7. Find 'x' if $2^{-3} = \frac{1}{2^x}$
8. Simplify $\left[\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{5}\right)^{-3}\right] \times \left(\frac{3}{5}\right)^{-2}$
9. If $m = 3$ and $n = 2$ find the value of
 (i) $9m^2 - 10n^3$ (ii) $2m^2 n^2$ (iii) $2m^3 + 3n^2 - 5m^2 n$ (iv) $m^n - n^m$
10. Simplify and give reasons $\left(\frac{4}{7}\right)^{-5} \times \left(\frac{7}{4}\right)^{-7}$

4.3 Application of Exponents to Express numbers in Standard Form

In previous class we have learnt how to express very large numbers in standard form.

For example $300,000,000 = 3 \times 10^8$

Now let us try to express very small number in standard form.

Consider, diameter of a wire in a computer chip is 0.000003m

$$0.000003 \text{ m} = \frac{3}{1000000} \text{ m}$$

$$\begin{aligned} &= \frac{3}{10^6} \text{ m} \\ &= 3 \times 10^{-6} \text{ m} \end{aligned}$$

Therefore $0.000003 = 3 \times 10^{-6} \text{ m}$

Similarly consider the size of plant cell which is 0.00001275m

$$\begin{aligned} 0.00001275\text{m} &= \frac{1275}{100000000} \\ &= 1.275 \times \frac{10^3}{10^8} \\ &= 1.275 \times 10^{-5} \text{ m} \end{aligned}$$



Do This

- Change the numbers into standard form and rewrite the statements.
 - The distance from the Sun to earth is 149,600,000,000m
 - The average radius of Sun is 695000 km
 - The thickness of human hair is in the range of 0.005 to 0.001 cm
 - The height of Mount Everest is 8848 m
- Write the following numbers in the standard form

(i) 0.0000456	(ii) 0.000000529	(iii) 0.0000000085
(iv) 6020000000	(v) 35400000000	(vi) 0.000437×10^4

4.4 Comparing very large and very small numbers

We know that the diameter of the Sun is 1400000000 m. and earth is 12750000 m. If we want to know how bigger the Sun than the Earth, we have to divide the diameter of Sun by the diameter of the Earth.

$$\text{i.e. } \frac{1400000000}{12750000}$$

Do you not find it difficult. If we write these diameters in standard form then it is easy to find how bigger the Sun. Let us see

$$\text{Diameter of the Sun} = 1400000000 \text{ m} = 1.4 \times 10^9 \text{ m}$$

$$\text{Diameter of the Earth} = 12750000 = 1.275 \times 10^7 \text{ m}$$

$$\begin{aligned} \text{Therefore we have } \frac{\text{Diameter of the sun}}{\text{Diameter of the earth}} &= \frac{1.4 \times 10^2 \times 10^7}{1.275 \times 10^7} \\ &= \frac{1.4 \times 10^2}{1.275} \\ &\approx 10^2 = 100 \quad (\text{Approximately}) \end{aligned}$$

Thus the diameter of the Sun is approximately 100 times the diameter of the Earth.

i.e. Sun is 100 times bigger than the Earth.

Let us consider one more illustration

The mass of the earth is 5.97×10^{24} kg and the mass of the moon is 7.35×10^{22} kg.

What is their total mass?

$$\text{The mass of the earth} = 5.97 \times 10^{24} \text{ kg}$$

$$\text{The mass of the moon} = 7.35 \times 10^{22} \text{ kg}$$

$$\begin{aligned} \text{Total Mass} &= 5.97 \times 10^{24} \text{ Kg} + 7.35 \times 10^{22} \text{ kg} \\ &= (5.97 \times 10^2 \times 10^{22} \text{ Kg}) + 7.35 \times 10^{22} \text{ kg} \\ &= (5.97 \times 10^2 + 7.35) \times 10^{22} \text{ kg} \\ &= (597 + 7.35) \times 10^{22} \text{ kg} \\ &= 604.35 \times 10^{22} \text{ kg} \\ &= 6.0435 \times 10^{24} \text{ kg} \end{aligned}$$

When we have to add numbers in the standard form we convert them in numbers with same exponents

Example 11 : Express the following in the standard form.

(i) 4.67×10^4 (ii) 1.0001×10^9 (iii) 3.02×10^{-6}

Solution:

(i) $4.67 \times 10^4 = 4.67 \times 10000 = 46700$

(ii) $1.0001 \times 10^9 = 1.0001 \times 1000000000 = 1000100000$

(iii) $3.02 \times 10^{-6} = 3.02/10^6 = 3.02/1000000 = 0.00000302$



Exercise - 4.2

1. Express the following numbers in the standard form.

(i) 0.000000000947 (ii) 543000000000

(iii) 48300000 (iv) 0.00009298 (v) 0.0000529

2. Express the following numbers in the usual form.

(i) 4.37×10^5 (ii) 5.8×10^7 (iii) 32.5×10^{-4} (iv) 3.71529×10^7

(v) 3789×10^{-5} (vi) 24.36×10^{-3}

3. Express the following information in the standard form

(i) Size of the bacteria is 0.0000004 m

(ii) The size of red blood cells is 0.000007mm

- (iii) The speed of light is 300000000 m/sec
- (iv) The distance between the moon and the earth is 384467000 m(app)
- (v) The charge of an electron is 0.000000000000000016 coulombs
- (vi) Thickness of a piece of paper is 0.0016 cm
- (vii) The diameter of a wire on a computer chip is 0.000005 cm
4. In a stack, there are 5 books, each of thickness 20 mm and 5 paper sheets each of thickness 0.016mm. What is the total thickness of the stack.
5. Rakesh solved some problems of exponents in the following way. Do you agree with the solutions? If not why? Justify your argument.

(i) $x^{-3} \times x^{-2} = x^{-6}$ (ii) $\frac{x^3}{x^2} = x^4$ (iii) $(x^2)^3 = x^{2^3} = x^8$

(iv) $x^{-2} = \sqrt{x}$ (v) $3x^{-1} = \frac{1}{3x}$

Project :

Refer science text books of 6th to 10th classes in your school and collect some scientific facts involving very small numbers and large numbers and write them in standard form using exponents.



What we have discussed

1. Numbers with negative exponents holds the following laws of exponents

(a) $a^m \times a^n = a^{m+n}$ (b) $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$ (c) $(a^m)^n = a^{mn}$

(c) $a^m \times b^m = (ab)^m$ (d) $a^0 = 1$ (e) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

2. Very small numbers can be expressed in standard form using negative exponents.
3. Comparison of smaller and larger numbers.
4. Identification of common errors.