Chapter **8**

Exploring Geometrical Figures

8.0 Introduction

We come across various figures of geometry in our daily life. There are objects that have direct and indirect connection with geometry. These objects or actions have geometrical properties and applications.

Look at the following pictures, what are the various geometrical figures and patterns involved in it? You might have found some shapes are similar in nature, some are of congruent and some geometrical patterns that are evenly spread on the floor.

Can you identify such congruent shapes, similar shapes and symmetric shapes or patterns in the pictures?

The shapes of windows in the picture are congruent; the triangular elevations are similar and the tile patterns that spread on the floor are of symmetric figures.

Let us study how these principles of geometrical shapes and patterns are influencing our dialy life.

8.1 Congruency

You may have seen various objects with same size and shape which we use in our daily life. For example blades of a fan are of same size and shape.

Another example for congruency of shapes in daily life.

Go to an audio shop and find a Compact Disc (CD) there, what do you notice?

The CDs are of same size and shape. If you place them one above the other, they cover each other exactly. We can say that the faces of CDs are congruent to one another.

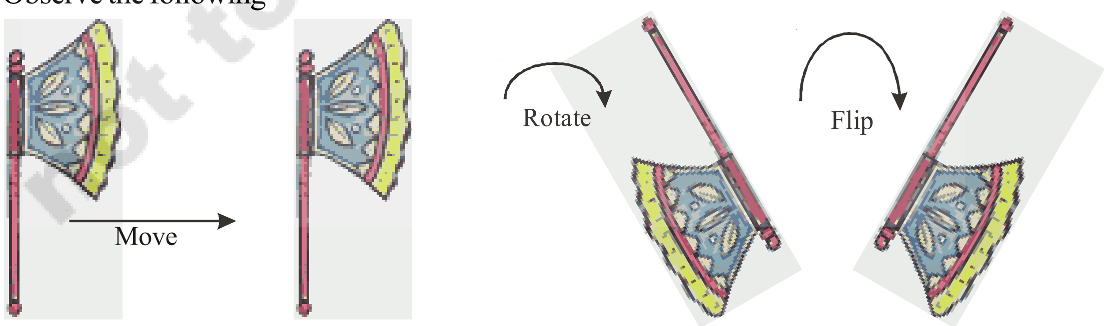
Now put the post cards one above the other. You will find that all post cards have same size and shape; they are all congruent to one another.

You too name certain objects with congruent faces.

8.1.1 Congruency of shapes

Observe the following

(i)



In the above, do all the figures represent the same object irrespective of their position?

Here the same figure is moved, rotated and flipped to get figures. They represent the same hand fan.

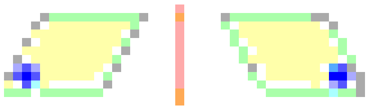
If we place all figures one above the other, what do you find?

They all cover each other exactly i.e. they have same size and shape.

Do you remember what we call the figures with same size and shape?

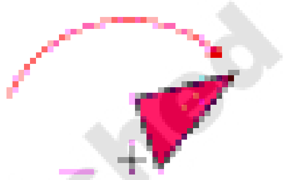
Figures with same size and shape are called congruent figures.

Flip : Flip is a transformation in which a plane figure is reflected across a line, creating a mirror image of the original figure.



After a figure is flipped or reflected, the distance between the line of reflection and each point on the original figure is the same as the distance between the line of reflection and the corresponding point on the mirror image.

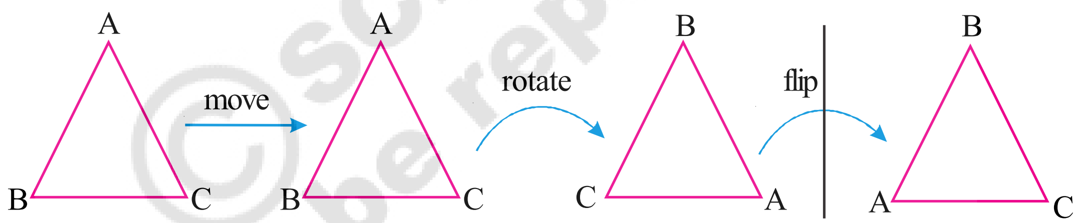
Rotation : "Rotation" means turning around a center. The distance from the center to any point on the shape stays the same. Every point makes a circle around the center.



There is a central point that stays fixed and everything else moves around that point in a circle. A "Full Rotation" is 360°

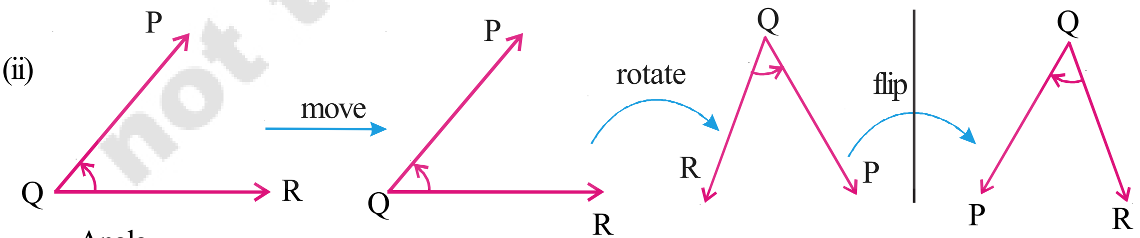
Now observe the following geometrical figures.

(i)



Triangle

(ii)



Angle

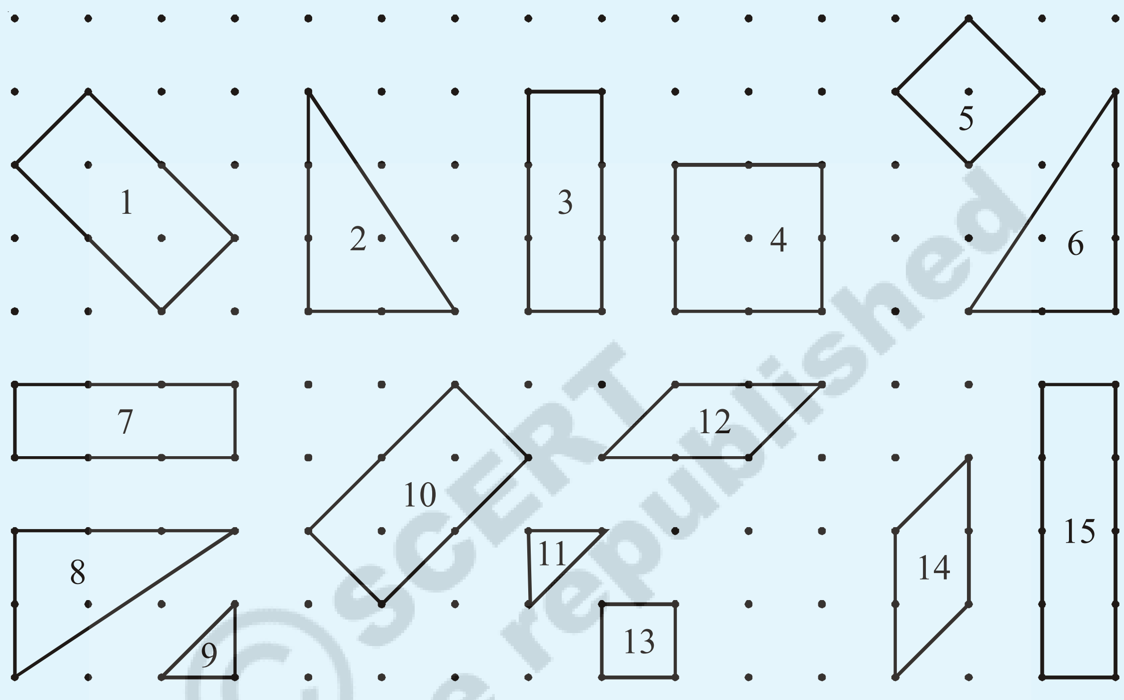
In all the cases if the first figure in the row is moved, rotated and flipped do you find any change in size and shape? No, the figures in every row are congruent they represent the same figure but oriented differently.

If two shapes are congruent, still they remain congruent if they are moved or rotated. The shapes would also remain congruent if we reflect the shapes by producing their mirror images.

We use the symbol ≅ to represent congruency.

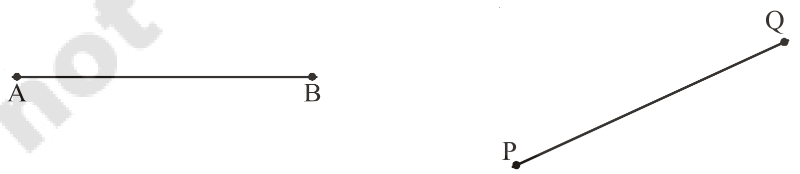
Do This

Identify which of the following pairs of figures are congruent.



Can you say when do two (a) Line segments (b) angles and (c) triangles are congruent?

(a) We know that two line segments are congruent if they have same lengths.



Length of AB = length of PQ then AB ≅ PQ

(b) Two angles are congruent if they have same measure.

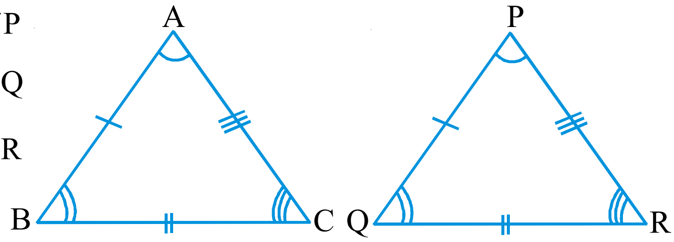
(c) Two triangles ΔABC and ΔPQR are congruent if all the pairs of corresponding sides are equal.

i.e. AB = PQ and ∠A = ∠P

BC = QR ∠B = ∠Q

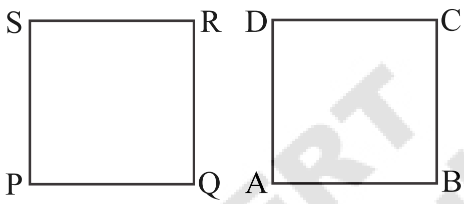
CA = RP ∠C = ∠R

ΔABC ≅ ΔPQR.



Now how can you say that two polygons are congruents?

Let us discuss this with an example. Suppose two squares ABCD and PQRS. If we place one square (i.e.) ABCD on the other i.e. PQRS, they should cover each other exactly



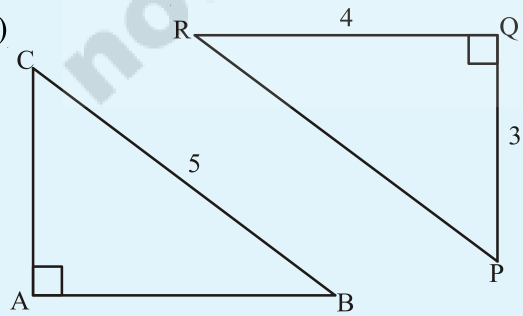
i.e. the edges must coincide with each other, only then we say that the two squares are congruent.

If two polygons are congruent then their corresponding sides are equal and corresonding angles are equal. Thus the two geometrical shapes are said to be congruent if they cover each other exactly.

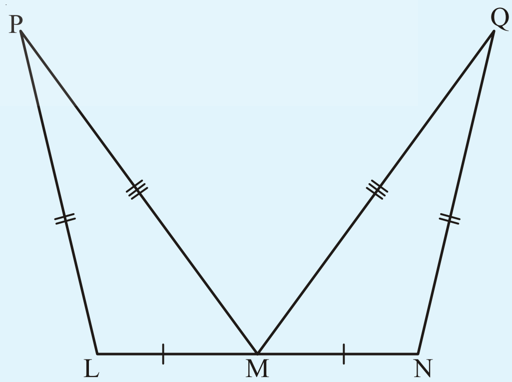
Do This:

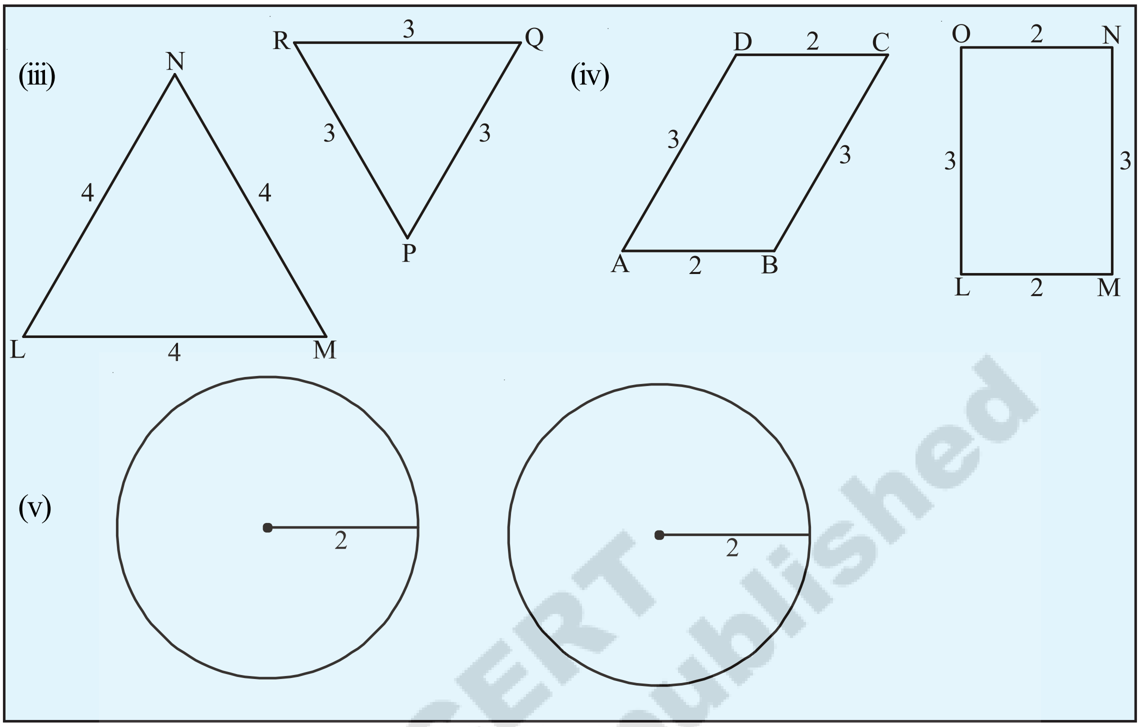
Look at the following pairs of figures and find whether they are congruent. Give reasons. Name them.

(i)



(ii)





8.1.2 Similar shapes

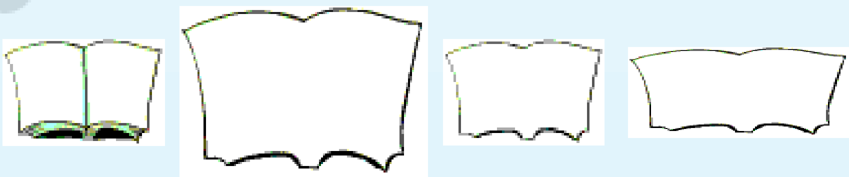
In our books, we have pictures of many object from our sorroundings. For example pictures of elephants, tigers, elevation plan of a huge building, block diagram of a microchip etc.

Are they drawn to their original size? No, it is not possible. Some of them are drawn smaller than the real object and some of them are drawn larger.

Do This

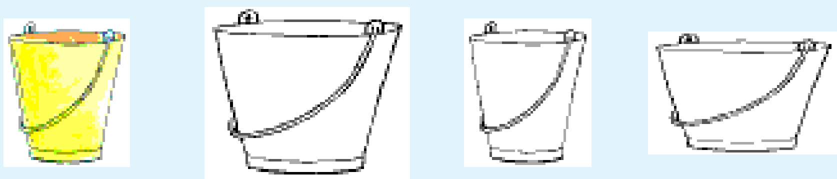
1. Identify the out line figures which are similar to those given first.

(a)



(i) (ii) (iii)

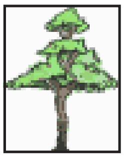
(b)



(i) (ii) (iii)

A picture of a tree is drawn on a paper. How do you say the picture drawn is similar to its original?

Tree



Picture

Here is an object and is reduced different proportions. Which of the following reductions resembles the original object?

Original object Reduction -1 Reduction - 2 Reduction - 3 Reduction -4

By comparing the dimensions, we say that reduction-3 resembles the original object. How?

Let us, find the ratio of corresponding sides of original object and reduction - 3, what do you notice?

$$\frac{\text{Length of the original}}{\text{length of the reduction -3}} = \frac{4}{3}$$

$$\frac{\text{breadth of the original}}{\text{breadth of the reduction -3}} = \frac{3}{2.25} = \frac{3 \times 4}{2.25 \times 4} = \frac{12}{9} = \frac{4}{3}$$

We notice that the ratios of corresponding sides are equal.

Here all the corresponding angles are right angles and are equal.

Hence we conclude that "two polygons are similar if their corresponding angles are congruent and lengths of corresponding sides are proportional".

Find the ratio of corresponding sides for all other reductions.

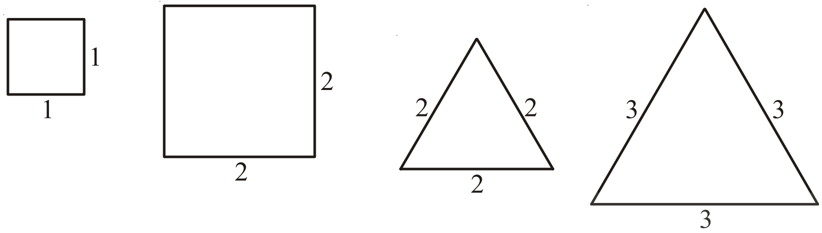
8.1.3 Where do we find the application of similarly?

Engineers draw elevation plans, similar to the building to be constructed, D.T.P operators draw diagrams on the computer which can be magnified in proportion to make banners. Photographer makes photo prints images of same by enlarging or reducing without distortion is based on principle of proportion. Diagrams of science apparatus and maps in social studies you have come a cross are in proportion i.e. similar to the original objects.

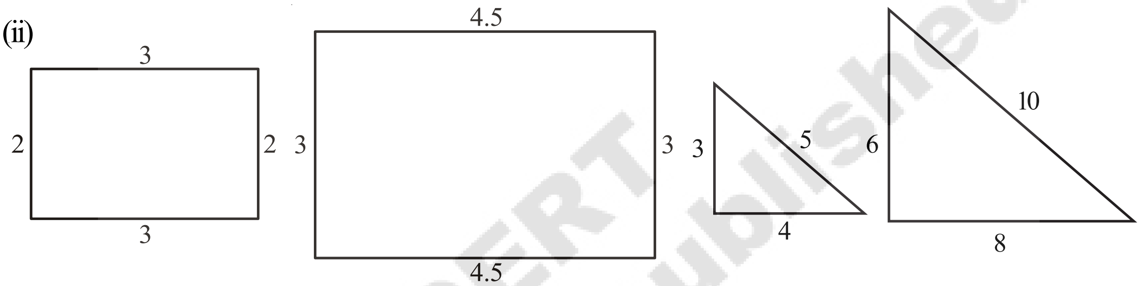
Checking the similarity

Observe the following pairs of similar figures. Measure their sides and find the ratio between corresponding sides, also find the corresponding angles, what do you observe?

(i)



(ii)



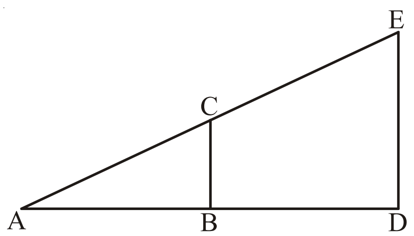
Complete the table based on the figures given in previous page.

Ratio of corresponding sides	Corresponding angles
(i) Square = $\frac{1}{2} = \frac{1}{2}$	$(90^o,90^o,90^o,90^o) = (90^o,90^o,90^o,90^o)$
(ii) Equilateral triangle = $\frac{2}{3} = \frac{2}{3} = \frac{2}{3}$	$(60^o,60^o,60^o) = (60^o,60^o,60^o)$
(iii) Rectangle = $\frac{2}{3}$ = ...	$(90^o,90^o,90^o,90^o) = (90^o,90^o,90^o,90^o)$
(iv) Right triangle = $\frac{3}{6}$ = ...	(......., ..., ...) = (......., ..., ...)

In every pair of these examples, we find the ratios of corresponding sides are equal and the pairs of corresponding angles are equal.

Consider another example.

In the adjacent figure if two triangles ΔABC and ΔADE are similar then we write it as $\Delta ABC \sim \Delta ADE$. If those two triangles are placed one over the other. You will find that the pairs of corresponding angles are equal



(i.e.) $\angle A \cong \angle A$

$\angle B \cong \angle D$ (why?)

$\angle C \cong \angle E$ (Why?)

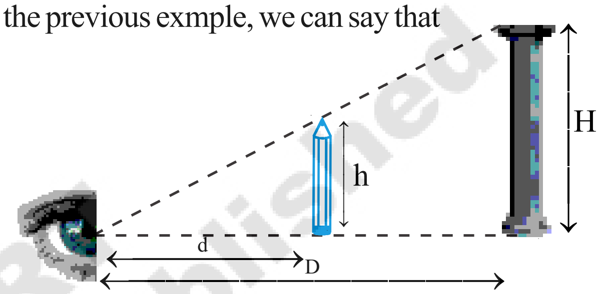
and the ratio of corresponding sides are equal

(i.e.) $\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$

Let us see an illustration how the principle of similar triangles helps to find out the heights of the objects located at for away.

Illustration: A girl stretched her arm towards a pillar, holding a pencil vertically in her arm by standing at a certain distance from the pillar. She found that the pencil exactly covers the pillar as in figure. If we compare this illustration with the previous exmple, we can say that

$$\frac{\text{Height of the pillar(H)}}{\text{Length of the pencil(h)}} = \frac{\text{Distance of pillar from the girl(D)}}{\text{Length of her arm(d)}}$$

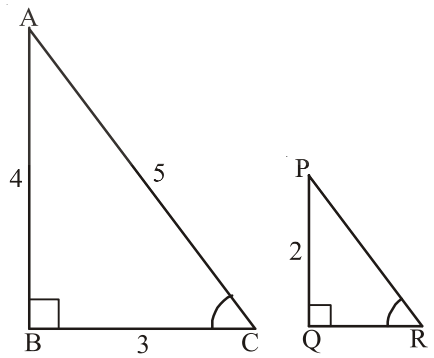


By measuring the length of the pencil, length of her arm and distance of the pillar, we can estimate the height of the pillar.

Try This

Stretch your hand, holding a scale in your hand vertically and try to cover your school building by the scale (Adjust your distance from the building). Draw the figure and estimate height of the school building.

Example 1: In the adjacent figure $\Delta ABC \sim \Delta PQR$, and $\angle C = 53^0$. Find the side PR and $\angle P$.



Solution: $\Delta ABC \sim \Delta PQR$

When two triangles are similar their corresponding angles are equal and corresponding sides are in proportion.

$$\frac{PR}{AC} = \frac{PQ}{AB} \Rightarrow \frac{PR}{5} = \frac{2}{4}$$

$$PR = \frac{2}{4} \times 5 = 2.5$$

Again

$\angle R = \angle C = 53^0$

Sum of all three angles in a triangle as 180^o

i.e. $\angle P + \angle Q + \angle R = 180^o$

$\angle P + 90^o + 53^o = 180^o$

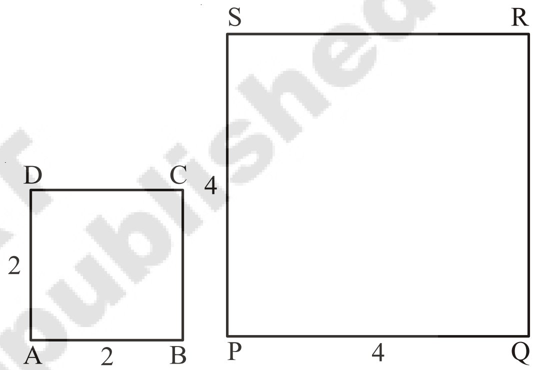
$\angle P = 180^o - 143^o = 37^o$

Example 2: Draw two squares of different sides. Can you say they are similar? Explain. Find the ratio of their perimeters and areas. What do you observe?

Solution: Let us draw two squares of sides 2 cm and 4 cm. As all the sides are in proportion

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SP} = \frac{2}{4}$

$= \frac{1}{2}$



And all the pairs of corresponding angles are 90^o

So square ABCD ~ square PQRS

Perimeter of □ ABCD = $4 \times 2 = 8$ cm

Perimeter of □ PQRS = $4 \times 4 = 16$ cm

Ratio of their perimeters = 8 : 16 = 1 : 2 Ratio of their perimeters is same as ratio of their corresponding sides.

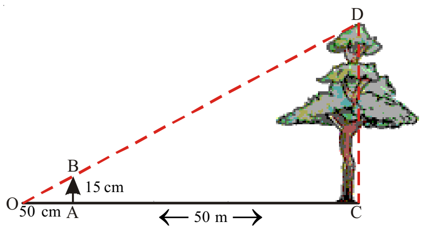
Area of ABCD $= 2 \times 2 = 4\ cm^2$

Area of PQRS $= 4 \times 4 = 16\ cm^2$

Ratio of their areas $= 4 : 16 = 1 : 4 = 1^2 : 2^2$

= Ratio of squares of the corresponding sides.

Example 3: Jagadeesh tried to estimate the height of a tree by covering the height with a vertical scale holding it at a distance of 50 cm from his eyes along horigental line and draw the figure as shown. If scale measurement of the tree is 15 cm and distance of the tree from him is 50 m. Find the actual height of the tree.



Solution: From the figure Δ OAB ~ Δ OCD

Corresponding sides of two similar triangles are in proportion.

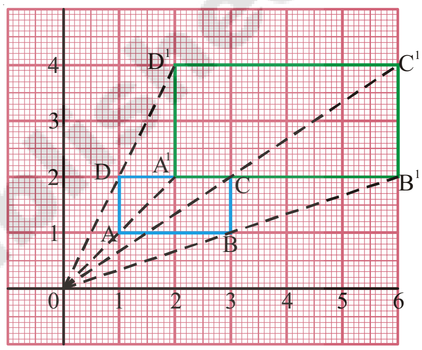
$$\therefore \frac{OA}{OC} = \frac{AB}{CD} = \frac{OB}{OD}$$

$$\therefore \frac{0.5}{50} = \frac{0.15}{CD} \Rightarrow CD = \frac{50 \times 0.15}{0.5} = 15 \text{ m}$$

$\therefore$ Height of the tree $= 15$ m

8.2 Dilations:

Some times we need to enlarge the figures say for example while making cutouts, and some times we reduce the figures during designing. Here in every case the figures must be similar to the original. This means we need to draw enlarged or reduced similar figures in daily life. This method of drawing enlarged or reduced similar figure is called 'Dilation'.



Observe the following dilation ABCD, it is a rectangle drawn on a graph sheet.

Every vertex A, B, C, D are joined from the sign 'O' and produced to double the length upto A^1, B^1, C^1 and D^1 respectively. Then A^1, B^1, C^1, D^1 are joined to form a rectangle which two times has enlarged sides of ABCD. Here, O is called centre of dilation and $\frac{OA^1}{OA} = \frac{2}{1} = 2$ is called scale factor.

Do This

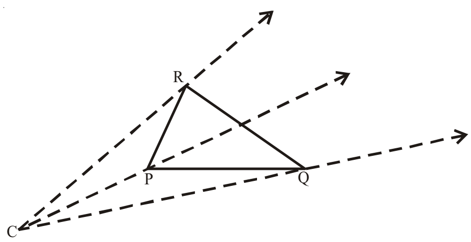
1. Draw a triangle on a graph sheet and draw its dilation with scale factor 3. Are those two figures are similar?
2. Try to extend the projection for anyother diagram and draw squares with scale factor 4, 5. What do you observe?

8.2.1 Constructing a Dilation:

Example 4: Construct a dilation, with scale factor 2, of a triangle using only a ruler and compasses.

Solution:

Step 1: Draw a Δ PQR and choose the center of dilation C which is not on the triangle. Join every vertex of the triangle from C and produce.

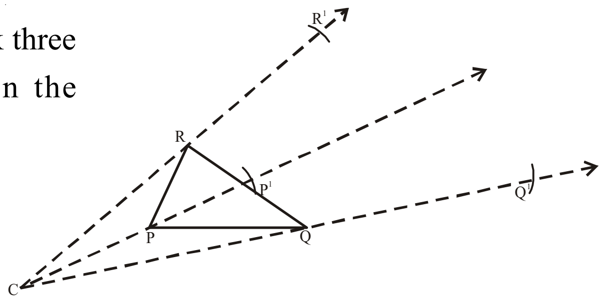


Step 2: By using compasses, mark three points P^1, Q^1 and R^1 on the projections so that

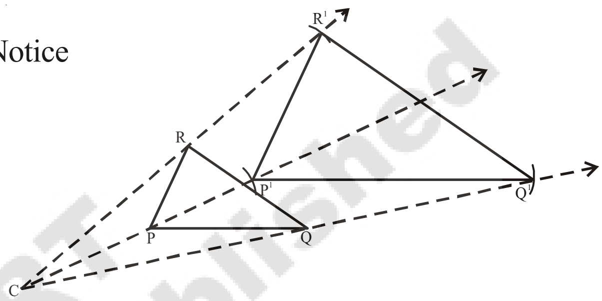
$CP^1 = k\,(CP) = 2\ CP$

$CQ^1 = 2\ CQ$

$CR^1 = 2\ CR$



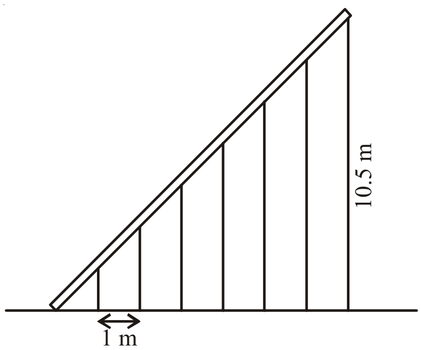
Step 3: Join P^1Q^1, Q^1R^1 and R^1P^1. Notice that $\Delta\, P^1Q^1R^1 \sim \Delta\, PQR$



Exercise - 8.1

1. Name five pairs of congruent objects, you use daily.
2. (a) Draw two congruent figures. Are they similar? Explain
   (b) Take two similar shapes. If you slide rotate or flip one of them, does the similarity remain?
3. If $\Delta\, ABC \cong \Delta\, NMO$, name the congruent sides and angles.
4. State whether the following statements are true. Explain with reason.
   (i) Two squares of side 3 cm each and one of them rotated through 45^o are congruent.
   (ii) Any two right triangles with hypotenuse 5 cm, are congruent.
   (iii) Any two circles of radii 4 cm each are congruent.
   (iv) Two equilateral triangles of side 4 cm each but labeled as Δ ABC and Δ LHN are not congruent.
   (v) Mirror image of a polygon is congruent to the original.
5. Draw a polygon on a square dot sheet. Also draw congruent figures in different directions and mirror image of it.
6. Using a square dot sheet or a graph sheet draw a rectangle and construct a similar figure. Find the perimeter and areas of both and compare their ratios with the ratio of their corresponding sides.

7. 7 pillars are used to hold a slant iron gudder as shown in the figure. If the distance between every two pillars is 1 m and height of the last piller is 10.5 m. Find the height of pillar.

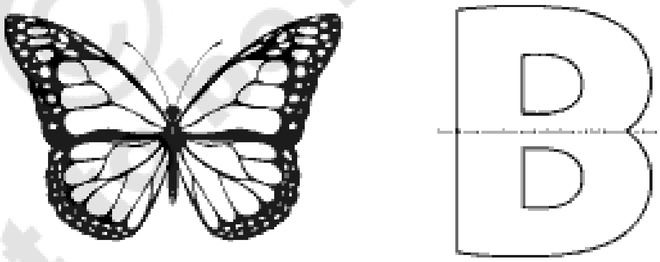


8. Standing at 5 m apart from a vertical pole of height 3 m, Sudha observed a building at the back of the piller that tip of the pillar is in line with the top of the building. If the distance between pillar and building is 10 m, estimate the height of the building. [Here height of Sudha is neglected]

9. Draw a quadrilateral of any measurements. Construct a dilation of scale factor 3. Measure their corresponding sides and verify whether they are similar.

8.3 Symmetry:

Look at the following figures. If we fold them exactly to their halves, one half of each figure exactly coinsides with other half.

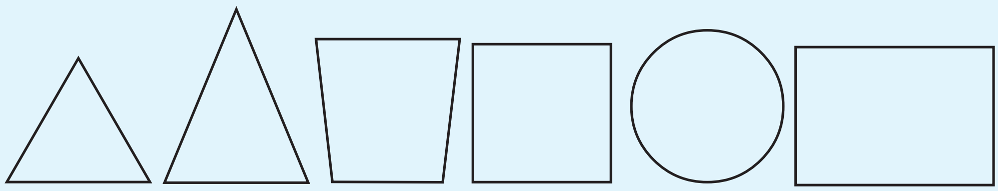


What do we call such figures? What do we call the line along which we fold the figures so that one half coincides with the other? Do you recollect from earlier classes.

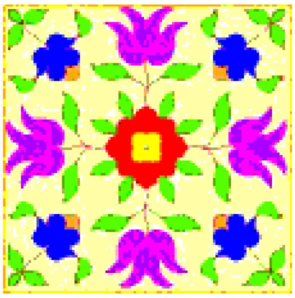
They are called symmetric figures and the line which cuts them exactly into two halves is called line of symmetry.

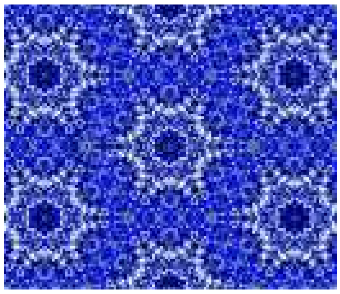
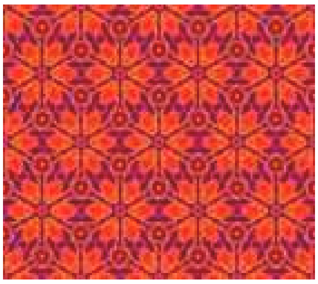
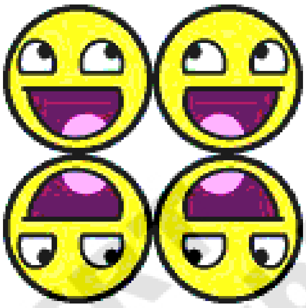
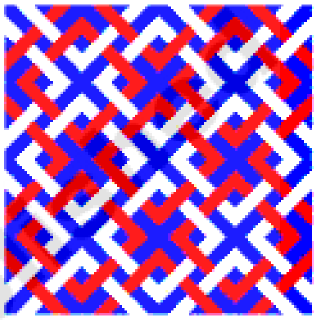
Do These

Draw all possible lines of symmetry for the following figures.



Observe the following symmetric designs which we see around us.



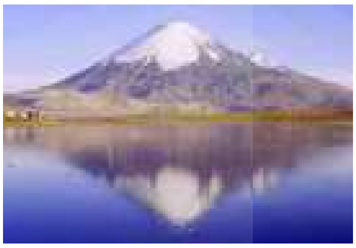





All these designs are products of different kinds of symmetry.

Here, the dog has her face made perfectly symmetrical with a bit of photo magic. Do you observe a vertical line at the center?

It is called 'line of symmetry' or 'mirror line'.

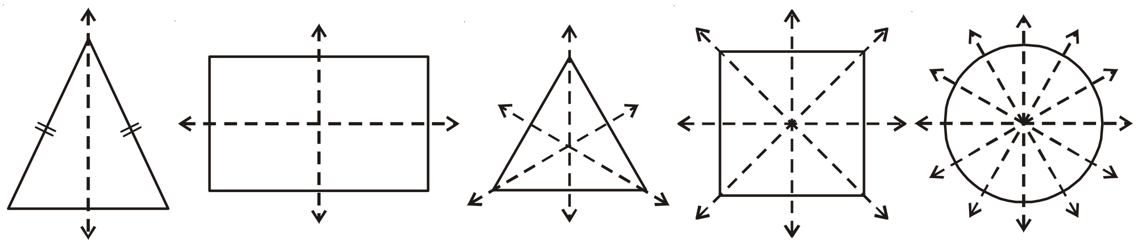
We call this symmetry as 'Reflection symmetry' or 'Mirror symmetry'.



Consider another example, reflection of a hill in a lake. It is also reflection symmetry and line of symmetry is a horizontal line that separating the hill and its image. This may not be perfectly symmetric because lower part is blurred by the lake surface.

8.3.1 Rotational symmetry

Observe the lines of symmetry in the following.



Different geometrical figures have different number of axes of symmetry.

Rotate each figure given above, about its centre and find out how many times it resembles its initial position during its one complete rotation.

For example, rectangle has two lines or axes of symmetry. When a rectangle is rotated about its center its shape resembles the initial position two times. We call this number as 'order of rotation'.

Tabulate your findings in the following table.

Geometrical figure	No. of axes of symmetry	No. of times resumes its initial position	Order of rotation
Isosceless triangle	....	....	....
Rectangle	2	2	2
Equilateral triangle	....	....	....
Square	....	....	....
Circle	....	....	....

Think, Discuss and Write

1. What is the relation between order of rotation and number of axes of symmetry of a geometrical figure?

2. How many axes of symmetry does a regular polygon has? Is there any relation between number of sides and order of rotation of a regular.polygon?

8.3.2 Point symmetry

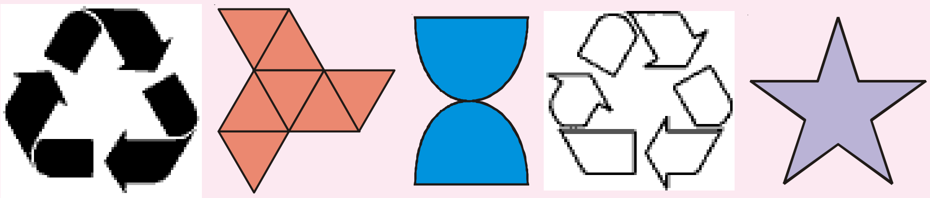
Observe the adjacent figure. Does it have line of symmetry? It does not have line symmetry, but it has another type of symmetry. The figure looks the same either you see it from upside or from down side. i.e., from any two opposite directions. This is called point symmetry. If you look at the figure you may observe that every part of it has a matching point. If you draw a line through its centre, it cuts the diagram on either sides of the center at equal distance. Draw some more lines through center and verify. Now this figure is said to have 'point symmetry'.

We also observe some letters of English alphabet have point symmetry too.

X H I S N Z

Try These

1. Identify which of the following have point symmetry.

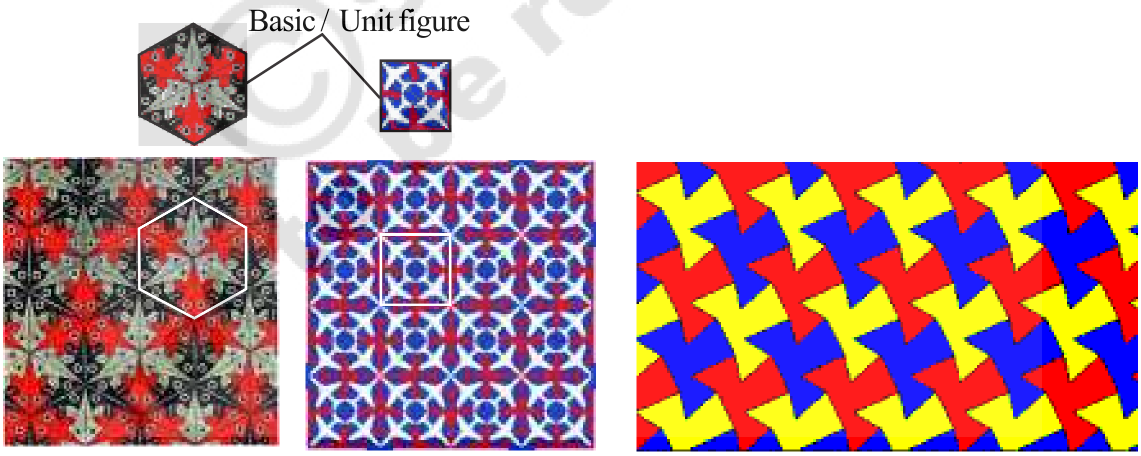


2. Which of the above figures are having symmetry?
3. What can you say about the relation between line symmetry and point symmetry?

8.3.3 Applications of symmetry

- Majority of the objects what we use have atleast one type of symmetry.
- Most of he Machine made products are symmetric. This speeds up the production.

Observe these patterns



Where do you find these? We find these patterns in floor designs and fabric painting etc.

How these patterns are formed?

Usually these patterns are formed by arranging congruent figures or mirror images side by side in all the directions to spread upon an area without any overlaps or gaps.

This is called tessellation. This enhances the beauty of the diagrams.

Are they symmetric as a whole?

Does the basic figure which is used to form the tessellation is symmetric?

You can observe that only some patterns have symmetry as a whole as in fig(b) and others does n't have any symmetry as a whole as in fig(a), through the basic figures/unit figures are symmetric.

Observe the following tessellations again.

What are the basic shapes used in these tessellations?

Fig. (a)

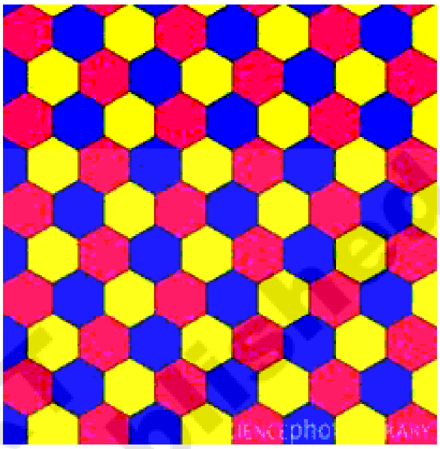
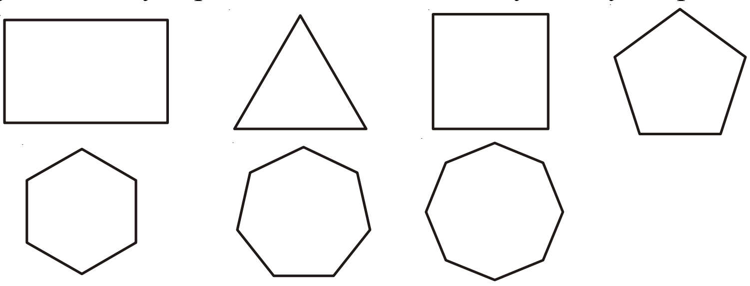


Fig. (b)

You may notice that the basic shapes used to draw tessellation are pentagon, rectangle, squares and equilateral triangle. Most tessellation can be formed with these shapes.

Exercise - 8.2

1. Cut the bold type English alphabets (capital) and paste in your note book. Draw possible number of lines of symmetry for each of the letter.
   (i) How many letters have no linear symmetry?
   (ii) How many letters have one line of symmetry?
   (iii) How many letters have two lines of symmetry?
   (iv) How many letters have more than two lines of symmetry?
   (v) Which of them have rotational symmetry?
   (vi) Which of then have point symmetry?
2. Draw lines of symmetry for the following figures. Identify which of them have point symmetry. Is there any implication between lines of symmetry and point symmetry?



3. Name some natural objects with faces which have atleast one line of symmetry.

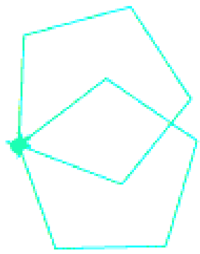
4. Draw three tessellations and name the basic shapes used on your tessellation.

What we have discussed

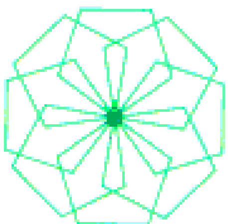
- Shapes are said to be congruent if they have same shape and size.
- Shapes are said to be similar if they have same shapes but in different size.
- If we flip, slide or turn the congruent/similar shapes their congruence/similarity remain the same.
- Some figures may have more than one line of symmetry.
- Symmetry is of three types namely line symmetry, rotational symmetry and point symmetry.
- With rotational symmetry, the figure is rotated around a central point so that it appears two or more times same as original. The number of times for which it appears the same is called the order.
- The method of drawing enlarged or reduced similar figures is called Dialation.
- The patterns formed by repeating figures to fill a plane without gaps or overlaps are called tessellations.

Rotating a Polygon

The following procedure draws a regular polygon with **n sides**.



Interesting pictures can be drawn by repeating these rotations through a full circle. The angle of rotation is found by dividing 360^0 by the number of times the figure is repeated. The number of repeatitions in the following diagram is 8.



If we take square what is the figure formed by rotating one of its vertex and by rotating its diagonal mid point.