Simple Equations

Chapter 4

4.1 A Mind-reading Game!

The teacher has said that she would be starting a new chapter in mathematics and it is going to be simple equations. Appu, Sarita and Ameena have revised what they learnt in algebra chapter in Class VI. Have you? Appu, Sarita and Ameena are excited because they have constructed a game which they call mind reader and they want to present it to the whole class.

The teacher appreciates their enthusiasm and invites them to present their game. Ameena begins; she asks Sara to think of a number, multiply it by 4 and add 5 to the product. Then, she asks Sara to tell the result. She says it is 65. Ameena instantly declares that the number Sara had thought of is 15. Sara nods. The whole class including Sara is surprised.

It is Appu's turn now. He asks Balu to think of a number, multiply it by 10 and subtract 20 from the product. He then asks Balu what his result is? Balu says it is 50. Appu immediately tells the number thought by Balu. It is 7, Balu confirms it.

Everybody wants to know how the 'mind reader' presented by Appu, Sarita and Ameena works. Can you see how it works? After studying this chapter and chapter 12, you will very well know how the game works.

4.2 Setting up of an Equation

Let us take Ameena's example. Ameena asks Sara to think of a number. Ameena does not know the number. For her, it could be anything 1, 2, 3, . . ., 11, . . . , 100, ... Let us denote this unknown number by a letter, say x. You may use y or t or some other letter in place of x. It does not matter which letter we use to denote the unknown number Sara has thought of. When Sara multiplies the number by 4, she gets $4x$. She then adds 5 to the product, which gives $4x + 5$. The value of $(4x + 5)$ depends on the value of x. Thus if $x = 1$, $4x + 5 = 4 \times 1 + 5 = 9$. This means that if Sara had 1 in her mind, her result would have been 9. Similarly, if she thought of 5, then for $x = 5$, $4x + 5 = 4 \times 5 + 5 = 25$; Thus if Sara had chosen 5, the result would have been 25.

To find the number thought by Sara let us work backward from her answer 65. We have to find x such that

$$4x + 5 = 65 \qquad (4.1)$$

Solution to the equation will give us the number which Sara held in her mind.

Let us similarly look at Appu's example. Let us call the number Balu chose as y. Appu asks Balu to multiply the number by 10 and subtract 20 from the product. That is, from y, Balu first gets $10y$ and from there $(10y - 20)$. The result is known to be 50.

Therefore, $$10y - 20 = 50 \qquad (4.2)$$

The solution of this equation will give us the number Balu had thought of.

4.3 Review of what We Know

Note, (4.1) and (4.2) are equations. Let us recall what we learnt about equations in Class VI. *An equation is a condition on a variable.* In equation (4.1), the variable is x; in equation (4.2), the variable is y.

The word *variable* means something that can vary, i.e. change. A **variable** *takes on different numerical values; its value is not fixed. Variables are denoted usually by letters of the alphabets, such as x, y, z, l, m, n, p, etc. From variables, we form expressions.* The expressions are formed by performing operations like addition, subtraction, multiplication and division on the variables. From x, we formed the expression $(4x + 5)$. For this, first we multiplied x by 4 and then added 5 to the product. Similarly, from y, we formed the expression $(10y - 20)$. For this, we multiplied y by 10 and then subtracted 20 from the product. All these are examples of expressions.

The value of an expression thus formed depends upon the chosen value of the variable. As we have already seen, when $x = 1$, $4x + 5 = 9$; when $x = 5$, $4x + 5 = 25$. Similarly,

when $x = 15$, $4x + 5 = 4 \times 15 + 5 = 65$;

when $x = 0$, $4x + 5 = 4 \times 0 + 5 = 5$; and so on.

Equation (4.1) is a condition on the variable x. It states that the value of the expression $(4x + 5)$ is 65. The condition is satisfied when $x = 15$. It is the solution to the equation $4x + 5 = 65$. When $x = 5$, $4x + 5 = 25$ and not 65. Thus $x = 5$ is not a solution to the equation. Similarly, $x = 0$ is not a solution to the equation. No value of x other than 15 satisfies the condition $4x + 5 = 65$.

Try These

The value of the expression $(10y - 20)$ depends on the value of y. Verify this by giving five different values to y and finding for each y the value of $(10y - 20)$. From the different values of $(10y - 20)$ you obtain, do you see a solution to $10y - 20 = 50$? If there is no solution, try giving more values to y and find whether the condition $10y - 20 = 50$ is met.

4.4 What Equation is?

In an equation there is always an **equality** sign. The equality sign shows that the value of the expression to the left of the sign (the left hand side or LHS) is equal to the value of the expression to the right of the sign (the right hand side or RHS). In equation (4.1), the LHS is $(4x + 5)$ and the RHS is 65. In equation (4.2), the LHS is $(10y – 20)$ and the RHS is 50.

> If there is some sign other than the equality sign between the LHS and the RHS, it is not an equation. Thus, $4x + 5 > 65$ is not an equation.
>
> It says that, the value of $(4x + 5)$ is greater than 65.
>
> Similarly, $4x + 5 < 65$ is not an equation. It says that the value of $(4x + 5)$ is smaller than 65.

In equations, we often find that the RHS is just a number. In Equation (4.1), it is 65 and in equation (4.2), it is 50. But this need not be always so. The RHS of an equation may be an expression containing the variable. For example, the equation

$$4x + 5 = 6x – 25$$

has the expression $(4x + 5)$ on the left and $(6x – 25)$ on the right of the equality sign.

In short, an equation is a condition on a variable. The condition is that two expressions should have equal value. Note that at least one of the two expressions must contain the variable.

We also note a simple and useful property of equations. The equation $4x +5 = 65$ is the same as $65 = 4x + 5$. Similarly, the equation $6x – 25 = 4x +5$ is the same as $4x + 5 = 6x – 25$. *An equation remains the same, when the expressions on the left and on the right are interchanged.* This property is often useful in solving equations.

Example 1 Write the following statements in the form of equations:

(i) The sum of three times x and 11 is 32.

(ii) If you subtract 5 from 6 times a number, you get 7.

(iii) One fourth of m is 3 more than 7.

(iv) One third of a number plus 5 is 8.

Solution

(i) Three times x is $3x$.

Sum of $3x$ and 11 is $3x + 11$. The sum is 32.

The equation is $3x + 11 = 32$.

(ii) Let us say the number is z; z multiplied by 6 is $6z$.

Subtracting 5 from $6z$, one gets $6z – 5$. The result is 7.

The equation is $6z – 5 = 7$

(iii) One fourth of m is $\frac{m}{4}$.

It is greater than 7 by 3. This means the difference ($\frac{m}{4}-7$) is 3.

The equation is $\frac{m}{4}-7=3$.

(iv) Take the number to be n. One third of n is $\frac{n}{3}$.

This one-third plus 5 is $\frac{n}{3}+5$. It is 8.

The equation is $\frac{n}{3}+5=8$.

EXAMPLE 2 Convert the following equations in statement form:

(i) $x-5=9$ (ii) $5p=20$ (iii) $3n+7=1$ (iv) $\frac{m}{5}-2=6$

SOLUTION (i) Taking away 5 from x gives 9.

(ii) Five times a number p is 20.

(iii) Add 7 to three times n to get 1.

(iv) You get 6, when you subtract 2 from one-fifth of a number m.

What is important to note is that for a given equation, **not just one, but many** statement forms can be given. For example, for Equation (i) above, you can say:

> **TRY THESE**
>
> Write atleast one other form for each equation (ii), (iii) and (iv).

Subtract 5 from x, you get 9.

or The number x is 5 more than 9.

or The number x is greater by 5 than 9.

or The difference between x and 5 is 9, and so on.

EXAMPLE 3 Consider the following situation:

Raju's father's age is 5 years more than three times Raju's age. Raju's father is 44 years old. Set up an equation to find Raju's age.

SOLUTION We do not know Raju's age. Let us take it to be y years. Three times Raju's age is $3y$ years. Raju's father's age is 5 years more than $3y$; that is, Raju's father is $(3y + 5)$ years old. It is also given that Raju's father is 44 years old.

Therefore, $$3y + 5 = 44 \tag{4.3}$$

This is an equation in y. It will give Raju's age when solved.

EXAMPLE 4 A shopkeeper sells mangoes in two types of boxes, one small and one large. A large box contains as many as 8 small boxes plus 4 loose mangoes. Set up an equation which gives the number of mangoes in each small box. The number of mangoes in a large box is given to be 100.

SOLUTION Let a small box contain m mangoes. A large box contains 4 more than 8 times m, that is, 8m + 4 mangoes. But this is given to be 100. Thus

$$8m + 4 = 100 \tag{4.4}$$

You can get the number of mangoes in a small box by solving this equation.

EXERCISE 4.1

1. Complete the last column of the table.

S. No.	Equation	Value	Say, whether the Equation is Satisfied. (Yes/ No)
(i)	$x + 3 = 0$	$x = 3$	
(ii)	$x + 3 = 0$	$x = 0$	
(iii)	$x + 3 = 0$	$x = -3$	
(iv)	$x - 7 = 1$	$x = 7$	
(v)	$x - 7 = 1$	$x = 8$	
(vi)	$5x = 25$	$x = 0$	
(vii)	$5x = 25$	$x = 5$	
(viii)	$5x = 25$	$x = -5$	
(ix)	$\frac{m}{3} = 2$	$m = -6$	
(x)	$\frac{m}{3} = 2$	$m = 0$	
(xi)	$\frac{m}{3} = 2$	$m = 6$	

2. Check whether the value given in the brackets is a solution to the given equation or not:

(a) $n + 5 = 19$ $(n = 1)$ (b) $7n + 5 = 19$ $(n = -2)$ (c) $7n + 5 = 19$ $(n = 2)$

(d) $4p - 3 = 13$ $(p = 1)$ (e) $4p - 3 = 13$ $(p = -4)$ (f) $4p - 3 = 13$ $(p = 0)$

3. Solve the following equations by trial and error method:

(i) $5p + 2 = 17$ (ii) $3m - 14 = 4$

4. Write equations for the following statements:

(i) The sum of numbers x and 4 is 9. (ii) 2 subtracted from y is 8.

(iii) Ten times a is 70. (iv) The number b divided by 5 gives 6.

(v) Three-fourth of t is 15. (vi) Seven times m plus 7 gets you 77.

(vii) One-fourth of a number x minus 4 gives 4.

(viii) If you take away 6 from 6 times y, you get 60.

(ix) If you add 3 to one-third of z, you get 30.

5. Write the following equations in statement forms:

(i) $p + 4 = 15$ (ii) $m - 7 = 3$ (iii) $2m = 7$ (iv) $\frac{m}{5} = 3$

(v) $\frac{3m}{5} = 6$ (vi) $3p + 4 = 25$ (vii) $4p - 2 = 18$ (viii) $\frac{p}{2} + 2 = 8$

6. Set up an equation in the following cases:

(i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. (Take m to be the number of Parmit's marbles.)

(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. (Take Laxmi's age to be y years.)

(iii) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. (Take the lowest score to be l.)

(iv) In an isosceles triangle, the vertex angle is twice either base angle. (Let the base angle be b in degrees. Remember that the sum of angles of a triangle is 180 degrees).

4.4.1 Solving an Equation

Consider an equality $8 - 3 = 4 + 1$ (4.5)

The equality (4.5) holds, since both its sides are equal (each is equal to 5).

- Let us now add 2 to both sides; as a result

  LHS = $8 - 3 + 2 = 5 + 2 = 7$ RHS = $4 + 1 + 2 = 5 + 2 = 7$.

  Again the equality holds (i.e., its LHS and RHS are equal).

  Thus *if we add the same number to both sides of an equality, it still holds.*

- Let us now subtract 2 from both the sides; as a result,

  LHS = $8 - 3 - 2 = 5 - 2 = 3$ RHS = $4 + 1 - 2 = 5 - 2 = 3$.

  Again, the equality holds.

  Thus *if we subtract the same number from both sides of an equality, it still holds.*

- Similarly, *if we multiply or divide both sides of the equality by the same non-zero number, it still holds.*

  For example, let us multiply both the sides of the equality by 3, we get

  LHS = $3 \times (8 - 3) = 3 \times 5 = 15$, RHS = $3 \times (4 + 1) = 3 \times 5 = 15$.

  The equality holds.

  Let us now divide both sides of the equality by 2.

  $$\text{LHS} = (8 - 3) \div 2 = 5 \div 2 = \frac{5}{2}$$

  $$\text{RHS} = (4+1) \div 2 = 5 \div 2 = \frac{5}{2} = \text{LHS}$$

  Again, the equality holds.

If we take any other equality, we shall find the same conclusions.

Suppose, we do not observe these rules. Specificially, suppose we add different numbers, to the two sides of an equality. We shall find in this case that the equality does not hold (i.e., its both sides are not equal). For example, let us take again equality (4.5),

$$8 - 3 = 4 + 1$$

add 2 to the LHS and 3 to the RHS. The new LHS is $8 - 3 + 2 = 5 + 2 = 7$ and the new RHS is $4 + 1 + 3 = 5 + 3 = 8$. The equality does not hold, because the new LHS and RHS are not equal.

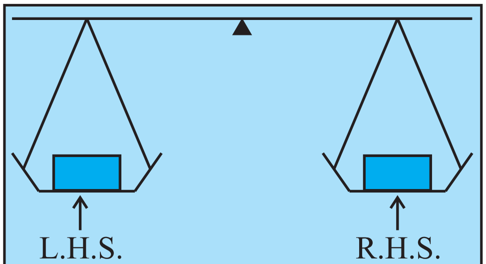
Thus if we fail to do the same mathematical operation on both sides of an equality, the equality does not hold.

The equality that involves variables is an equation.

These conclusions are also valid for equations, as in each equation variable represents a number only.

Often an equation is said to be like a weighing balance. Doing a mathematical operation on an equation is like adding weights to or removing weights from the pans of a weighing balance.

An equation is like a weighing balance with equal weights on both its pans, in which case the arm of the balance is exactly horizontal. If we add the same weights to both the pans, the arm remains horizontal. Similarly, if we remove the same weights from both the pans, the arm remains horizontal. On the other hand if we add different weights to the pans or remove different weights from them, the balance is tilted; that is, the arm of the balance does not remain horizontal.



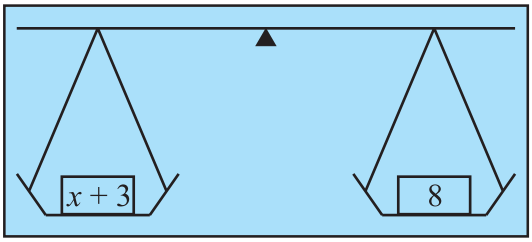
A balanced equation is like a weighing balance with equal weights in the two pans.

We use this principle for solving an equation. Here, ofcourse, the balance is imaginary and numbers can be used as weights that can be physically balanced against each other. This is the real purpose in presenting the principle. Let us take some examples.

- Consider the equation: $x + 3 = 8$ (4.6)

  We shall subtract 3 from both sides of this equation.

  The new LHS is $x + 3 - 3 = x$ and the new RHS is $8 - 3 = 5$



Why should we subtract 3, and not some other number? Try adding 3. Will it help? Why not?

It is because subtracting 3 reduces the LHS to x.

Since this does not disturb the balance, we have

$$\text{New LHS} = \text{New RHS} \quad \text{or} \quad x = 5$$

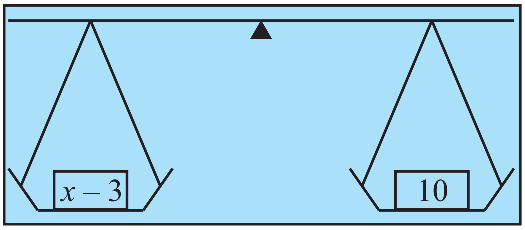
which is exactly what we want, the solution of the equation (4.6).

To confirm whether we are right, we shall put $x = 5$ in the original equation. We get LHS $= x + 3 = 5 + 3 = 8$, which is equal to the RHS as required.

By doing the right mathematical operation (i.e., subtracting 3) on both the sides of the equation, we arrived at the solution of the equation.

- Let us look at another equation $\qquad x - 3 = 10 \qquad (4.7)$

  What should we do here? We should add 3 to both the sides, By doing so, we shall retain the balance and also the LHS will reduce to just x.



New LHS $= x - 3 + 3 = x$, New RHS $= 10 + 3 = 13$

Therefore, $x = 13$, which is the required solution.

By putting $x = 13$ in the original equation (4.7) we confirm that the solution is correct:

LHS of original equation $= x - 3 = 13 - 3 = 10$

This is equal to the RHS as required.

- Similarly, let us look at the equations

$$5y = 35 \qquad (4.8)$$

$$\frac{m}{2} = 5 \qquad (4.9)$$

In the first case, we shall divide both the sides by 5. This will give us just y on LHS

$$\text{New LHS} = \frac{5y}{5} = \frac{5 \times y}{5} = y, \qquad \text{New RHS} = \frac{35}{5} = \frac{5 \times 7}{5} = 7$$

Therefore, $\qquad y = 7$

This is the required solution. We can substitute $y = 7$ in Eq. (4.8) and check that it is satisfied.

In the second case, we shall multiply both sides by 2. This will give us just m on the LHS

The new LHS $= \frac{m}{2} \times 2 = m$. The new RHS $= 5 \times 2 = 10$.

Hence, $m = 10$ (It is the required solution. You can check whether the solution is correct).

One can see that in the above examples, the operation we need to perform depends on the equation. Our attempt should be to get the variable in the equation separated. Sometimes, for doing so we may have to carry out more than one mathematical operation. Let us solve some more equations with this in mind.

EXAMPLE 5 Solve: (a) $3n + 7 = 25 \qquad (4.10)$

(b) $2p - 1 = 23 \qquad (4.11)$

SOLUTION

(a) We go stepwise to separate the variable n on the LHS of the equation. The LHS is $3n + 7$. We shall first subtract 7 from it so that we get $3n$. From this, in the next step we shall divide by 3 to get n. Remember we must do the same operation on both sides of the equation. Therefore, subtracting 7 from both sides,

$$3n + 7 - 7 = 25 - 7 \qquad \text{(Step 1)}$$

or $\qquad 3n = 18$

Now divide both sides by 3,

$$\frac{3n}{3} = \frac{18}{3} \qquad \text{(Step 2)}$$

or $\quad n = 6$, which is the solution.

(b) What should we do here? First we shall add 1 to both the sides:

$$2p - 1 + 1 = 23 + 1 \qquad \text{(Step 1)}$$

or $\quad 2p = 24$

Now divide both sides by 2, we get $\frac{2p}{2} = \frac{24}{2}$ (Step 2)

or $\quad p = 12$, which is the solution.

One good practice you should develop is to check the solution you have obtained. Although we have not done this for (a) above, let us do it for this example.

Let us put the solution $p = 12$ back into the equation.

$$\text{LHS} = 2p - 1 = 2 \times 12 - 1 = 24 - 1$$
$$= 23 = \text{RHS}$$

The solution is thus checked for its correctness.

Why do you not check the solution of (a) also?

We are now in a position to go back to the mind-reading game presented by Appu, Sarita, and Ameena and understand how they got their answers. For this purpose, let us look at the equations (4.1) and (4.2) which correspond respectively to Ameena's and Appu's examples.

- First consider the equation $4x + 5 = 65$. (4.1)

  Subtracting 5 from both sides, $4x + 5 - 5 = 65 - 5$.

  i.e. $\quad 4x = 60$

  Divide both sides by 4; this will separate x. We get $\frac{4x}{4} = \frac{60}{4}$

  or $\quad x = 15$, which is the solution. (Check, if it is correct.)

- Now consider, $10y - 20 = 50$ (4.2)

  Adding 20 to both sides, we get $10y - 20 + 20 = 50 + 20$ or $10y = 70$

  Dividing both sides by 10, we get $\frac{10y}{10} = \frac{70}{10}$

  or $\quad y = 7$, which is the solution. (Check if it is correct.)

You will realise that exactly these were the answers given by Appu, Sarita and Ameena. They had learnt to set up equations and solve them. That is why they could construct their mind reader game and impress the whole class. We shall come back to this in Section 4.7.

EXERCISE 4.2

1. Give first the step you will use to separate the variable and then solve the equation:

(a) $x - 1 = 0$ (b) $x + 1 = 0$ (c) $x - 1 = 5$ (d) $x + 6 = 2$

(e) $y - 4 = -7$ (f) $y - 4 = 4$ (g) $y + 4 = 4$ (h) $y + 4 = -4$

2. Give first the step you will use to separate the variable and then solve the equation:

(a) $3l = 42$ (b) $\frac{b}{2} = 6$ (c) $\frac{p}{7} = 4$ (d) $4x = 25$

(e) $8y = 36$ (f) $\frac{z}{3} = \frac{5}{4}$ (g) $\frac{a}{5} = \frac{7}{15}$ (h) $20t = -10$

3. Give the steps you will use to separate the variable and then solve the equation:

(a) $3n - 2 = 46$ (b) $5m + 7 = 17$ (c) $\frac{20p}{3} = 40$ (d) $\frac{3p}{10} = 6$

4. Solve the following equations:

(a) $10p = 100$ (b) $10p + 10 = 100$ (c) $\frac{p}{4} = 5$ (d) $\frac{-p}{3} = 5$

(e) $\frac{3p}{4} = 6$ (f) $3s = -9$ (g) $3s + 12 = 0$ (h) $3s = 0$

(i) $2q = 6$ (j) $2q - 6 = 0$ (k) $2q + 6 = 0$ (l) $2q + 6 = 12$

4.5 MORE EQUATIONS

Let us practise solving some more equations. While solving these equations, we shall learn about transposing a number, i.e., moving it from one side to the other. We can transpose a number instead of adding or subtracting it from both sides of the equation.

EXAMPLE 6 Solve: $12p - 5 = 25$ (4.12)

SOLUTION

- Adding 5 on both sides of the equation,

  $12p - 5 + 5 = 25 + 5$ or $12p = 30$

- Dividing both sides by 12,

  $\frac{12p}{12} = \frac{30}{12}$ or $p = \frac{5}{2}$

Note, adding 5 to both sides is the same as changing side of (– 5).

$12p - 5 = 25$

$12p = 25 + 5$

Changing side is called ***transposing****. While transposing a number, we change its sign.*

Check Putting $p = \frac{5}{2}$ in the LHS of equation 4.12,

$$\text{LHS} = 12 \times \frac{5}{2} - 5 = 6 \times 5 - 5$$

$$= 30 - 5 = 25 = \text{RHS}$$

As we have seen, while solving equations one commonly used operation is adding or subtracting the same number on both sides of the equation. *Transposing a number (i.e., changing the side of the number) is the same as adding or subtracting the number from both sides*. In doing so, the sign of the number has to be changed. What applies to numbers also applies to expressions. Let us take two more examples of transposing.

Adding or Subtracting on both sides	**Transposing**
(i) $3p - 10 = 5$ Add 10 to both sides $3p - 10 + 10 = 5 + 10$ or $3p = 15$	(i) $3p - 10 = 5$ Transpose (–10) from LHS to RHS (On transposing – 10 becomes + 10). $3p = 5 + 10$ or $3p = 15$
(ii) $5x + 12 = 27$ Subtract 12 from both sides $5x + 12 - 12 = 27 - 12$ or $5x = 15$	(ii) $5x + 12 = 27$ Transposing + 12 (On transposing + 12 becomes – 12) $5x = 27 - 12$ or $5x = 15$

We shall now solve two more equations. As you can see they involve brackets, which have to be solved before proceeding.

EXAMPLE 7 Solve

(a) $4(m + 3) = 18$ (b) $-2(x + 3) = 8$

SOLUTION

(a) $4(m + 3) = 18$

Let us divide both the sides by 4. This will remove the brackets in the LHS We get,

$$m + 3 = \frac{18}{4} \quad \text{or} \quad m + 3 = \frac{9}{2}$$

or $m = \frac{9}{2} - 3$ (transposing 3 to RHS)

or $m = \frac{3}{2}$ (required solution) $\left(\text{as } \frac{9}{2} - 3 = \frac{9}{2} - \frac{6}{2} = \frac{3}{2}\right)$

Check LHS = $4\left[\frac{3}{2} + 3\right] = 4 \times \frac{3}{2} + 4 \times 3 = 2 \times 3 + 4 \times 3$ [put $m = \frac{3}{2}$]

$= 6 + 12 = 18 =$ RHS

(b) $-2(x + 3) = 8$

We divide both sides by (– 2), so as to remove the brackets in the LHS, we get,

$$x + 3 = -\frac{8}{2} \quad \text{or} \quad x + 3 = -4$$

i.e., $x = -4 - 3$ (transposing 3 to RHS) or $x = -7$ (required solution)

Check LHS = $-2(-7+3) = -2(-4)$

$= 8 =$ RHS as required.

4.6 From Solution to Equation

Atul always thinks differently. He looks at successive steps that one takes to solve an equation. He wonders why not follow the reverse path:

Equation	$\longrightarrow$	Solution	(normal path)
Solution	$\longrightarrow$	Equation	(reverse path)

He follows the path given below:

Start with	$\downarrow$	$x = 5$	$\uparrow$	
Multiply both sides by 4,		$4x = 20$		Divide both sides by 4.
Subtract 3 from both sides,	$\downarrow$	$4x - 3 = 17$	$\uparrow$	Add 3 to both sides.

This has resulted in an equation. If we follow the reverse path with each step, as shown on the right, we get the solution of the equation.

Hetal feels interested. She starts with the same first step and builds up another equation.

Try These

Start with the same step $x = 5$ and make two different equations. Ask two of your classmates to solve the equations. Check whether they get the solution $x = 5$.

	$x = 5$
Multiply both sides by 3	$3x = 15$
Add 4 to both sides	$3x + 4 = 19$

Start with $y = 4$ and make two different equations. Ask three of your friends to do the same. Are their equations different from yours?

> *Is it not nice that not only can you solve an equation, but you can make equations? Further, did you notice that given an equation, you get one solution; but given a solution, you can make many equations?*

Now, Sara wants the class to know what she is thinking. She says, "I shall take Hetal's equation and put it into a statement form and that makes a puzzle. For example, think of a number; multiply it by 3 and add 4 to the product. Tell me the sum you get.

If the sum is 19, the equation Hetal got will give us the solution to the puzzle. In fact, we know it is 5, because Hetal started with it."

She turns to Appu, Ameena and Sarita to check whether they made their puzzle this way. All three say, "Yes!"

We now know how to create number puzzles and many other similar problems.

Try These

Try to make two number puzzles, one with the solution 11 and another with 100

EXERCISE 4.3

1. Solve the following equations:

   (a) $2y+\frac{5}{2}=\frac{37}{2}$ (b) $5t + 28 = 10$ (c) $\frac{a}{5}+3=2$ (d) $\frac{q}{4}+7=5$

   (e) $\frac{5}{2}x = \quad 10$ (f) $\frac{5}{2}x=\frac{25}{4}$ (g) $7m+\frac{19}{2}=13$ (h) $6z + 10 = –2$

   (i) $\frac{3l}{2}=\frac{2}{3}$ (j) $\frac{2b}{3} \quad 5=3$

2. Solve the following equations:

   (a) $2(x + 4) = 12$ (b) $3(n – 5) = 21$ (c) $3(n – 5) = – 21$

   (d) $– 4(2 + x) = 8$ (e) $4(2 – x) = 8$

3. Solve the following equations:

   (a) $4 = 5(p – 2)$ (b) $– 4 = 5(p – 2)$ (c) $16 = 4 + 3(t + 2)$

   (d) $4 + 5(p – 1) =34$ (e) $0 = 16 + 4(m – 6)$

4. (a) Construct 3 equations starting with $x = 2$

   (b) Construct 3 equations starting with $x = – 2$

4.7 APPLICATIONS OF SIMPLE EQUATIONS TO PRACTICAL SITUATIONS

We have already seen examples in which we have taken statements in everyday language and converted them into simple equations. We also have learnt how to solve simple equations. Thus we are ready to solve puzzles/problems from practical situations. The method is first to form equations corresponding to such situations and then to solve those equations to give the solution to the puzzles/problems. We begin with what we have already seen [Example 1 (i) and (iii), Section 4.2].

EXAMPLE 8 The sum of three times a number and 11 is 32. Find the number.

SOLUTION

- If the unknown number is taken to be x, then three times the number is $3x$ and the sum of $3x$ and 11 is 32. That is, $3x + 11 = 32$

This equation was obtained earlier in Section 4.2, Example 1.

- To solve this equation, we transpose 11 to RHS, so that

  $3x = 32 – 11$ or $3x = 21$

  Now, divide both sides by 3

  So $x = \frac{21}{3} = 7$

The required number is 7. (We may check it by taking 3 times 7 and adding 11 to it. It gives 32 as required.)

EXAMPLE 9 Find a number, such that one-fourth of the number is 3 more than 7.

SOLUTION

- Let us take the unknown number to be y; one-fourth of y is $\frac{y}{4}$.

  This number $\left(\frac{y}{4}\right)$ is more than 7 by 3.

  Hence we get the equation for y as $\frac{y}{4} - 7 = 3$

- To solve this equation, first transpose 7 to RHS We get,

  $\frac{y}{4} = 3 + 7 = 10.$

  We then multiply both sides of the equation by 4, to get

  $\frac{y}{4} \times 4 = 10 \times 4$ or $y = 40$ (the required number)

  Let us check the equation formed. Putting the value of y in the equation,

  LHS $= \frac{40}{4} - 7 = 10 - 7 = 3 =$ RHS, as required.

TRY THESE

(i) When you multiply a number by 6 and subtract 5 from the product, you get 7. Can you tell what the number is?

(ii) What is that number one third of which added to 5 gives 8?

EXAMPLE 10 Raju's father's age is 5 years more than three times Raju's age. Find Raju's age, if his father is 44 years old.

SOLUTION

- As given in Example 3 earlier, the equation that gives Raju's age is

  $3y + 5 = 44$

- To solve it, we first transpose 5, to get $3y = 44 - 5 = 39$

  Dividing both sides by 3, we get $y = 13$

  That is, Raju's age is 13 years. (You may check the answer.)

TRY THESE

There are two types of boxes containing mangoes. Each box of the larger type contains 4 more mangoes than the number of mangoes contained in 8 boxes of the smaller type. Each larger box contains 100 mangoes. Find the number of mangoes contained in the smaller box?

EXERCISE 4.4

1. Set up equations and solve them to find the unknown numbers in the following cases:
   (a) Add 4 to eight times a number; you get 60.
   (b) One-fifth of a number minus 4 gives 3.
   (c) If I take three-fourths of a number and add 3 to it, I get 21.
   (d) When I subtracted 11 from twice a number, the result was 15.
   (e) Munna subtracts thrice the number of notebooks he has from 50, he finds the result to be 8.
   (f) Ibenhal thinks of a number. If she adds 19 to it and divides the sum by 5, she will get 8.
   (g) Anwar thinks of a number. If he takes away 7 from $\frac{5}{2}$ of the number, the result is 23.
2. Solve the following:
   (a) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. What is the lowest score?
   (b) In an isosceles triangle, the base angles are equal. The vertex angle is 40°. What are the base angles of the triangle? (Remember, the sum of three angles of a triangle is 180°).
   (c) Sachin scored twice as many runs as Rahul. Together, their runs fell two short of a double century. How many runs did each one score?
3. Solve the following:
   (i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. How many marbles does Parmit have?
   (ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. What is Laxmi's age?
   (iii) People of Sundargram planted trees in the village garden. Some of the trees were fruit trees. The number of non-fruit trees were two more than three times the number of fruit trees. What was the number of fruit trees planted if the number of non-fruit trees planted was 77?
4. Solve the following riddle:

I am a number,
Tell my identity!
Take me seven times over
And add a fifty!
To reach a triple century
You still need forty!

What have We Discussed?

1. An equation is a condition on a variable such that two expressions in the variable should have equal value.
2. The value of the variable for which the equation is satisfied is called the solution of the equation.
3. An equation remains the same if the LHS and the RHS are interchanged.
4. In case of the balanced equation, if we

   (i) add the same number to both the sides, or (ii) subtract the same number from both the sides, or (iii) multiply both sides by the same number, or (iv) divide both sides by the same number, the balance remains undisturbed, i.e., the value of the LHS remains equal to the value of the RHS
5. The above property gives a systematic method of solving an equation. We carry out a series of identical mathematical operations on the two sides of the equation in such a way that on one of the sides we get just the variable. The last step is the solution of the equation.
6. Transposing means moving to the other side. Transposition of a number has the same effect as adding same number to (or subtracting the same number from) both sides of the equation. When you transpose a number from one side of the equation to the other side, you change its sign. For example, transposing +3 from the LHS to the RHS in equation $x + 3 = 8$ gives $x = 8 – 3$ (= 5). We can carry out the transposition of an expression in the same way as the transposition of a number.
7. We have learnt how to construct simple algebraic expressions corresponding to practical situations.
8. We also learnt how, using the technique of doing the same mathematical operation (for example adding the same number) on both sides, we could build an equation starting from its solution. Further, we also learnt that we could relate a given equation to some appropriate practical situation and build a practical word problem/puzzle from the equation.