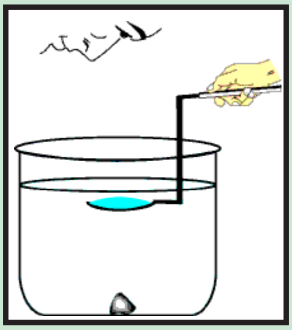
Chapter

6

Refraction of Light at Curved Surfaces



It is a common observation that some people use spectacles for reading. The watch repairer uses a small magnifying glass to see tiny parts of a watch.

- Have you ever touched a magnifying glass with your hand?
- Have you touched the glass in the spectacles used for reading with your hand?
- Is it a plane or curved surface?
- Is it thicker in the middle or at the edge?

We have learnt about refraction of light at a plane surface in the previous chapter. Now let us understand refraction of light at curved surfaces.

Let us do an activity to understand refraction of light at curved surfaces.

Refraction of light at a curved surface

Activity 1

Draw an arrow of length 4 cm using a black sketch pen on a thick sheet of paper. Take an empty cylindrical-shaped transparent vessel such as glass tumbler. Keep it on the table. Ask your friend to bring the sheet of paper on which arrow was drawn behind the vessel while you look at it from the other side (The arrow mark should be in horizontal position).

- What do you see?

  You will see a diminished (small-sized) image of the arrow
- Why do you see a diminished image?
- Is the image real or virtual?
- Can you draw a ray diagram showing how it is formed?

Ask your friend to fill the vessel with water. Look at the figure of arrow from the same position as before.

- What do you see now?
- Do you get an inverted image?
- How could this happen?

In the first case, when the vessel is empty, light from the arrow refracts at the curved interface, moves through the glass and enters into air then it again undergoes refraction on the opposite curved surface of the vessel (at the other end from where we are looking) and comes out into the air. In this way light travels through two media and comes out of the vessel and forms a diminished image.

In the second case, light enters the curved surface, moves through water, comes out of the glass and forms an inverted image.

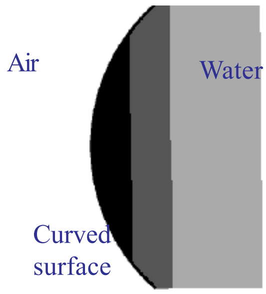


fig-1

When the vessel is filled with water, there is a curved interface between two different media (air and water). Assume that the refractive indices of both water and glass are the same (they really are not equal). This setup of air and water separated by a curved surface is shown in figure 1.

- What happens to a ray that is incident on a curved surface separating the two media?
- Are the laws of refraction still valid?

Let us find out.

Consider a curved surface seperating two different media as shown in figure 2. The centre of the sphere, of which curved surface is a part, is called as *centre of curvature*. It is denoted by letter 'C'.

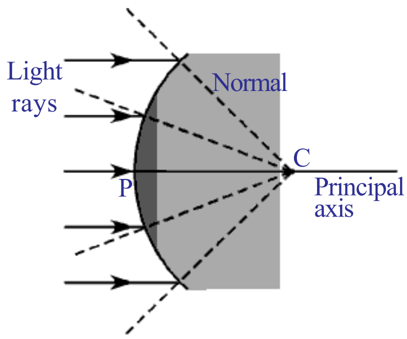


fig-2

Any line drawn from the centre of curvature to a point on the curved surface becomes normal to the curved surface at that point. The direction of the normal changes from one point to another point on the curved surface. The centre of the curved surface is called the *pole* (P) of the curved surface. The *line that joins the centre of curvature and the pole is called 'principal axis'*.

- How do rays bend when they are incident on a curved surface?

As in the case of plane surfaces, a ray will bend towards the normal if it travels from a rarer to denser medium and it bends away from the normal if it travels from a denser to a rarer medium .

Let us see how we can draw ray diagrams for certain useful cases.

- What happens to a ray that travels along the principal axis? Similarly, a ray that travels through the centre of curvature?

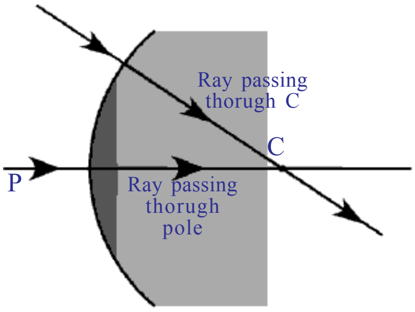


fig-3

According to Snell's law the ray which travels along the normal drawn to the surface does not deviate from its path. Hence both rays mentioned above travel along the normal (see figure 3), so they do not deviate.

- What happens to a ray travelling parallel to the principal axis?

Observe the following figures 4a, 4b, 4c, and 4d. In all the cases as represented by the diagrams, the incident ray is parallel to the principal axis.

Case1: A ray travelling parallel to the principal axis strikes a convex surface and passes from a rarer medium to denser medium. (see figure 4a)

Case2: A ray travelling parallel to the principal axis strikes a convex surface and passes from a denser medium to rarer medium. (see figure 4b)

Case3: A ray travelling parallel to the principal axis strikes a concave surface and passes from a denser medium to rarer medium. (see figure 4c)

Case4: A ray travelling parallel to the principal axis strikes a concave surface and passes from a rarer medium to denser medium. (see figure 4d)

- What difference do you notice in the refracted rays in 4a and 4b?

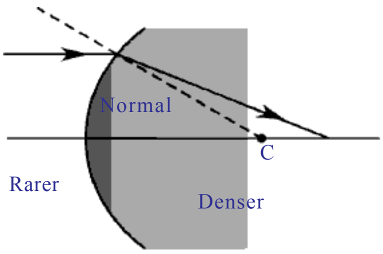


fig-4(a)

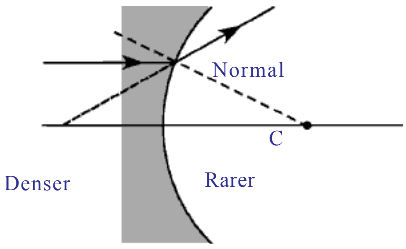


fig-4(b)

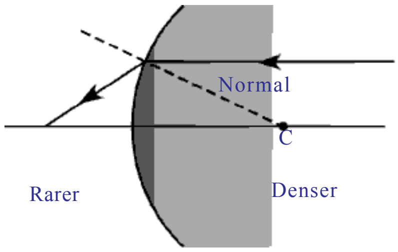


fig-4(c)

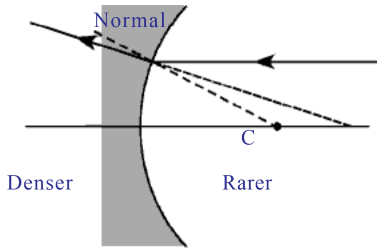


fig-4(d)

- What could be the reason for that difference?
- What difference do you notice in refracted rays in figure 4c and 4d?
- What could be the reasons for that difference?

You might have noticed that in figures 4(a) and 4 (c) the refracted ray reaches a particular point on the principal axis. In figures 4(b) and 4(d) the refracted ray moves away from the principal axis. When you extend the refracted ray backwards along the ray as shown in 4b and 4d, the extended ray intersects the principal axis at some point. The point where refracted ray intersects the axis in all the above cases is called the *focal point*.

You might have observed that a lemon in a glass of water appears bigger than its actual size, when viewed from the sides of tumbler.

- How can you explain this change in the size of lemon?
- Is the lemon that appears bigger in size an image of lemon or is it the real lemon?
- Can you draw a ray diagram to explain this phenomenon?

Let us find out.

Image formation

Consider a curved surface separating two media of refractive indices n_1 and n_2 (figure 5). A point object is placed on the principal axis at point O. The ray, which travels along the principal axis passes through the pole undeviated. The second ray, which forms an angle α with principal axis, meets the interface (surface) at A. The angle of incidence is θ_1. The ray bends and passes through the second medium along the line AI. The angle of refraction is θ_2. The two refracted rays meet at I and the image is formed there. Let the angle made by the second refracted ray with principal axis be γ and the angle between the normal and principal axis be β. (see figure 5)

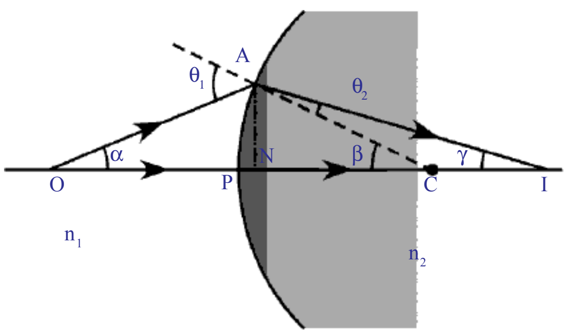


fig-5

In figure 5,

PO is the object distance which we denote as 'u'

PI is image distance which we denote as 'v'

PC is radius of curvature which we denote as 'R'

n_1, n_2 are refractive indices of two media.

Can you establish any relation between the above mentioned quantities?

In the triangle ACO, $\theta_1 = \alpha + \beta$

and in the triangle ACI, $\beta = \theta_2 + \gamma \Rightarrow \beta - \gamma = \theta_2$

According to Snell's law, we know

$n_1 \sin \theta_1 = n_2 \sin \theta_2$

substituting the values of θ_1 and θ_2, we get,

$$n_1 \sin(\alpha + \beta) = n_2 \sin(\beta - \gamma) \quad ....\ (1)$$

If the rays move very close to the principal axis, the rays can be treated as parallel and are called *paraxial rays*. Then the angles α, β and γ become very small. This approximation is called *paraxial approximation*.

$\sin(\alpha + \beta) = \alpha + \beta$ and $\sin(\beta - \gamma) = \beta - \gamma$

Substituting in equation (1)

$$n_1 (\alpha + \beta) = n_2 (\beta - \gamma) \Rightarrow n_1\alpha + n_1 \beta = n_2 \beta - n_2 \gamma \quad ....(2)$$

since all angles are small, we can write

$\tan \alpha = AN/NO = \alpha$

$\tan \beta = AN/NC = \beta$

$\tan \gamma = AN/NI = \gamma$

Substitute these in equation (2), we get,

$$n_1 AN/NO + n_1 AN/NC = n_2 AN/NC - n_2 AN/NI \quad ....\ (3)$$

As the rays move very close to the principal axis, the point N coincides with pole of the interface (P). Therefore NI, NO, NC can be replaced by PI, PO and PC respectively.

After substituting these values in equation (3), we get,

$n_1/PO + n_1/PC = n_2/PC - n_2/PI$

$$n_1/PO + n_2/PI = (n_2 - n_1)/PC \quad ....(4)$$

Equation (4) shows the relation between refractive indices of two media, object distance, image distance and radius of curvature.

The above equation is true for the case we considered.

We can generalize equation (4) if we use the following sign convention.

For all purposes of applications of refraction at curved surfaces and through lenses following conventions are used.

- All distances are measured from the pole (or optic centre).
- Distances measured along the direction of the incident light ray are taken as positive
- Distances measured opposite to the direction of the incident light ray are taken as negative
- The heights measured vertically above from the points on axis are taken as positive

– The heights measured vertically down from points on axis are taken as negative.

Here PO is called the *object distance* (u)

PI is called the *image distance* (v)

PC is called *radius of curvature* (R)

According to sign convention mentioned above, we have

PO = -u ; PI = v ; PC = R

Substituting these values in equation (4) we get,

$n_2/v - n_1/u = (n_2 - n_1)/R$ .... (5)

This formula can also be used for plane surfaces. In the case of a plane surface, radius of curvature (R) approaches infinity. Hence 1/R becomes zero. Substituting this in equation 5, we get formula for the plane surfaces

$n_2/v - n_1/u = 0 \Rightarrow n_2/v = n_1/u$

NOTE: The distances u and v are measured from the plane interface.

Let us consider the following examples.

Example 1

A bird is flying down vertically towards the surface of water in a pond with constant speed. There is a fish inside the water. If that fish is exactly vertically below the bird, then the bird will appear to the fish to be:

a. farther away than its actual distance.

b. closer than its actual distance.

c. moving faster than its actual speed.

d. moving slower than its actual speed.

Which of the four options are true? How can you prove it?

Solution: For refraction at a plane surface,

we use $n_2/v = n_1/u$ ....(1)

Let x be the height of the bird above the water surface at an instant and n be the refractive index of water.

n_1 = refractive index of air,

Then $n_1 = 1$, $n_2 = n$, u= $-x$ and let v = -y, (see figure E-1)

Substituting these values in equation (1)

$n/(-y) = 1/(-x) \Rightarrow y = nx$

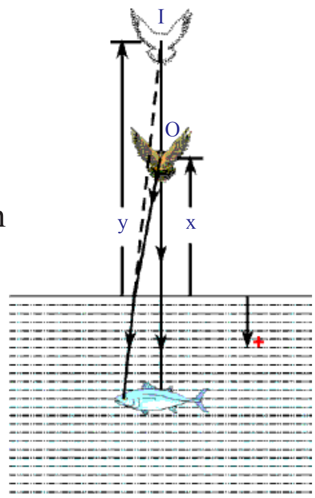


fig-E1

In the above equation, we know that n is greater than 1. Hence y is greater than x. Thus the bird appears to the fish to be farther away than its actual distance. We have assumed that bird is flying vertically down with constant speed. For the observer on the ground, bird appears that it has covered 'x' distance for certain time. But for fish, it appears that bird has covered a distance 'y' in the same time. As y is greater than x , we can conclude that the speed of the bird, observed by the fish, is greater than its actual speed.

So, options (a) and (c) are correct.

Example 2

A transparent sphere of radius R and refractive index n is kept in air. At what distance from the surface of the sphere should a point object be placed on the principal axis so as to form a real image at the same distance from the second surface of the sphere?

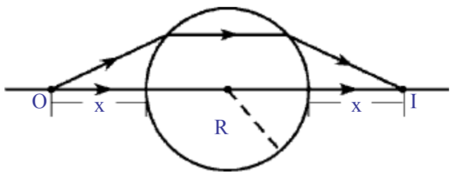


fig-E2

Solution: From the symmetry of figure E2, the rays must pass through the sphere parallel to the principal axis.

From the figure,

$u = -x$, $v = \infty$ (refracted ray is parallel to the optical axis after refraction at first surface)

$n_1 = 1$ and $n_2 = n$, (where n_1 is refractive index of air)

Using $n_2/v - n_1/u = (n_2 - n_1)/R$

$n/\infty - 1/(-x) = (n-1)/R \Rightarrow 1/x = (n-1)/R$

$\Rightarrow x = R/(n-1)$

Object distance from the first surface of the sphere is $x = R/(n-1)$

Example 3

A transparent (glass) sphere has a small, opaque dot at its centre. Does the apparent position of the dot appear to be the same as its actual position when observed from outside?

Solution: Let refractive index of glass $n_1 = n$

refractive index of air $n_2 = 1$

Then $u = -R$ (radius of sphere) ; Radius of curvature $R = -R$

Using $n_2/v - n_1/u = (n_2 - n_1)/R$

$1/v - n/(-R) = (1-n)/(-R) \Rightarrow 1/v + n/R = (n-1)/R$

After solving this equation, we get

Image distance $v = -R$

Thus we can say that the image distance and object distance are equal and that the apparent position of dot is the same as its actual position.

It is independent of the refractive index of the material of the sphere.

Till now we have discussed refraction of light at a single curved surface either convex or concave. Let us suppose that a transparent material has two curved surfaces.

- What happens to the light ray when a transparent material with two curved surfaces is placed in its path?
- Have you heard about lenses?
- How does a light ray behave when it is passed through a lens?

Let us learn about refraction of light through lenses.

Lenses

A lens is formed when a transparent material is bounded by two surfaces of which one (or) both surfaces are spherical. That is a lens is bounded by atleast one curved surface. Lenses can be of various types. Some typical lenses along with their names are shown in figure 6.

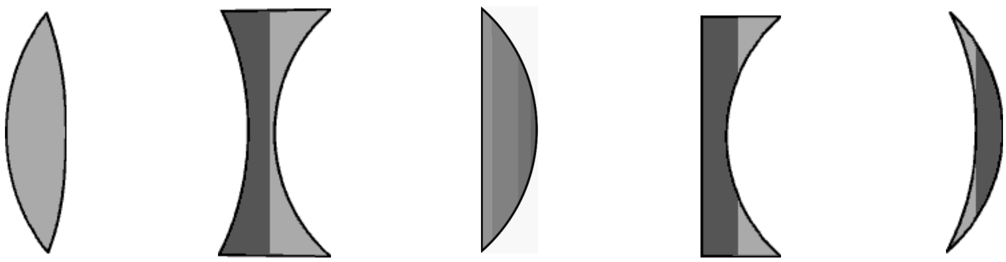


fig-6(a): **Biconvex** *fig-6(b):* **Biconcave** *fig-6(c):* **Plano-convex** *fig-6(d):* **Plano-concave** *fig-6(e):* **Concavo-convex**

figure 6 : Different types of lenses

A lens may have two spherical surfaces bulging outwards. Such a lens is called double convex lens (Biconvex lens, see figure 6(a)). It is thick at the middle as compared to edges.

Similarly, a double concave lens is bounded by two spherical surfaces curved inwards. (Biconcave lens, see figure 6(b)). It is thin at the middle and thicker at the edges.

Observe the 6(c), 6(d) and 6(e) to understand structure of Plano-Convex lens, Plano-Concave lens; Concavo-Convex lens.

Here we are concerned only with thin lenses i.e. the thickness of the lens is negligible.

Let us learn the terminology used in the case of lenses.

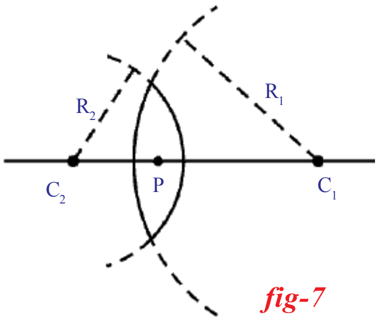


fig-7

Each curved surface of a lens is part of a sphere. The centre of the sphere which contains the part of the curved surface is called *centre of curvature*. It is denoted by a letter 'C'. If a lens contains two curved surfaces then their centres of curvature are denoted as C_1 and C_2. The distance between the centre of curvature and curved surface is called *radius of curvature (R)*. Radii of curvature are represented by R_1 and R_2 respectively. Let us consider a double convex lens as shown in figure 7.

The line joining the points C_1 and C_2 is called *principal axis*. The *midpoint of a thin lens* is called *optic centre* of lens (P).

Focal length of the lens

A parallel beam of light incident on a lens converges to a point as shown in figure 8(a) or seems to emanate from a point on the principal axis as shown in figure 8(b). The point of convergence (or) the point from which rays seem to emanate is called *focal point* or *focus* (F). Every lens has two focal points. The distance between the focal point and optic centre is called the *focal length* of lens denoted by 'f'.

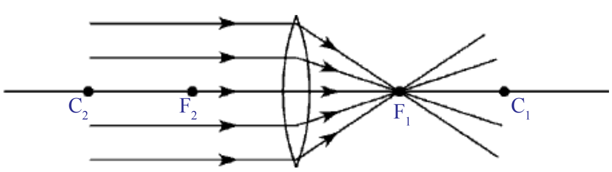


fig-8(a)

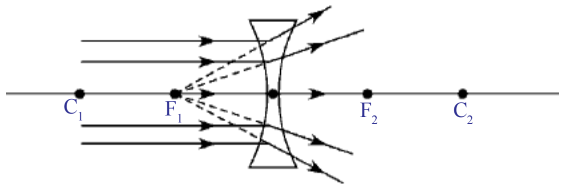


fig-8(b)

For drawing ray diagrams related to lenses we represent convex lens with a symbol and concave lens with . See figures 8(c) and 8(d).

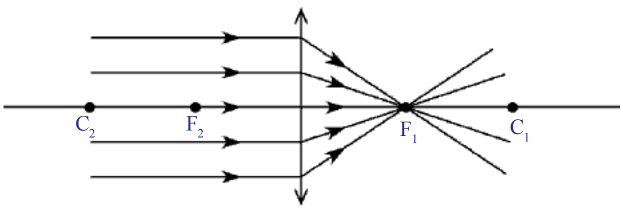


fig-8(c)

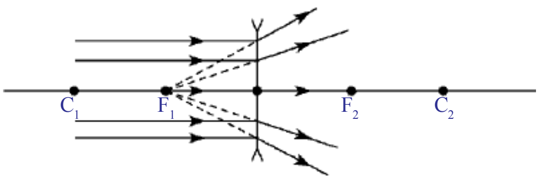


fig-8(d)

- How does the lens form an image?

To know the formation of image by lenses, we need to know the behavior of light rays when they meet a lens.

Though we know that the lens has two surfaces; while drawing ray diagrams, we can consider the lens as a single surface element because we assume that the thickness of the lens is very small and show the net refraction at only one of the surfaces, as shown in the figure 8(c) and 8(d).

Behaviour of certain light rays when they are incident on a lens

The behaviour of a light ray when it passes through a lens can be understood by observing its behaviour in the following situations:

Situation I: Ray passing along the principal axis

Any ray passing along the principal axis is undeviated. (see Figures 9a and 9b)

Situation II: Ray passing through the optic centre.

Any ray passing through the optic centre is also undeviated. (see Figure 10a and 10b)

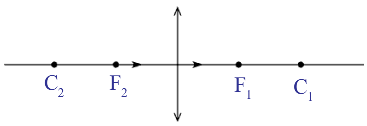


fig-9(a)

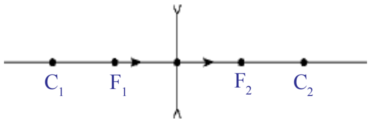


fig-9(b)

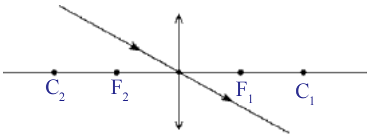


fig-10(a)

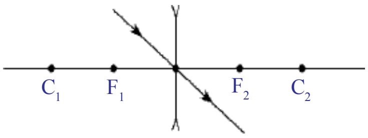


fig-10(b)

Situation III: Rays travelling parallel to the principal axis.

We know that the rays passing parallel to the principal axis converge at the focus or appear to diverge from the focus as shown in figure 8(c) and 8(d).

- If we allow a light ray to pass through the focus, which path does it take?

Situation IV: Ray passing through focus.

Light rays obey the principle of least time. Hence the ray passing through the focus will take a path parallel to principal axis after refraction. (see Figure 11(a) and 11(b))

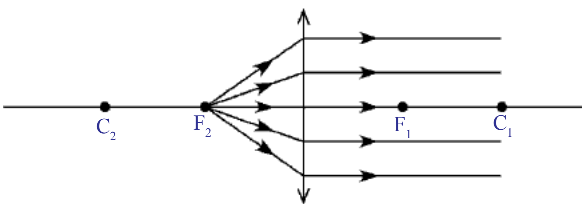


fig-11(a)

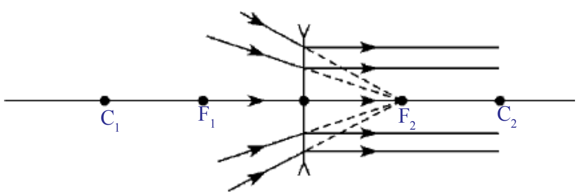


fig-11(b)

- What happens when parallel rays of light fall on a lens making some angle with the principal axis?

Let us observe the following figures

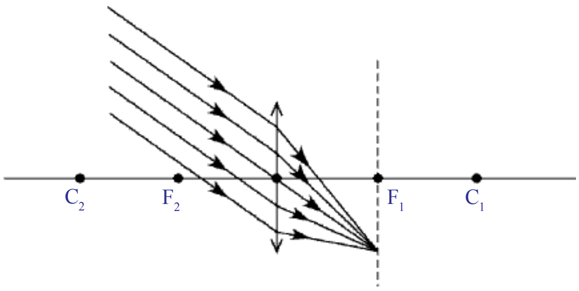


fig-12(a)

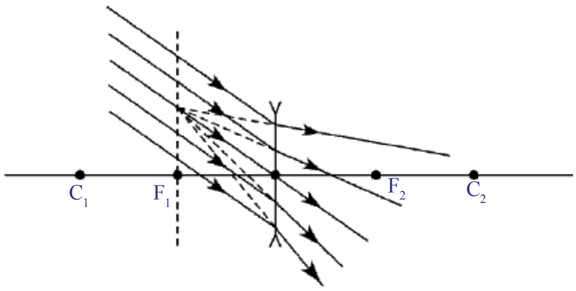


fig-12(b)

When parallel rays, making an angle with principal axis, fall on a lens, as shown in figures 12(a) and 12(b), the rays converge at a point or appear to diverge from a point lying on the focal plane. Focal plane is the plane perpendicular to the principal axis at the focus.

Rules to draw ray diagrams for image formation by lenses

Let us learn a few basic rules to draw ray diagrams to locate the position of images.

For drawing a ray diagram to find position and size of the image formed by lens for any position of object on the principal axis you need to follow the rules mentioned below:

For locating position and to find the size of image, we need two rays out of four rays that were mentioned in the situations I to IV.

- Select a point on the object placed at a point on the principal axis.
- Draw two rays that were chosen by you from rays mentioned in situations I to IV.
- Extend both rays to intersect at a point. This point gives position of the image.
- Draw a normal from point of intersection to the principal axis.
- Length of this normal represents the size of the image.

Observe the following diagrams. They represent image formation by a convex lens for various positions of the object.

1. Object at infinity

- What do you mean by an object at infinity?
- What type of rays fall on the lens?

You know that the rays falling on the lens from an object at infinity are parallel to the principal axis.

They converge to the focal point. So a point sized image is formed at the focal point. This can be seen in figures 8(a).

2. Object placed beyond the centre of curvature on the principal axis

In figure (13) you notice that when object is placed beyond the centre of curvature (C_2), a *real, inverted* and *diminished* image is formed on the principal axis between the points F_1 and C_1.

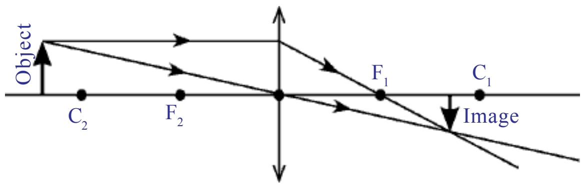


fig-13

In figure (13) we have chosen two rays, one ray passing parallel to the principal axis and another ray passing through the optic centre to locate the position of the image.

Try to draw ray diagram using the pair of rays, one passing parallel to the axis another passing through the focus.

3. Object placed at the centre of curvature

When an object is placed at the centre of curvature (C_2) on the principal axis, you will get an image at C_1 which is *real*, *inverted* and of the *same size* as that of object. See figure 14.

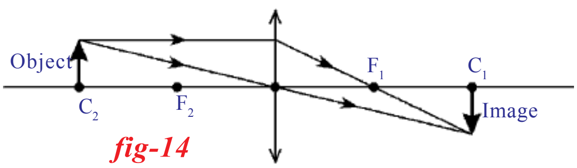


fig-14

4. Object placed between the centre of curvature and focal point

When an object is placed between centre of curvature (C_2) and focus (F_2), you will get an image which is *real*, *inverted* and *magnified*. See figure 15. The image will form beyond C_1.

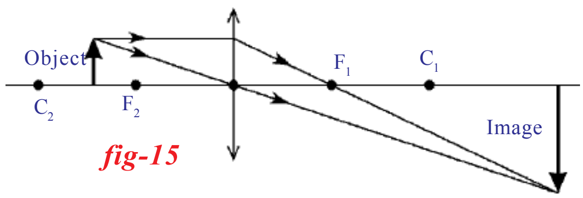


fig-15

5. Object located at the focal point

When an object is placed at focus (F_2), the image will be at infinity. See figure 16. When the image is formed at an infinite distance away we can not discuss the size and nature of the image.

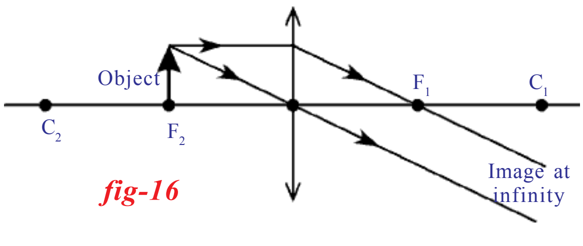


fig-16

6. Object placed between focal point and optic centre

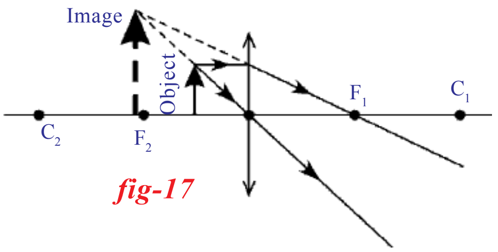


fig-17

If we place an object between focus and optic centre, we will get an image which is *virtual*, *erect* and *magnified*.

From the ray diagram shown in figure 17, you will notice that the image formed is a *virtual*, *erect* and is formed on the same side of the lens where the object is placed. The size of the image is larger than that of the object. It is a *magnified image*.

In the above situation of image formation, we understand two things:

1. As the image formed is virtual, we can see it with our eyes. In all other cases the image is real which we can't see with our eyes but can be viewed if the image is captured on a screen.
2. A magnified virtual image is formed on the same side of the lens where the object is placed. Thus the image you are seeing through a lens is not real, it is a virtual image of the object.

This particular behaviour of convex lens helps to construct a microscope, which gives a magnified image. You might remember that the magnification of the virtual image is possible only when the object is at the distance less than the focal length of the lens.

Till now we have drawn ray diagrams for various positions of object placed on principal axis using convex lens. Draw ray diagram for an object placed between C_1 and F_1 for a concave lens.

- What do you notice?

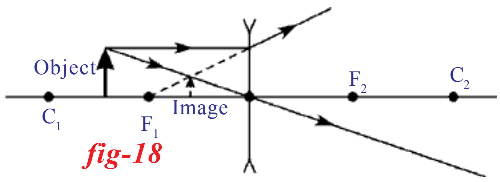


fig-18

Verify your ray diagram with the ray diagram we have drawn for a convex lens. See figure 18. Try to draw ray diagrams for other positions of an object. You will notice that irrespective of the position of object, on the principal axis, you will get an *erect, virtual image*, *diminished in size* in between the focal point and optic centre for concave lens.

Let us see a few examples of ray diagrams

Example 4

Draw a ray diagram to locate the position of image when a point source (S) is placed on optical axis MN of a convex lens, in such a way that it is beyond focal point (F). See figure E(4).

Solution

- Draw a perpendicular line to principal axis passing through the focus (F^I).

- Draw a ray from point source (S) in any direction to meet lens at point (P^I).
- Now draw another line parallel to the ray drawn from the point source (S) such that it passes through the optic centre (P); This line intersects the normal at point F_O.

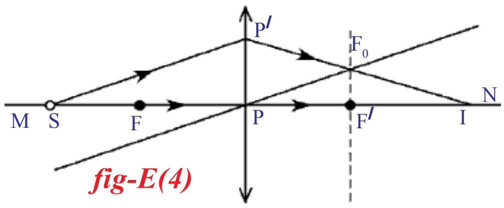


fig-E(4)

- Now draw a line passing from point P^I to pass through the point F_O such that it meets principal axis at a point say (I).
- 'I' is the image point for the point source (S).

Example 5

Complete the ray diagram to show the paths of the rays after refraction through the lenses shown in the figures E5(a) and E5(b)?

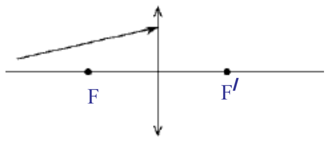


fig-E5(a)

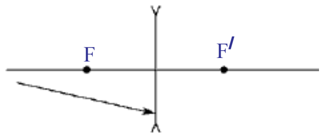


fig-E5(b)

Solution

Follow the steps mentioned in Example (4) to complete the ray diagrams.

You will notice that the paths of the rays are shown in figures E5(c) and 5(d).

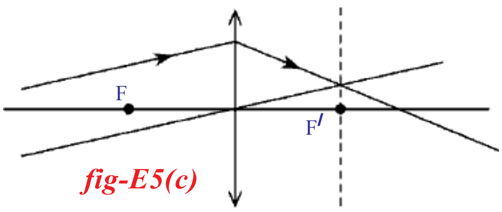


fig-E5(c)

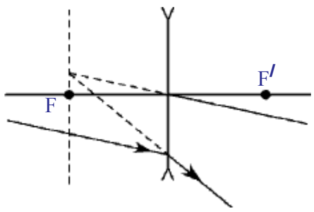


fig-E5(d)

- Can we realise in practice the results obtained in the ray diagrams when we perform experiments with a lens?

  Let us see

Activity 2

Take a v-stand and place it on a long (nearly 2m) table at the middle. Place a convex lens on the v-stand. Imagine the principal axis of the lens. Light a candle and ask your friend to take the candle far away from the lens along the principal axis. Adjust a screen (a sheet of white paper placed perpendicular to the axis) which is on other side of the lens until you get an image on it.

- Why are we using a screen to view this image? Why don't we see it directly with our eye?

Measure the distance of the image from the v-stand of lens and also measure the distance between the candle and stand of lens.

Record the values in a table 1.

Table 1

Object distance (u)	Image distance (v)	Focal length (f)

Now place the candle at a distance of 60 cm from the lens, such that the flame of the candle lies on the principal axis of the lens. Try to get an image of the candle flame on the other side on a screen. Adjust the screen till you get a clear image. Measure the image distance (v) from lens and record the values of 'u' and 'v' in table 1. Repeat this for various object distances like 50 cm, 40 cm, 30 cm, etc. Measure image distances in all the cases and note them in table 1.

- Could you get an image on the screen for every object distance?
- Why don't you get an image for certain object distances?
- Can you find the minimum limiting object distance for obtaining a real image?
- What do you call this minimum limiting object distance for real images?

When you do not get an image on the screen, try to see the image with your eye directly from the place of the screen.

- Could you see the image?
- What type of image do you see?

You will see a magnified image on the same side where we kept the object. This is a *virtual* image of the object which we cannot capture on the screen.

- Can you find the image distance of this virtual image?

In table-1, you got different values of 'v' for various position of candle (u).

- Could you find focal length of the lens from the values recorded in table-1?
- Can we establish a relation between 'u', 'v' and 'f'?

Let us find out.

Consider an object OO^{I} placed on the principal axis in front of a convex lens as shown in figure(19). Let II^{I} be the real image formed by the lens on the other side of it. Observe the figure(19).

- How is the image formed?

Lens formula

The ray, starting at O' and moving parallel to the principal axis and which falls on the lens, should pass through the focal point F_1 as shown in figure (19). To locate the point of image (I') for the object point (O'), consider another ray that passes through the optic centre P. We know that any ray passing through the optic centre P will not deviate.

The ray starting from O' and passing through optic centre P, will meet the refracted ray (first ray) at the point I'. This point is the image of the point O' of the object. Similarly, the image of the point O on the principal axis is formed at point I on the principal axis (see Figure 19). We get the inverted image II' of object OO'.

PO, PI and PF_1 are the object distance, image distance and focal length respectively. From figure 19, triangle $PP'F_1$ and triangle F_1II' are similar triangles,

⇨ $PP'/II' = PF_1/F_1I$ ....(1)

But from the figure 19,

$F_1I = PI - PF_1$

substituting F_1I in equation (1) above, we get

$PP'/II' = PF_1/(PI - PF_1)$ ....(2)

We have another set of similar triangles $OO'P$ and PII'.

From these triangles we get, $OO'/II' = PO/PI$

but from figure (19), $OO' = PP'$, hence

we have

$PP'/II' = PO/PI$ ....(3)

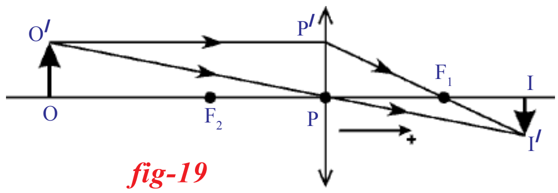


fig-19

From (2) and (3), we get

$PO/PI = PF_1/(PI-PF_1)$

$PI/PO = (PI-PF_1)/PF_1$

$PI/PO = PI/PF_1 - 1$

On dividing the equation by PI, we get

$1/PO = 1/PF_1 - 1/PI$

$1/PO + 1/PI = 1/PF_1$ ....(4)

The above equation is derived for a particular case of the object while using a convex lens. To convert this into a general equation, we need to use the sign convention.

According to the sign convention

$PO = -u$; $PI = v$; $PF_1 = f$

Substituting these values in equation 4, we get

$1/v - 1/u = 1/f$

This equation is called *lens formula*. It can be used for any lens. But remember to use the sign convention while using this equation.

We have 'u' and 'v' values in table – 1 that were measured during activity – 2. Find focal length of the lens from the values of the table for each set of values of 'u' and 'v'.

- Is the focal length same for each set of values?

You might have noticed that irrespective of object distance and image distance, you will get same focal length. If you do not get the same value of focal length, there may be some experimental error while doing the experiment. In such a case, find the average of all the values. This will be equal to the focal length of the lens.

Let us see an example

Example 6

An electric lamp and a screen are placed on the table, in a line at a distance of 1m. In what positions of convex lens of focal length of f = 21 cm will the image of lamp be sharp?

Solution

Let 'd' be distance between the lamp and screen and 'x' be the distance between lamp and lens. From figure E-6, we have $u = -x$ and $v = d-x$

By substituting these in lens formula,

We get

$1/f = 1/(d-x) + 1/x$

After solving this equation, we get

$x^2 - dx + fd = 0$

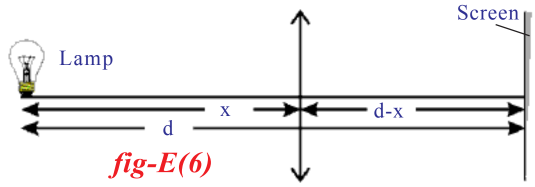


fig-E(6)

It is a quadratic equation. Hence we get two solutions. The solutions of the above equation are

$$x = [d \pm \sqrt{(d^2-4fd)}] / 2 \quad ....(1)$$

Given that f = 21cm and d= 1m =100cm.

Substituting these values in equation 1, we get

x_1 =70 cm and x_2 = 30cm.

NOTE: Image of the lamp can be sharp only when f is less than or equal to 25cm.

Discuss the reason for this using equation (1). Take the help of your teacher.

- On what factors does the focal length of the lens depend?

  Let us find out.

Activity 3

Take the same lens that was used in activity 2. Note the average focal length of the lens that was calculated in the activity. Take a cylindrical vessel such as glass tumbler. Its height must be much greater than the focal length of the lens. (We require the vessel which has a length (depth) nearly equal to four times of the focal length of lens). Keep a black stone inside the vessel at its bottom. Now pour water into the vessel up to a height such that the height of the water level from the top of the stone is greater than focal length of lens. Now dip the lens horizontally using a circular lens holder as shown in the figure (20) above the stone. Set the distance between stone and lens that is equal to or less than focal length of lens measured in activity 2. Now look at the stone through the lens. (Do this in open ground)

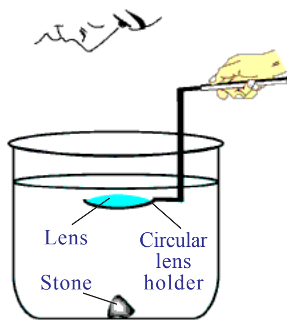


fig-20

- Can you see the image of the stone?
- If Yes / Not, why? Give your reasons.

You can see the image of the stone if the distance between lens and stone is less than the focal length of the lens (in air). Now increase the distance between lens and stone until you cannot see the image of the stone.

- What do you conclude from this activity?
- Does the focal length of the lens depend on surrounding medium?

You have dipped the lens to a certain height which is greater than the focal length of lens in air. But you can see the image (when lens is raised further, you could not see the image). This shows that the focal length of lens has increased in water. Thus we conclude that the focal length of lens depends upon the surrounding medium in which it is kept.

Lens maker's formula

Imagine a point object 'O' placed on the principal axis of the thin lens as shown in figure 21. Let this lens be placed in a medium of refractive index n_a and let refractive index of lens medium be n_b.

Consider a ray, from 'O' which is incident on the convex surface of the lens with radius of curvature R_1 at A as shown in figure 21.

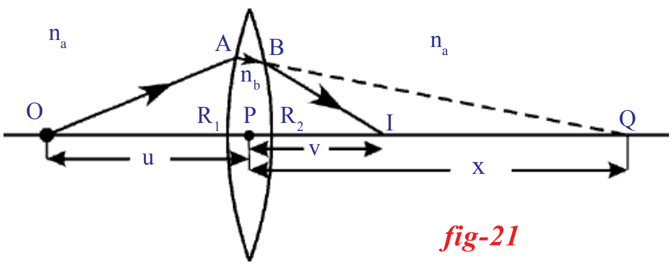


fig-21

The incident ray refracts at A.

Let us assume that, it forms image at Q, if there were no concave surface.

From the figure(21), Object distance $PO = -u$;

Image distance $v = PQ = x$

Radius of curvature $R = R_1$

$n_1 = n_a$ and $n_2 = n_b$

Substitute the above values in the equation, $n_2 / v - n_1 / u = (n_2 - n_1) / R$

$\Rightarrow \; n_b / x + n_a / u = (n_b - n_a) / R_1$ ....(1)

But the ray that has refracted at A suffers another refraction at B on the concave surface with radius of curvature (R_2). At B the ray is refracted and reaches I on the principal axis.

The image Q of the object due to the convex surface is taken as object for the concave surface. So, we can say that I is the image of Q for concave surface. See figure 21.

Object distance $u = PQ = +x$

Image distance $PI = v$

Radius of curvature $R = -R_2$

For refraction, the concave surface of the lens is considered as medium-1 and surrounding medium is considered as medium2. Hence the suffixes of refractive indices interchange. Then we get,

$n_1 = n_b$ and $n_2 = n_a$

Substituting the above values in equation $n_2 / v - n_1 / u = (n_2 - n_1) / R$

$n_a / v - n_b / x = (n_a - n_b) / (-R_2)$ ....(2)

By adding (1) and (2) we get,

$\Rightarrow \; n_a / v + n_a / u = (n_b - n_a)(1 / R_1 + 1 / R_2)$

Dividing both sides by n_a, We get

$\Rightarrow \; 1 / v + 1 / u = (n_b / n_a - 1)(1 / R_1 + 1 / R_2)$

We know $n_b / n_a = n_{ba}$ called refractive index of lens with respect to surrounding medium.

$1 / v + 1 / u = (n_{ba} - 1)(1 / R_1 + 1 / R_2)$

This is derived for specific case for the convex lens so we need to generalize this relation. For this we use sign convention. Applying sign convention to this specific case we get,

$1/v - 1/u = (n_{ba} - 1)(1/R_1 - 1/R_2)$

We know that

$1/v - 1/u = 1/f$

So, we get

$1/f = (n_{ba} - 1)(1/R_1 - 1/R_2)$ ....(3)

If the surrounding medium is air, then the relative refractive index could be absolute refractive index of the lens.

$1/f = (n - 1)(1/R_1 - 1/R_2)$ ....(4)

This can be used only when the lens is kept in air.

Where n is absolute refractive index and this equation is called lens maker's formula.

NOTE: Always use sign convention while using any formula derived in this chapter and the above formula can be used for any thin lens.

The convex lens behaves as a converging lens, if it is kept in a medium with refractive index less than the refractive index of the lens. It behaves like a diverging lens when it is kept in a transparent medium with greater refractive index than that of the lens.

For example an air bubble in water behaves like a diverging lens.

Let us see an example for lens maker formula.

Example 7

What is the focal length of double concave lens kept in air with two spherical surfaces of radii $R_1 = 30$cm and $R_2 = 60$cm. Take refractive index of lens as n = 1.5.

Solution:

From the figure E-7 using sign convention we get

$R_1 = -30$cm, $R_2 = 60$cm and also given that n = 1.5

using $1/f = (n-1)(1/R_1 - 1/R_2)$

$1/f = (1.5 - 1)[1/(-30) - 1/60]$

Solving this, we get

f = -120cm

Here minus indicates that the lens is divergent.

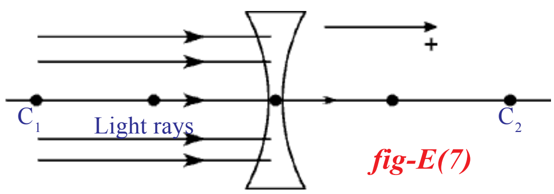


fig-E(7)

Key words

Lens, Focal length, Focus, Optic centre, Principal axis, Radius of curvature, Centre of curvature.

What we have learnt

- The formula used when a light ray enters a medium with refractive index n_2 from a medium with refractive index n_1 at curved interface with a radius of curvature R is
  $n_2/v - n_1/u = (n_2 - n_1)/R$
- A lens is formed when one medium is separated from another medium by two surfaces, one of which is curved.
- Lens formula is $1/f = 1/v - 1/u$
  where f is the focal length of lens, u is the object distance and v is the image distance.
- Lens maker's formula is
  $1/f = (n-1)(1/R_1 - 1/R_2)$
  where R_1 and R_2 are radii of curvature , n is the refractive index and f is the focal length.

Improve your learning

1. A man wants to get a picture of a zebra. He photographed a white donkey after fitting a glass, with black stripes on to the lens of his camera. What photo will he get? Explain. (AS1)
2. Two converging lenses are to be placed in the path of parallel rays so that the rays remain parallel after passing through both lenses. How should the lenses be arranged? Explain with a neat ray diagram. (AS1)
3. The focal length of a converging lens is 20cm. An object is 60cm from the lens. Where will the image be formed and what kind of image is it? (AS1)

   (Ans: A real, diminished, inverted image formed at 30cm from the lens)
4. A double convex lens has two surfaces of equal radii 'R' and refractive index n = 15. Find the focal length 'f'. (AS1)
5. Write the lens maker's formula and explain the terms in it. (AS1)
6. How do you verify experimentally that the focal length of a convex lens is increased when it is kept in water? (AS1)
7. How do you find the focal length of a lens experimentally? (AS1)
8. Harsha tells Siddhu that the double convex lens behaves like a convergent lens. But Siddhu knows that Harsha's assertion is wrong and corrected Harsha by asking some questions. What are the questions asked by Siddhu? (AS2)
9. **Assertion (A):** A person standing on the land appears taller than his actual height to a fish inside a pond. (AS2)

   Reason (R): Light bends away from the normal as it enters air from water.

   Which of the following is correct? Explain.

   a) Both A and R are true and R is the correct explanation of A.

   b) Both A and R are true and R is not the correct explanation of A.

   c) A is true but R is false. d) Both A and R are false.

   e) A is false but R is true.

10. A convex lens is made up of three different materials as shown in the figure Q-10. How many of images does it form? (AS2)


fig-Q(10)

11. Can a virtual image be photographed by a camera?(AS2)
12. You have a lens. Suggest an experiment to find out the focal length of the lens. (AS3)
13. Let us assume a system that consists of two lenses with focal length f_1 and f_2 respectively. How do you find the focal length of the system experimentally, when
    i) two lenses are touching each other
    ii) they are separated by a distance 'd' with common principal axis. (AS3)
14. Collect the information about the lenses available in an optical shop. Find out how the focal length of a lens may be determined by the given 'power' of the lens. (AS4)
15. Collect the information about lenses used by Galileo in his telescope. (AS4)
16. Use the data obtained by activity-2 in table-1 of this lesson and draw the graphs of u *vs* v and 1/u *vs* 1/v. (AS5)

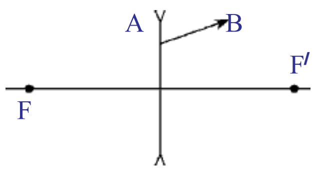


fig-Q(17)

fig- Q(18)

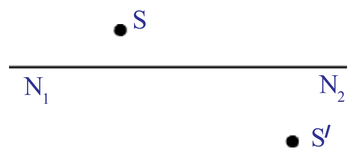


fig-Q(19)

17. Figure Q-17 shows ray AB that has passed through a divergent lens. Construct the path of the ray up to the lens if the position of its foci is known. (AS5)
18. Figure Q-18 shows a point light source and its image produced by a lens with an optical axis N_1N_2. Find the position of the lens and its foci using a ray diagram. (AS5)
19. Find the focus by drawing a ray diagram using the position of source S and the image S' given in the figure Q-19. (AS5)
20. A parallel beam of rays is incident on a convergent lens with a focal length of 40cm. Where should a divergent lens with a focal length of 15 cm be placed for the beam of rays to remain parallel after passing through the two lenses? Draw a ray diagram. (AS5)
21. Draw ray diagrams for the following positions and explain the nature and position of image.
    i. Object is placed at C_2
    ii. Object is placed between F_2 and optic centre P. (AS5)
22. How do you appreciate the coincidence of the experimental facts with the results obtained by a ray diagram in terms of behaviour of images formed by lenses? (AS6)
23. Find the refractive index of the glass which is a symmetrical convergent lens if its focal length is equal to the radius of curvature of its surface. (AS7) (Ans:1.5)
24. Find the radii of curvature of a convexo –concave convergent lens made of glass with refractive index n=1.5 having focal length of 24cm. One of the radii of curvature is double the other. (Ans:R_1=6cm,R_2=12cm) (AS7)

25. The distance between two point sources of light is 24cm .Where should a convergent lens with a focal length of f=9cm be placed between them to obtain the images of both sources at the same point? (AS7)

26. Suppose you are inside the water in a swimming pool near an edge. A friend is standing on the edge. Do you find your friend taller or shorter than his usual height? Why?(AS7)

Fill in the blanks

1. The rays from the distant object, falling on the convex lens pass through ....
2. The ray passing through the .... of the lens is not deviated.
3. Lens formula is given by ....
4. The focal length of the plano convex lens is 2R where R is the radius of curvature of the surface. Then the refractive index of the material of the lens is....................
5. The lens which can form real and virtual images is ....

Multiple choice questions

1. Which one of the following materials cannot be used to make a lens? [ ]

   a) water b) glass

   c) plastic d) clay

2. Which of the following is true? [ ]

   a) the distance of virtual image is always greater than the object distance for convex lens

   b) the distance of virtual image is not greater than the object distance for convex lens

   c) convex lens always forms a real image

   d) convex lens always forms a virtual image

3. Focal length of the plano-convex lens is .... when its radius of curvature of the surface is R and n is the refractive index of the lens. [ ]

   a) f = R b) f = R/2

   c) f = R/(n–1) d) f = (n–1)/R

4. The value of the focal length of the lens is equal to the value of the image distance when the rays are [ ]

   a) passing through the optic centre b) parallel to the principal axis

   c) passing throught he focus d) in all the cases

5. Which of the following is the lens maker's formula [ ]

   a) $1/f = (n-1)(1/R_1 + 1/R_2)$ b) $1/f = (n+1)(1/R_1 - 1/R_2)$

   c) $1/f = (n-1)(1/R_1 - 1/R_2)$ d) $1/f = (n+1)(1/R_1 + 1/R_2)$