6. Progressions

Section-3

$8 \times 2 = 16$

21. Determine the AP whose 3^{rd} term is 5 and 7^{th} term is 9?

Sol:

We have $\boxed{a_n = a + (n-1)d.}$

$a_3 = a + (3-1)d = a + 2d = 5.$ — (1)

$a_7 = a + (7-1)d = a + 6d = 9$ — (2)

Solving the pair of linear equations (1) & (2)

$$\begin{array}{r} a + 2d = 5 \\ a + 6d = 9 \\ (-)\ (-)\quad (-) \\ \hline -4d = -4 \end{array}$$

$\boxed{d = 1}$

Substitute $d = 1$ in (1)

$a + 2(1) = 5$

$\boxed{a = 3}$

Hence the required AP is

$\boxed{3, 4, 5, 6, 7}$

22. How many two digit numbers are divisible by 3?

Sol:

The list of two-digit numbers divisible by 3 is:

12, 15, 18, ---- 99

Here $a = 12$ $d = 15-12 = 3$ $a_n = 99$.

Hence the Common difference is Same

the Series is in A.P

$$\boxed{a_n = a+(n-1)d.}$$

$$99 = 12 + (n-1)3$$

$$99-12 = (n-1)3$$

$$87 = (n-1)3$$

$$\Rightarrow n-1 = \frac{87}{3}$$

$$n-1 = 29$$

$$\Rightarrow n = 29+1 = 30$$

$$\boxed{n = 30}$$

So, there are 30- two digit number divisible by 3.

23. Which term of AP 3, 8, 13, 18 --- is 78?

Sol:

3, 8, 13, 18, --- are in A.P.

$a = 3 \quad d = 8-3 = 5 \quad a_n = 78$. (Say)

$$\boxed{a_n = a + (n-1)d.}$$

$$78 = 3 + (n-1)5$$

$$78 - 3 = (n-1)5$$

$$\Rightarrow \frac{75}{5} = (n-1)$$

$$\Rightarrow 15 = n-1$$

$$\Rightarrow \boxed{n = 16}$$

$\therefore$ 78 is the 16^{th} term of the given A.P

24. Find the 31^{st} term of an AP whose 11^{th} term is 38 and 16^{th} term is 73 ?

Sol:

11^{th} term is 38 $\Rightarrow a + 10d = 38$. —— (1)

16^{th} term is 73 $\Rightarrow a + 15d = 73$ —— (2)

$$\begin{array}{rcl} (-)\ \ (-) & & (-) \\ \hline -5d & = & -35 \end{array}$$

$$\boxed{d = 7}$$

Substitute $d = 7$ in equation (1)

$$a + 10(7) = 38.$$

$$a + 70 = 38$$

$$\boxed{a = -32}$$

Now, the 31^{st} term $= a+30d$

$= -32+30\times 7$

$= -32+210$

$= 178.$

$\boxed{\text{So, the } 31^{st} \text{ term} = 178.}$

25. Which term of GP $2, 2\sqrt{2}, 4 ----$ IS 128?

Sol:

$2, 2\sqrt{2}, 4 -----$

$a = 2 \quad r = \frac{2\sqrt{2}}{2} = \sqrt{2}.$

let 128 be the n^{th} term of the G.p.

Then $\boxed{a_n = ar^{n-1}} = 128.$

$2(\sqrt{2})^{n-1} = 128.$

$(\sqrt{2})^{n-1} = 64$

$2^{\frac{n-1}{2}} = 2^6$

$\Rightarrow \frac{n-1}{2} = 6$

$\boxed{n = 13}$

$\boxed{\text{Hence 128 is the } 13^{th} \text{ term of the Gp.}}$

26. In a G.p the third term is 24 and 6^{th} term is 192..

Find the 10^{th} term ?

Sol:

Here third term is 24 $\Rightarrow a_3 = ar^2 = 24$ —(1)

6^{th} term is 192 $\Rightarrow a_6 = ar^5 = 192$ —(2)

Dividing (2) by (1) we get

$$\frac{ar^5}{ar^2} = \frac{192}{24}$$

$$\Rightarrow r^3 = 8$$

$$\Rightarrow r^3 = 2^3$$

$$\Rightarrow \boxed{r = 2}$$

Substituting $r = 2$ in (1)

$$a(2)^2 = 24$$

$$\boxed{a = 6.}$$

$\therefore$ 10^{th} term $= ar^9$

$$= (6)(2)^9$$

$$\boxed{10^{th} \text{ term} = 3072}$$

27. Find the 12^{th} term of a G.p whose 8^{th} term is 192 and Common ratio is 2 ?

Sol: Given a G.p Such that 8^{th} term is 192

$a_8 = 192$ and $r = 2$

$$\boxed{a_n = a \cdot r^{n-1}}$$

$$a_8 = a \cdot 2^{8-1}$$

$$192 = a 2^7$$

$$\Rightarrow a = \frac{192}{2^7} = \frac{192}{128}$$

$$a = \frac{12}{8} = \frac{3}{2}.$$

$$\boxed{\therefore \quad a = \frac{3}{2}}$$

$$12^{th} \text{ term} = ar^{11}$$

$$= \frac{3}{2}(2)^{11}$$

$$= 3 \times 2^{10}$$

$$\boxed{12^{th} \text{ term} = 3072.}$$

28 If the Sum of the first 14 terms of an Ap is 1050 and its first term is 10. Find the 20^{th} term?

Sol: Sum of the first 14 terms of an A.p is 1050

$$S_n = 1050 \quad ; \quad n = 14.$$

first term is 10

$$\Rightarrow \boxed{a = 10}$$

$$\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$$

$$1050 = \frac{14}{2}\left[20 + 13d\right]$$

$$1050 = 7\left[20 + 13d\right]$$

$$1050 = 140 + 91d$$

$$910 = 91d.$$

$$\boxed{\therefore d = 10}$$

$$\therefore 20^{th} \text{ term} = a_{20}$$

$$= a + 19d.$$

$$= 10 + 19 \times 10$$

$$= 10 + 190$$

$$\boxed{\therefore 20^{th} \text{ term} = 200}$$

Section – 4 $\quad$ $5 \times 4 = 20$ m

29. A Sum of Rs. 1000 is invested at 8% Simple interest per year. Calculate the interest at the end of each year Do these interests form an AP? If So, find the interest at the end of 30 years?

Sol: We know that the formula to Calculate Simple interest is given by

$$\boxed{\text{Simple Interest} = \frac{P \times R \times T}{100}}$$

So, the interest at the end of the

1^{st} year $= \frac{₹\, 1000 \times 8 \times 1}{100} = ₹\, 80$

The interest at the end of the 2^{nd} year

$$= \frac{₹\, 1000 \times 8 \times 2}{100} = ₹\, 160$$

The interest at the end of the 3^{rd} year

$$= \frac{₹\, 1000 \times 8 \times 3}{100} = ₹\, 240$$

Similarly, we can obtain the interest at the end of the 4^{th} year, 5^{th} year, and so on. So, the interest (in Rs) at the end of 1^{st}, 2^{nd}, 3^{rd} --- years, respectively are

80, 160, 240 ---

It is an A.P, as the difference between the consecutive terms in the list is 80.

ie $d = 80$. Also $a = 80$.

Interest at end of 30 years

$$a_{30} = a + (30-1)d.$$

$$= 80 + 29 \times 80 = 2400$$

$\boxed{\text{So, the interest at end of 30 years will be ₹ 2400}}$

(OR)

In a flower bed, there are 23 rose plants in the first row, 21 in the Second, 19 in the third and So on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Sol: The number of rose plants in the $1^{st}, 2^{nd}, 3^{rd}$... rows are:

$$23, 21, 19 \ldots 5$$

It forms an A.P

let the number of rows in the flower bed be n.

Then $a = 23$ $\quad d = 21 - 23 = -2$ $\quad a_n = 5$

As $\boxed{a_n = a + (n-1)d.}$

$$5 = 23 + (n-1)(-2)$$

$$-18 = (n-1)(-2)$$

$$\boxed{n = 10}$$

So there are 10 rows in the flower bed.

---

30. A manufacturer of TV Sets produced 600 Sets in the third year and 700 Sets in the Seventh year. Assuming that the production increases uniformly by a fixed number every year, find:

i) the production in the last year.

ii) the production in the 10^{th} year.

iii) the total production in first 7 years

Sol:-

i) Since the production increases uniformly by a fixed number every year, the number of TV Sets manufactured in 1^{st}, 2^{nd}, 3^{rd} --- years will form an A.P

let us denote the number of TV Sets manufactured in the n^{th} year be a_n

then $a_3 = 600$ and $a_7 = 700$

$\Rightarrow$ $a + 2d = 600$ ——— ①

$a + 6d = 700$ ——— ②

(-) (-) (-)

$-4d = -100$

$\boxed{d = 25}$

Substitute $d = 25$ in ①

$a + 2(25) = 600$

$\boxed{a = 550}$

$\therefore$ production of TV Sets in the first year is 550.

ii) Now, $a_{10} = a + 9d$

$= 550 + 9 \times 25$

$\boxed{a_{10} = 775}$

So, the production of TV Sets in the 10^{th} year is 775.

iii) Also,

$$\boxed{S_n = \frac{n}{2}\left[2a+(n-1)d\right]}$$

$$S_7 = \frac{7}{2}\left[2\times 550 + (7-1)25\right]$$

$$= \frac{7}{2}\left[1100 + 6\times 25\right]$$

$$= \frac{7}{2}\left[1100 + 150\right]$$

$$\boxed{S_7 = 4375}$$

Thus the total production of TV Sets in the first 7 years is 4375.

(OR)

A Sum of ₹ 700 is to be used to give Seven cash prizes to Students of a School for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the Value of each of the prizes.

Sol:

Given

Total / Sum of all cash prizes = ₹ 700

Each prize differs by ₹ 20

let the prizes (in ascending order) be

$x, x+20, x+40, x+60, x+80, x+100, x+120$

$\therefore$ Sum of the prizes $= S_7 = \frac{n}{2}[a+l]$

$$\Rightarrow 700 = \frac{7}{2}[x+x+120]$$

$$\Rightarrow 700 = \frac{7}{2}[2x+120]$$

$$\Rightarrow 100 = x+60$$

$$\boxed{\Rightarrow x = 40}$$

$\therefore$ The prizes are 160, 140, 120, 100, 80, 60, 40.

31. The 4^{th} term of a Gp is $2/3$ and 7^{th} term is $16/81$. Find the Gp?

Sol:

4^{th} term of a Gp is $\frac{2}{3}$

$$a_4 = \frac{2}{3}$$

$$\Rightarrow ar^3 = \frac{2}{3} \quad \text{--- (1)}$$

7^{th} term is $\frac{16}{81}$

$$a_7 = \frac{16}{81}$$

$$ar^6 = \frac{16}{81} \quad \text{--- (2)}$$

dividing (2) by (1)

$$\frac{ar^6}{ar^3} = \frac{16}{81} \times \frac{3}{2}$$

$\Rightarrow r^{6-3} = \frac{8}{27}$

$\Rightarrow r^3 = \left(\frac{2}{3}\right)^3$

$\Rightarrow \boxed{r = \frac{2}{3}}$

Now Substituting $r = \frac{2}{3}$ in equation (1)

$a\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)$

$\Rightarrow a = \frac{2}{3} \times \left(\frac{3}{2}\right)^3$

$\Rightarrow \boxed{a = \frac{9}{4}}$

$\therefore$ The G.p is $a, ar, ar^2, ar^3 - - -$

$\frac{9}{4}, \frac{9}{4} \times \frac{2}{3}, \frac{9}{4} \times \left(\frac{2}{3}\right)^2, - - -$

$\boxed{\frac{9}{4}, \frac{3}{2}, 1 - - -}$

(or)

If the geometric progressions 162, 54, 18 - - - and $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}$ - - have their n^{th} term equal, find the Value of n ?

Sol: Given G.p :

162, 54, 18 - - - and $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}$ - - -

32. Find the Sum of the first 40 positive integers divisible by 6.

Sol: The given numbers are the first 40 positive multiples of 6.

$\Rightarrow 6\times1,\ 6\times2,\ 6\times3 - - -\ 6\times40$

$\Rightarrow 6,\ 12,\ 18 - - - -\ 240.$

$a=6 \quad d=12-6=6 \quad l=240. \quad n=40$

$$\boxed{S_n = \frac{n}{2}[a+l]}$$

$$= \frac{40}{2}[6+240]$$

$$= 20\times246$$

$$\boxed{S_{40} = 4920}$$

(OR)

The Sum of 4th and 8th terms of an A.p is 24 and the Sum of the 6th and 10th terms is 44. Find the first three terms of the A.p ?

Sol: Given an A.p in which.

$$a_4 + a_8 = 24$$

$$(a+3d) + (a+7d) = 24.$$

Here $a = 162$

$r = \frac{a_2}{a_1} = \frac{54}{162} = \frac{3}{9}$

$r = \frac{1}{3}$

Here $a = \frac{2}{81}$

$r = \frac{a_2}{a_1} = \frac{\frac{2}{27}}{\frac{2}{81}}$

$r = \frac{2}{27} \times \frac{81}{2}$

$r = 3$

Given n^{th} terms are equal

$$\boxed{a_n = a \cdot r^{n-1}}$$

$\Rightarrow 162 \times \left(\frac{1}{3}\right)^{n-1} = \frac{2}{81} \times (3)^{n-1}$

$\Rightarrow 3^{n-1} \times 3^{n-1} = 162 \times \frac{81}{2}$

$\Rightarrow 3^{n-1+n-1} = 81 \times 81$

$\Rightarrow 3^{2n-2} = 3^4 \times 3^4$

$\Rightarrow 3^{2n-2} = 3^{4+4} \quad [\because a^m \cdot a^n = a^{m+n}]$

$\Rightarrow 2n-2 = 8$

$2n = 10$

$\boxed{n = 5}$

The 5^{th} term of the two G.Ps are equal.

$a_2 = 3 + 2 \times 2 = 7$

$a_3 = 3 + 2 \times 3 = 9$

$\vdots$

list of numbers becomes

$5, 7, 9, 11 - - -$

Here $7-5 = 9-7 = 11-9 = 2$ and So on

So it forms an Ap with Common difference $d = 2$

Sum of first 24 terms $= S_{24}$

$n = 24 \quad a = 5 \quad d = 2$

$S_{24} = \frac{24}{2}\left[2 \times 5 + (24-1) \times 2\right] \qquad \left[\because S_n = \frac{n}{2}\left[2a + (n-1)d\right]\right]$

$= 12(10 + 46)$

$= 672$

$\boxed{\therefore \text{Sum of first 24 terms of the list of numbers is } 672.}$

33. Subba Rao Started work in 1995 at an annual Salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000 ?

Sol:

Given Salary of Subba Rao in 1995 = ₹ 5000

Annual increment = ₹ 200

Clearly $a = ₹ 5000$ $d = ₹ 200$

let $a_n = ₹ 7000$

$$\boxed{a_n = a + (n-1)d.}$$

$$7000 = 5000 + (n-1)200$$

$$2000 = (n-1)200$$

$$n = 10 + 1$$

$$n = 11$$

∴ In 11^{th} year his Salary reached ₹ 7000

(OR)

Find the Sum of first 24 terms of the list of numbers whose n^{th} term is given by $a_n = 3 + 2n$.

Sol:

AS $a_n = 3 + 2n$

So $a_1 = 3 + 2 = 5$

$2a + 10d = 24$

$a + 5d = 12$ —— (1)

and $a_6 + a_{10} = 44$

$(a+5d) + (a+9d) = 44$

$2a + 14d = 44$

$a + 7d = 22$ —— (2)

Solving (1) and (2)

$a + 5d = 12$

$a + 7d = 22$

$(-)\ (-)\ (-)$

$-2d = -10$

$\boxed{d = 5}$

Substitute $d = 5$ in (1)

$a + 5(5) = 12$

$\boxed{a = -13}$

$\therefore$ The A.p is $a, a+d, a+2d \ldots$

ie $-13, (-13+5), (-13+2\times5 \ldots)$

$\Rightarrow \boxed{-13, -8, -3 \ldots}$