Chapter: 4 Linear Equations

Section: 3.

21. For what value of p, The equations $2x+py=-5$ and $3x+3y=-6$ have a unique solution?

sol:- given equations are: $2x+py+5=0 \longrightarrow (1)$

$3x+3y+6=0 \longrightarrow (2)$

$a_1=2$; $b_1=p$; $c_1=5$

$a_2=3$; $b_2=3$; $c_2=6$

given system of linear equations have unique solution.

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{2}{3} \neq \frac{p}{3} \neq \frac{5}{6}$$

on solving 1 & 2 to get; $\frac{2}{\cancel{3}} \neq \frac{p}{\cancel{3}} \Rightarrow p \neq 2$.

on solving 2 & 3 to get; $p \neq 5/2$.

22. For what value of k, The equations $2x-ky+3=0$ and $4x+6y-5=0$ represent parallel lines?

sol:- given equations are: $2x-ky+3=0 \longrightarrow (1)$

$4x+6y-5=0 \longrightarrow (2)$

$a_1=2$ $b_1=-k$ $c_1=3$

$a_2=4$ $b_2=6$ $c_2=-5$.

system of linear equations are parallel lines.

condition for parallel lines is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

$$\Rightarrow \frac{2}{4} = \frac{-k}{6} \neq \frac{3}{-5}$$

$$\Rightarrow \frac{2}{4} = \frac{-k}{6} \Rightarrow k=-3$$

and $\frac{-k}{6} \neq \frac{3}{-5} \Rightarrow k \neq \frac{18}{5}$.

23. For what value of k, The equations $3x+4y+2=0$ and $9x+12y+k=0$ represent coincident lines?

sol:- given equations are $3x+4y+2=0 \longrightarrow (1)$

$9x+12y+k=0 \longrightarrow (2)$

$a_1=3$ $b_1=4$ $c_1=2$

$a_2=9$ $b_2=12$ $c_2=k$.

given, the linear equations represent coincident lines.

Then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

$$\Rightarrow \frac{\cancel{3}^{1}}{\cancel{9}_{3}} = \frac{\cancel{4}^{1}}{\cancel{12}_{3}} = \frac{2}{k}$$

on equating (2) & (3) $\boxed{k=6}$

24. solve $3x+2y=11$ and $2x+3y=4$ by Elimination method?

Sol:- given; $3x+2y=11 \longrightarrow (1)$

$2x+3y=4 \longrightarrow (2)$

$$\begin{aligned} (1)\times 2 &= \cancel{6x}+4y=22 \\ (2)\times 3 &= \cancel{6x}+9y=12 \\ \hline & -5y=10 \end{aligned}$$

$y=\frac{\cancel{10}}{-\cancel{5}}=-2.$

$\boxed{y=-2}$

substute $y=-2$ in Eq (1).

$3x+2(-2)=11$

$3x-4=11$

$3x=11+4=15$

$\boxed{x=5}$

$\therefore x=5;\ y=-2.$

25 solve: $2x-y=5$ and $3x+2y=11$ by substitution method.

Sol:- given; $2x-y=5 \longrightarrow (1)$

$3x+2y=11 \longrightarrow (2)$

Eq (1) can be written as; $y=2x-5 \longrightarrow (3)$

substute "y" value in Eq (2).

$3x+2(2x-5)=11$

$\Rightarrow 3x+4x-10=11$

$\Rightarrow 7x=21 \Rightarrow x=\frac{\cancel{21}}{\cancel{7}}=3$

$\boxed{x=3}$

substute $x=3$ in Eq (3).

$y=2(3)-5=6-5=1.$

$\boxed{y=1}$

$\therefore x=3$ and $y=1.$

26 The larger of two supplementary angles exceeds the smaller by 18. find the angles?

Sol: Let the larger of the two supplementary angles be "y" and the smaller angle be "x".

$\Rightarrow x+y=180^\circ \longrightarrow (1)$

Also, $y=x+18$ (given) $\longrightarrow (2)$.

substitute ② in ① we get,

$\Rightarrow x + x + 18 = 180^\circ$

$\Rightarrow 2x + 18 = 180^\circ$

$\Rightarrow 2x = 180^\circ - 18 = 162^\circ$

$\Rightarrow x = 81^\circ \longrightarrow (3)$

substitute ③ in ②

$y = 81 + 18^\circ = 99^\circ$

$\therefore$ Larger angle is 99°; smaller angle is 81°

---

27. Two angles are complementary. The larger angle is 3° less than twice the measure of smaller angle. find the measure of each angle?

Sol: Let the measure of ~~larger~~ larger angle be x°; smaller angle be y°

given condition is, Larger angle is 3° less than twice the smaller.

so; $x = 2y - 3^\circ$

W.K.T $x + y = 90^\circ$

$2y - 3 + y = 90^\circ$

$\Rightarrow 3y = 93^\circ$

$\Rightarrow y = 31^\circ$

Larger angle $x = 2y - 3^\circ = 2(31^\circ) - 3^\circ = 62^\circ - 3^\circ = 59^\circ$.

smaller angle, $y = 31^\circ$

---

28. Half the perimeter of a Rectangular garden, whose length is 4m more than its width, is 36m. find the dimensions of the garden?

Sol: Let length, and width of a rectangular garden be "l" & "b" respectively.

Perimeter $= 2(l+b)$

Half the perimeter $= \frac{2(l+b)}{2} = l+b$.

$\Rightarrow l + b = 36 \longrightarrow ①$

given; $l = 4 + b$ (4m more than width)

sustute in ① $\Rightarrow 4 + b + b = 36$

$2b = 32$

$b = \frac{32}{2} = 16$

so; $l = 36 - 16 = 20$

Hence length of the garden is 20m and width is 16m.

Section: 4

29 (i) In a garden there are some bees and flowers. If one bee sits on each flower then one bee will be left. If two bees sit on each flower, one flower will be left. Find the number of bees and number of flowers?

Sol: Let the number of bees $= x$ and

number of flowers $= y$.

If one bee sits on each flower then one bee will be left.

$$so, \; x = y + 1$$

$$\Rightarrow x - y - 1 = 0 \longrightarrow (1)$$

If two bees sits on each flower, one flower will be left.

$$so, \; x = 2(y-1)$$

$$\Rightarrow x - 2y + 2 = 0 \longrightarrow (2)$$

solving (1) & (2)

$$\begin{array}{r} x - y = 1 \\ \cancel{x} - 2y = -2 \\ \hline y = 3 \end{array}$$

substitute $y = 3$ in Eq (1)

$$\Rightarrow x - 3 = 1$$

$$x = 4$$

$\therefore$ Therefore, There are 4 bees and 3 flowers.

(ii) The Perimeter of a rectangular plot is 32m. If the length is increased by 2m and the breadth is decreased by 1m, the area of the plot remains the same. Find the length and breadth of the plot?

Sol:- Let the length and breadth of a rectangular plot be "l" and "b" respectively, Then

Area $=$ length $\times$ breadth $= lb$. and

perimeter $= 2(l+b) = 32$

$$\Rightarrow l + b = 16 \quad (or) \quad l + b - 16 = 0 \longrightarrow (1)$$

When length is increased by 2m and breadth is decreased by 1met

$\Rightarrow (l+2)$ met & $(b-1)$ met.

Then Area $= (l+2)(b-1)$.

since there is no change in Area.

$$(l+2)(b-1) = lb$$

$$\Rightarrow \cancel{lb} - l + 2b - 2 = \cancel{lb}$$

$$\Rightarrow l - 2b + 2 = 0 \longrightarrow (2).$$

on solving ① & ②

$$\cancel{l}+b=16$$
$$\cancel{l}-2b=-2$$
$$3b=18$$
$$\boxed{b=6}$$

substutute $b=6$ in eq ①

$$l+6=16$$
$$\Rightarrow l=16-6=10 \text{ met}$$

$\therefore$ original length of The plot is 10m and breadth is 6m.

30: Mary told her daughter "seven years ago, I was seven times as old as you were Then. Also Three years from now, I shall be three times as old as you will be". find The present age of Mary and her daughter?

Sol:- Let Mary's present age be x years and her daughter's age is "y" years.

7 years ago:-

Mary's age = $(x-7)$ years

Daughter's age = $(y-7)$ years

A/Q ; $x-7=7(y-7)$

$$\Rightarrow x-7=7y-49$$
$$\Rightarrow x-7y+42=0 \longrightarrow ①$$

3 years hence:-

Mary's age = $(x+3)$ years

Daughter's age = $(y+3)$ years

A/Q ; $x+3=3(y+3)$

$$\Rightarrow x+3=3y+9$$
$$\Rightarrow x-3y-6=0 \longrightarrow ②$$

on solving,

$$\cancel{x}-7y=-42$$
$$\cancel{x}-3y=+6$$
$$-4y=-48$$
$$\boxed{y=12}$$

substutute $y=12$ in eq ②

$$x-3(12)=6$$
$$x=6+36=42.$$

$\therefore$ Mary's present age = 42 years and her daughters age = 12 years.

30. A fraction becomes 4/5 if 1 is added to both numerator and denominator. If however 5 is subtracted from both numerator and denominator, the fraction becomes 1/2. What is the fraction?

Sol: Let Numerator of fraction = x
Denominator of fraction = y

$\therefore$ Required fraction = $\frac{x}{y}$

A/Q, $\frac{x+1}{y+1} = \frac{4}{5}$

$\Rightarrow 5x+5 = 4y+4$

$\Rightarrow 5x-4y+1=0 \longrightarrow (1)$

A/Q $\frac{x-5}{y-5} = \frac{1}{2}$

$\Rightarrow 2x-10 = y-5$

$\Rightarrow 2x-y-5=0 \longrightarrow (2)$

on solving 1 & 2

$5x-4y+1=0 \longrightarrow (1)$

$2x-y-5=0 \longrightarrow (2)$

eq (1) = $5x - \cancel{4y} + 1 = 0$

(2) × 4 = $8x - \cancel{4y} - 20 = 0$

$\quad - \quad + \quad +$

$-3x \quad +21 = 0$

$\Rightarrow -3x = -21$

$\boxed{x=7}$

substitute $x=7$ in eq (2)

$2x-y=5$

$\Rightarrow 2(7)-y=5$

$\Rightarrow 14-y=5$

$\Rightarrow 14-5=y$

$\boxed{y=9}$

$\therefore$ Hence the required fraction = $\frac{x}{y} = \frac{7}{9}$

31. Solve $\frac{2}{x}+\frac{3}{y}=13$ and $\frac{5}{x}-\frac{4}{y}=-2$.

Sol: We have $2(1/x)+3(1/y)=13 \longrightarrow (1)$

$5(1/x)-4(1/y)=-2 \longrightarrow (2)$

If we substute $\frac{1}{x}=p$ and $\frac{1}{y}=q$; we get The following equations:

$2p+3q=13 \longrightarrow (3)$

$5p-4q=-2 \longrightarrow (4)$

coefficient of q are 3 & 4. and Their l.c.m is 12.

$(3)\times 4 = 8p+\cancel{12q}=52$

$(4)\times 3 = 15p-\cancel{12q}=-6$

$23p = 46$

$p=\frac{46}{23}=2$. ; $\boxed{p=2}$

substute $p=2$ in eq (3)

$\Rightarrow 2(2)+3q=13$

$3q=13-4$

$\Rightarrow 3q=9$

$\Rightarrow q=9/3=3$

$\boxed{q=3}$

NOW $\frac{1}{x}=p=2 \Rightarrow x=1/2$

$\frac{1}{y}=q=3 \Rightarrow y=1/3$.

---

ii. $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2 \longrightarrow (1)$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1 \longrightarrow (2)$

$\Rightarrow 2\left(\frac{1}{\sqrt{x}}\right)+3\left(\frac{1}{\sqrt{y}}\right)=2 \longrightarrow (3)$

$4\left(\frac{1}{\sqrt{x}}\right)-9\left(\frac{1}{\sqrt{y}}\right)=-1 \longrightarrow (4)$

If We sustute $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$

Now we get

$2p + 3q = 2 \longrightarrow (3)$

$4p - 9q = -1 \longrightarrow (4)$

$$\text{III} \times 2 = \begin{array}{r} \cancel{4p} + 6q = 4 \\ \cancel{4p} \underset{+}{-} 9q = \underset{+}{-1} \\ \hline 15q = 5 \end{array}$$

$q = \frac{\cancel{5}}{\cancel{15}} = \frac{1}{3}$

$\boxed{q = \frac{1}{3}}$

substute the "q" value in eq ③

$\Rightarrow 2p + \cancel{3}\left(\frac{1}{\cancel{3}}\right) = 2$

$\Rightarrow 2p + 1 = 2$

$\Rightarrow 2p = 2 - 1 = 1$

$\Rightarrow \boxed{p = 1/2}$

But $\frac{1}{\sqrt{x}} = p \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow \sqrt{x} = 2$. (s.b.s)

$\Rightarrow x = 4$

$\frac{1}{\sqrt{y}} = q \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow \sqrt{y} = 3$ (s.b.s)

$y = 9$

$\therefore x = 4$ and $y = 9$.

---

33 (i) $\frac{5}{x-1} + \frac{1}{y-2} = 2$.

$\frac{6}{x-1} - \frac{3}{y-2} = 1$.

$5\left(\frac{1}{x-1}\right) + 1\left(\frac{1}{y-2}\right) = 2 \longrightarrow (1)$

$6\left(\frac{1}{x-1}\right) - 3\left(\frac{1}{y-2}\right) = 1 \longrightarrow (2)$.

If we substute $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$ we get the following pair of linear equations.

$$5p + q = 2 \longrightarrow (3)$$
$$6p - 3q = 1 \longrightarrow (4)$$

$(3) \times 3 = 15p + \cancel{3q} = 6$

$(4) = 6p - \cancel{3q} = 1$

$21p = 7$

$p = 7/21 = 1/3$

$\boxed{p = 1/3}$

substute $p = 1/3$ in Eq (3)

$5(1/3) + q = 2$

$q = 2 - \frac{5}{3} = \frac{6-5}{3} = \frac{1}{3}$.

$\boxed{q = 1/3}$

But $\frac{1}{x-1} = p$

$\Rightarrow \frac{1}{x-1} = \frac{1}{3} \Rightarrow x - 1 = 3$

$\Rightarrow x = 4$.

$\frac{1}{y-2} = q$

$\Rightarrow \frac{1}{y-2} = \frac{1}{3} \Rightarrow y - 2 = 3$

$\Rightarrow y = 5$

$\therefore x = 4$ and $y = 5$.

ii) $\frac{x+y}{xy} = 2$ and $\frac{x-y}{xy} = 6$.

Sol:- $\frac{x+y}{xy} = 2 \Rightarrow \frac{x}{xy} + \frac{y}{xy} = 2 \Rightarrow \frac{1}{x} + \frac{1}{y} = 2 \longrightarrow (1)$

$\frac{x-y}{xy} = 6 \Rightarrow \frac{x}{xy} - \frac{y}{xy} = 6 \Rightarrow -\frac{1}{x} + \frac{1}{y} = 6 \longrightarrow (2)$

If we substute $\frac{1}{x} = p$ and $\frac{1}{y} = q$ Then linear eq'ns be:

$$\cancel{p}+q=2 \longrightarrow 3$$
$$-\cancel{p}+q=6 \longrightarrow (4)$$

$$2q=8$$
$$q=\frac{8}{2}=4.$$
$$\boxed{q=4}$$

substitute $q=4$ in eq (3)

$$p+4=2$$
$$\Rightarrow p=2-4=-2.$$

But $\frac{1}{x}=p \Rightarrow \frac{1}{x}=-2 \Rightarrow x=\frac{1}{-2}.$

$\frac{1}{y}=q \Rightarrow \frac{1}{y}=4 \Rightarrow y=\frac{1}{4}$

$\therefore x=-1/2$ and $y=1/4.$

32 Solve $2x+3y=1$ and $3x-y=7$ graphically.

Sol: $2x+3y-1=0 \longrightarrow (1)$

$3x-y-7=0 \longrightarrow (2)$

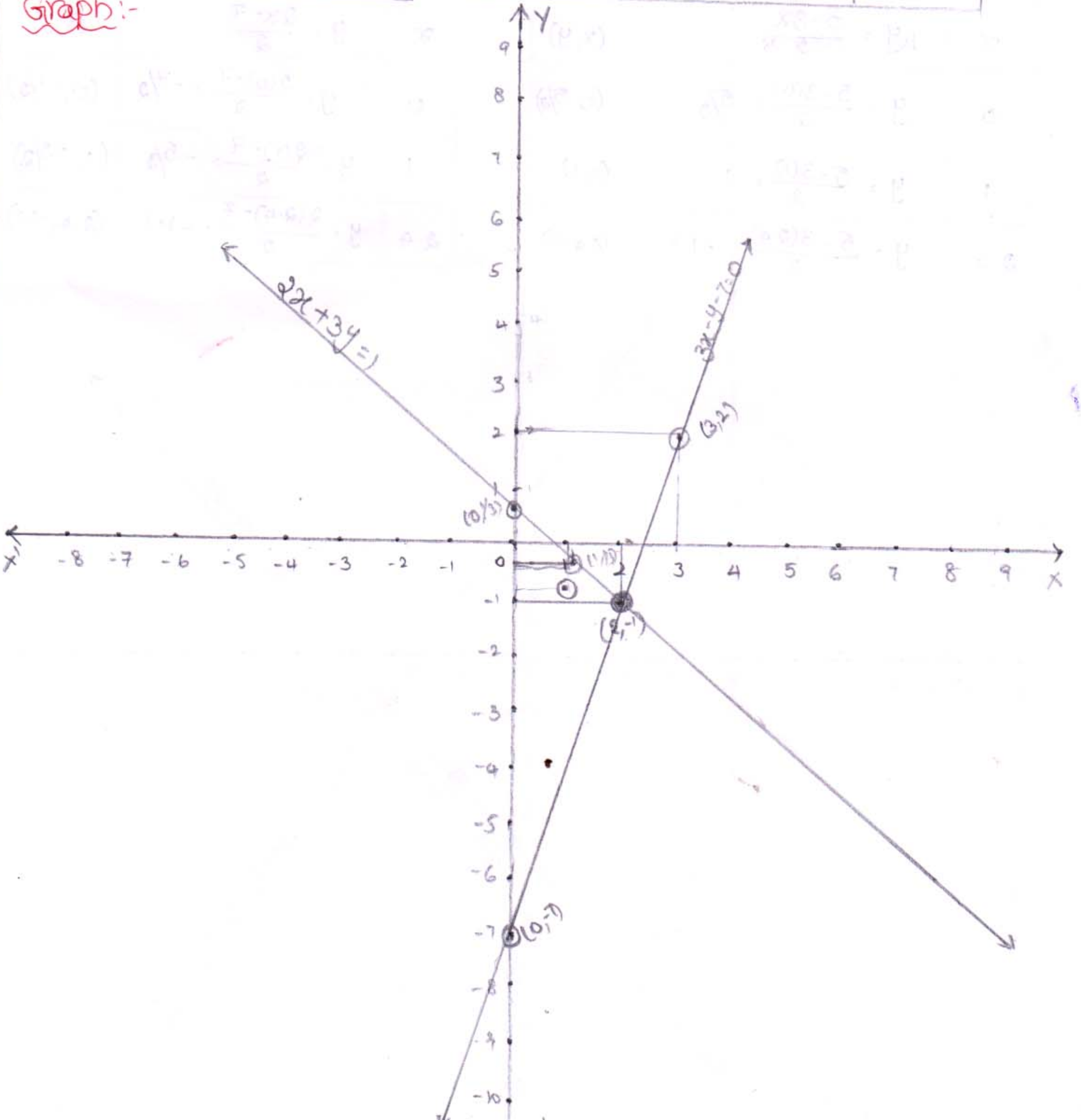
$\frac{a_1}{a_2}=\frac{2}{3}$; $\frac{b_1}{b_2}=\frac{3}{-1}$, $\frac{c_1}{c_2}=\frac{-1}{-7}.$

since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Therefore They are intersecting lines and hence consistent pair of linear equations. They have unique solution.

for the equation $2x+3y=1$		
x	$y=\frac{1-2x}{3}$	(x,y)
0	$y=\frac{1-2(0)}{3}=1/3$	$(0,1/3)$
1	$y=\frac{1-2(1)}{3}=-1/3$	$(1,-1/3)$
2	$y=\frac{1-2(2)}{3}=-1$	$(2,-1)$ ✓

for the equation $3x-y=7$		
x	$y=3x-7$	(x,y)
0	$y=3(0)-7=-7$	$(0,-7)$
2	$y=3(2)-7=-1$	$(2,-1)$ ✓
3	$y=3(3)-7=2$	$(3,2)$

Graph:-



32. Solve $3x+2y=5$ and $2x-2y=7$ graphically.

$3x+2y-5=0 \longrightarrow (1)$

$2x-2y-7=0 \longrightarrow (2)$

$\frac{a_1}{a_2} = \frac{3}{2}$; $\frac{b_1}{b_2} = \frac{2}{-2}$ $\frac{c_1}{c_2} = \frac{-5}{-7}$.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. Therefore they are intersecting lines. and hence they are consistent.

They have unique solution. and also they

For Equation: $3x+2y-5=0$

x	$y = \frac{5-3x}{2}$	(x,y)
0	$y = \frac{5-3(0)}{2} = 5/2$	$(0, 5/2)$
1	$y = \frac{5-3(1)}{2} = 1$	$(1, 1)$
2.4	$y = \frac{5-3(2.4)}{2} = -1.1$	$(2.4, -1.1)$

For The Equation $2x-2y-7=0$

x	$y = \frac{2x-7}{2}$	(x,y)
0	$y = \frac{2(0)-7}{2} = -7/2$	$(0, -7/2)$
1	$y = \frac{2(1)-7}{2} = -5/2$	$(1, -5/2)$
2.4	$y = \frac{2(2.4)-7}{2} = -1.1$	$(2.4, -1.1)$