Dr.K.K.R GOWTHAM (E.M) HIGH SCHOOL :: GUDIVADA

Class : X – State **Marks : 50**

Sub : Mathematics *PRE-FINAL* **Time: 2 ½ hrs**

Paper – II Key

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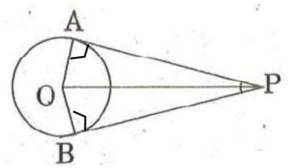
SECTION – A



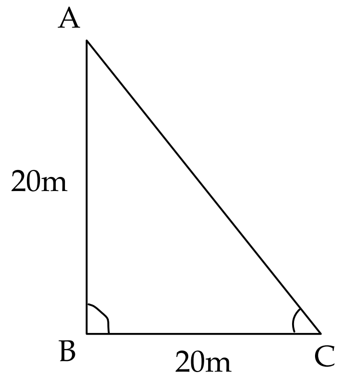
1. Here given $\angle APB = 85^0$

$\angle APB + \angle AOB = 180^0$

$\angle AOB = 180^0 - 85^0$

$= 95^0$

2. One Tangent



3. From the given data,

Angle elevation of boy to the top end of a pillar is

Tan C = $\frac{AB}{BC}$

AB = height of pillar = 20m

BC=distance between pillar and boy = 20m

Tan C = $\frac{AB}{BC} \Rightarrow \tan C = \frac{20}{20} \Rightarrow \tan C = 1$

$= \tan 45^0$

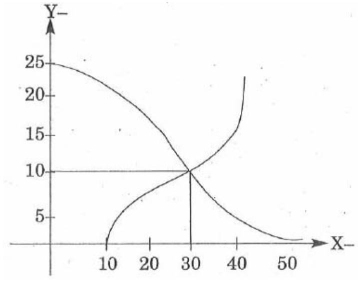
$\angle C = 45^0$

$\therefore$ angle of elevation from boy to top end of a pillar i 45^0

4.

x_i	f_i	$f_i\ x_i$

5. Median from graph is 30



6. (B) P (E) + P $\left(\overline{E}\right) = 1$ is correct

7. both are correct

8. ΔABC , $\angle B = 90^0$

$\angle A + \angle B + \angle C = 180^0$

$\angle A + \angle C = 180^0 - \angle B$

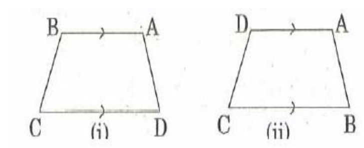
$\angle A + \angle C = 180^0 - 90^0$

$\angle A = 90^0 - \angle C$

$\sin A = \sin(90^0 - C)$

$= \cos C$

9. (ii) figure shows the given condition



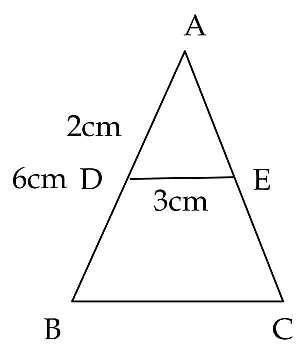
10. By given data ,

$\Delta ABC \sim \Delta ADE$

$\therefore \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$

$\frac{AB}{AD} = \frac{BC}{DE}$

$\frac{3\not{6}}{\not{2}} = \frac{BC}{3} \Rightarrow BC = 9cm$



11. Slop of line PQ = $\frac{y_2 - y_1}{x_2 - x_1}$

12. The distance between two pints (0, 0) and (a, b) is $\sqrt{a^2 + b^2}$

SECTION – B (8× 1 = 8 M)

13. Given P (2, 5) , Q(x, 3)

Slope of PQ = $\frac{3-5}{x-2}$

$= \frac{-2}{x-2}$

Given slope of PQ = 2

$\therefore \frac{-2}{x-2} = 2$

$-1 = x - 2$

$x = 2 + (-1)$

$x = 1$

14. Given probability of Navya winning = 0.82

Probability of Rekha winning = 1 – probability of Naavya winning

= 1-0.82

= 0.18

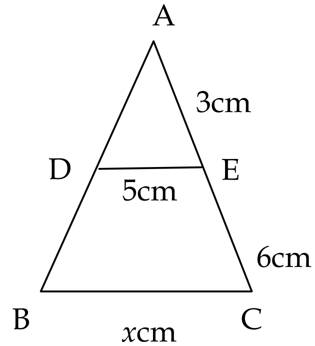
15. $coec^2\theta - \cot^2\theta = 1$

$coec^2\theta = 1 + \cot^2\theta$

$= 1 + \frac{1}{\tan^2\theta}$

$= \frac{1+\tan^2\theta}{\tan^2\theta}$

16. Given Δ ABC ~ΔADE



$\therefore \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$

$\frac{BC}{DE} = \frac{AC}{AE} \Rightarrow \frac{x}{5} = \frac{AE+EC}{AE}$

$$\frac{x}{5} = \frac{6+3}{3}$$

$$x = \frac{\cancel{9}^{3}}{\cancel{3}} \times 5$$

$$x = 15 \text{ cm}$$

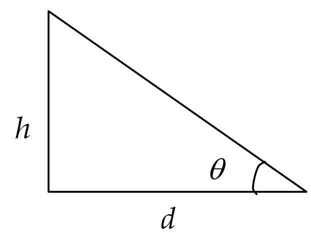
17. fi=frequency of given class interval

    di=x_i – a

18. Given height of tower be 'h'

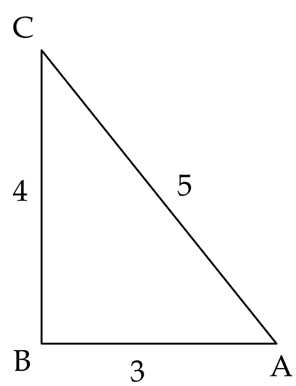
    Distance between tower and observer is 'd'

    Angle between observer and top of tower is θ



$$\therefore \tan\theta = \frac{h}{d}$$

19. In ΔABC, $\tan A = \frac{4}{3} = \frac{BC}{AB}$



$$AC^2 = AB^2 + BC^2$$

$$= 4^2 + 3^2$$

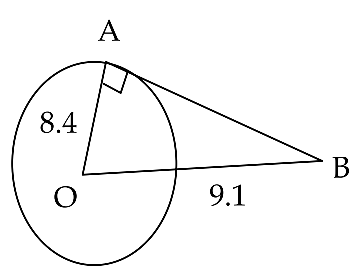
$$= 16 + 9 = 25$$

$$AC = 5$$

$$\sin A = \frac{BC}{AC}$$

$$= \frac{4}{5}$$

20. ΔAOB, ∠A=90^0



$OB^2 = AB^2 + OA^2$

$AB^2 = OB^2 - OA^2 = (9.1)^2 - (8.4)^2$

$= 82.81 - 70.56$

$= 12.25$

$AB = \sqrt{12.25}$

$= 3.5$ cm

SECTION – C

21. Given points A (2, -5) and B(-2, 9)

Let P (x, 0) be the point on x – axis which is equidistant from

A and B., i.e., PA = PB [Distance two points (x_1, y_1) & (x_2, y_2) = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$]

$$\sqrt{(2-x)^2+(-5-0)^2}=\sqrt{(-2-x)^2+(9-0)^2}$$

$$x^2-4x+4+25=x^2+4x+4+81$$

$$-4x+29=4x+85$$

$$4x+4x=29-85$$

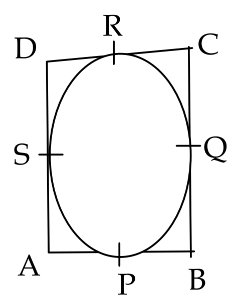
$$8x=-56$$

$$x=\frac{-56}{8}$$

$$x = -7$$

22. The circle touches the sides AB, BC, CD and DA of quadrilateral ABCD at the points P, Q, R, S respectively

$\therefore$ the two tangents to a circle. Drawn from a point out side it are equal



AP = AS

BP = BQ

DR = DS

CR = CQ

By adding, we get

AP + BP + DR + CR = AS + BQ + DS + CQ

(AP + PB) + (CR+DR) = (BQ + QC) + (DS +SA)

AB + CD = BC + DA

Hence proved

23. Total number of cards = 52

i) No. of favourable outcomes = No . of an ace cards

= 4

$$\text{P (E)} = \frac{\text{No. of favourableoutcomes}}{\text{Total No. outcomes}} = \frac{4}{52}=\frac{1}{13}$$

ii) No. of favourable out comes = No. of Red kings

= 2

$$\text{P(E)} = \frac{\text{No. of favourableout comes}}{\text{Total No. out comes}}=\frac{2}{52}=\frac{1}{26}$$

24. Median of grouped data

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Where l = lower boundary of median class

n = number of observations

cf = cumulative frequency of preceding median class

h = size of the median class.

25. Given $\tan A = \frac{1}{\sqrt{3}}$ | $\tan B = \sqrt{3}$

$= \tan 30^0$ | $= \tan 60^0$

$A = 30^0$ | $B = 60^0$

$\sin A \cos B + \cos A \sin B = \sin 30^0 \cos 60^0 + \cos 30^0 \sin 60^0$

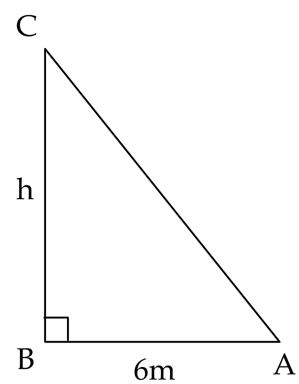
$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

26. From the figure , In triangle ABC

AB = 6 meters

$\angle$ CAB = 60^0



Let the height of the pole = BC = h meters

(We know the adjacent sides and we need to find the opposite side of $\angle$AOB in the triangle Δ OAB Hence we need to consider the triangonometric ration "tanθ" to solve the problem)

$$\text{Tan}60^0 = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{h}{6}$$

$h = 6\sqrt{3}$ m.

27.

$$\frac{\sec 15^0}{\cos ec 75^0}+\frac{\sin 72^0}{\cos 18^0}-\frac{\tan 33^0}{\cot 57^0}=\frac{\sec 15^0}{\cos ec\left(90^0-15\right)}+\frac{\sin 72^0}{\cos\left(90^0-72^0\right)}-\frac{\tan 33}{\cot\left(90^0-33^0\right)}=\frac{\sec 15^0}{\sec 15^0}+\frac{\sin 72^0}{\sin 72^0}-\frac{\tan 33^0}{\tan 33^0}=1+1-1=1$$

28. Δ ABC ~ Δ PQR

So $\frac{ar(\Delta ABC)}{ar(\Delta PQR)}=\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{AC}{PR}\right)^2$

But $\frac{ar(\Delta ABC)}{ar(\Delta PQR)}=1(\because$ areas are equal)

$\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{AC}{PR}\right)^2=1$

So $AB^2 = PQ^2$, $BC^2 = QR^2$, $AC^2 = PR^2$

From which we get AB = PQ

BC = QR

AC = PR

$\therefore$ Δ ABC $\cong$ Δ PQR (by sss congruency)

Hence proved

SECTION – D

29. a) **Given :** In ΔABC, D is mid point of AB A line 'l' through D, parallel to BC meeting AC at E.

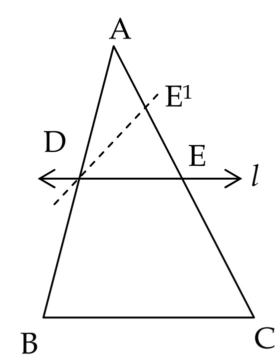
R.T.P : E is the midpoint of AC

Proof : Case (i) If 'E' is the midpoint then the given statement becomes true and the proof is obvious

Case (ii) If 'E' is not the midpoint of AC

Let E' be the midpoint of AC then , $\frac{AD}{DB}=\frac{AE}{EC}$

(A line drawn parallel to one side of a triangle divides the other two sides in the same ratio D DE || BC)



and given that

$1=\frac{AE}{EC}$ (D is mid point of AB, AD=DB)

$\Rightarrow$ AE = EC

$\Rightarrow$ E is midpoint of AC

i.e., E must coincide with E′

in any case 'l' bisects AC and hence the proof

(Or)

b) Given A (-5, -1) , B (3, -5) , C(5,3) are vertices of Δ ABC

Let D, E & F be the midpoints of the sides $\overline{AB}, \overline{BC}$ & $\overline{AC}$

(Midpoint of (x_1 , y_1) & (x_2 , y_2) = $\left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$)

$\therefore$ D is midpoint of AB, D $= \left[\frac{-5+3}{2}, \frac{-1-5}{2}\right]$

$= \left[\frac{-2}{2}, \frac{-6}{2}\right]$

$= [-1, -3]$

E is midpoint of BC, E = $\left[\frac{3+5}{2}, \frac{-5+3}{2}\right]$

$= \left[\frac{8}{2}, \frac{-2}{2}\right] = [4, -1]$

F is midpoint of AC, F= $\left[\frac{-5+5}{2}, \frac{-1+3}{2}\right]$

$= \left[\frac{0}{2}, \frac{2}{2}\right] = [0,1]$

Area of ΔABC = $\frac{1}{2}\left|x_,(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

$= \frac{1}{2}\left|-5(-5-3)+3(3+1)+5(-1+5)\right|$

$= \frac{1}{2}\left|40+12+20\right| = \frac{72}{2} = 36$ squints

Area of ΔDEF = $\frac{1}{2}\left|-1(-1,-1)+4(1+3)+0(-3+1)\right|$

$= \frac{1}{2}\left|-1(-2)+4(4)+0\right| = \frac{1}{2}\left|2+16\right| = \frac{18}{2} = 9$ squints

Ratio of areas = ar (ΔABC) : ar (ΔDEF)= 36: 9 = 4 : 1

30. a) Given

$\tan x = \frac{5}{12}$

$\sec^2 x = 1 + \tan^2 x$

$= 1 + \left(\frac{5}{12}\right)^2$

$= 1 + \frac{25}{144}$

$= \frac{144+25}{144}$

$$\sec^2 x = \frac{169}{144} \Rightarrow \sec^2 x = \frac{13^2}{12^2}$$

$$\Rightarrow \sec x = \frac{13}{12}$$

$$\sqrt{\frac{\sec x + 1}{\sec x - 1}} = \sqrt{\frac{\frac{13}{12}+1}{\frac{13}{12}-1}}$$

$$= \sqrt{\frac{\sec x + 1}{\sec x - 1}} = \sqrt{\frac{\frac{13+1}{12}}{\frac{13-1}{12}}} = \sqrt{\frac{25}{1}} = \sqrt{5^2} = 5$$

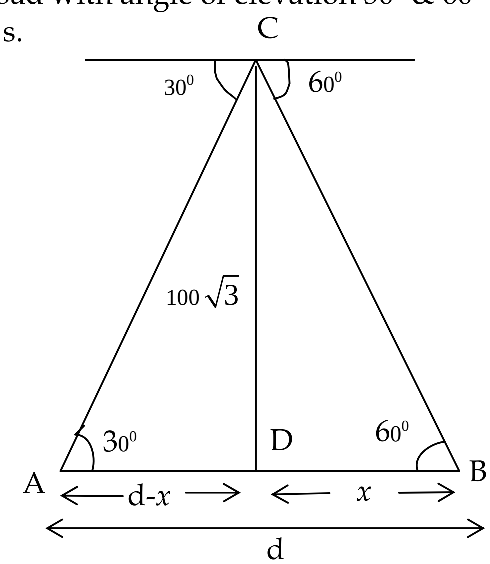
$\therefore$ sec$x$ = $\frac{13}{12}, \sqrt{\frac{\sec x + 1}{\sec x - 1}} = 5$

(Or)

b) Given height of tower CD = 100 $\sqrt{3}$

also given two cars A & B on either side of road with angle of elevation 30^0 & 60^0 traveling with uniform speed 10m/s & 5 m/s.

Let distance between A & B be 'd' m



In ΔACD

$$\tan A = \frac{CD}{AD}$$

$$\tan 30^0 = \frac{100\sqrt{3}}{d-x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{100\sqrt{3}}{d-x} \Rightarrow d - x\ 300m$$

In Δ ΔBCD

$$\tan B = \frac{CD}{BD}$$

$$\tan 60^0 = \frac{100\sqrt{3}}{x} \Rightarrow \sqrt{3} = \frac{100\sqrt{3}}{x} \Rightarrow x = 100m$$

Time taken for A car $= \frac{dis\tan ce}{speed} = \frac{300}{10} = 30\sec$

Time taken for B car = $= \frac{dis\tan ce}{speed} = \frac{100}{5} = 20\sec$

$\therefore$ 'A' car late by 10 sec.

31. a) Given No. of yellow balls is '6'

Let No. of green balls be 'x'

$\therefore$ total no. of balls = 6 +x

So, probability of drawing a green ball = $\frac{x}{6+x}$

Probability of drawing a yellow ball = $\frac{6}{6+x}$

Given, probability of drawing a green ball is thrice the yellow ball

$$\Rightarrow \frac{x}{6+x} = \frac{3 \times 6}{6+x}$$

$\Rightarrow x = 18$

$\therefore$ No. of green balls = 18

Total No. of balls = 6+ 18 = 24

Probability of yellow ball = $\frac{6}{24} = \frac{1}{4}$

Probability of green ball = $\frac{18}{24} = \frac{3}{4}$

(Or)

b) Given A (6, 0) and B (0, -4)

Let P, Q and R be the points which divides devides $\overline{AB}$ into four equal parts

P divides AB in ratio 1 : 3

Q divides AB in ratio 2 : 2 $\Rightarrow$ 1 : 1

R divides AB in ratio 3 : 1

[The coordinates of P (x, y) divides the linesegment joining the points A (x_1, y_1) B (x_2, y_2) in the ratio m : n is $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$]

$$P = \left[\frac{1(0)+3(6)}{1+3}, \frac{1(-4)+3(0)}{1+3}\right] = \left[\frac{18}{4}, \frac{-4}{4}\right]$$

$$= \left[\frac{9}{2}, -1\right]$$

$$Q = \left[\frac{6+0}{2}, \frac{0-4}{2}\right] = \left[\frac{6}{2}, \frac{-4}{2}\right] = [3, -2]$$

$$R = \left[\frac{3(0)+1(6)}{1+3}, \frac{3(-4)+1(0)}{1+3}\right] = \left[\frac{6}{4}, \frac{-12}{4}\right]$$

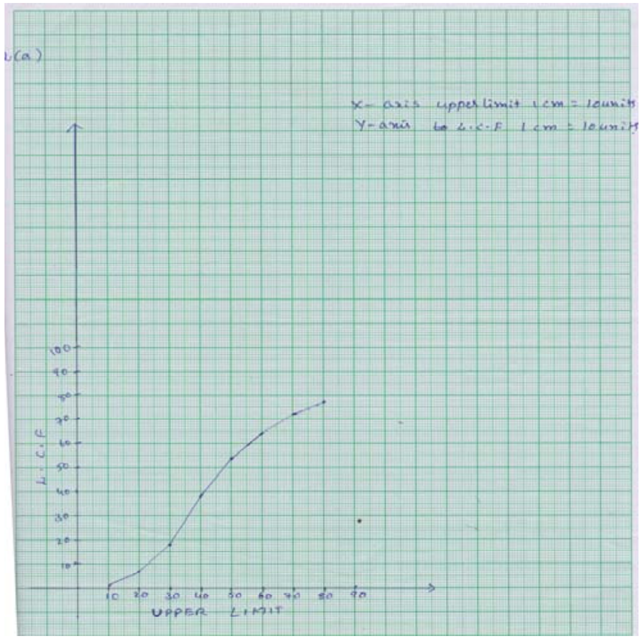
$$= \left[\frac{3}{2}, -3\right]$$

32. a) Since the curve is a less than type the data changes to

Marks Scored	Class interval	f	2.C.F	Points
0-10	Less than 10	01	01	(10,01)
10-20	Less than 20	06	07	(20,07)
20-30	Less than 30	11	18	(30,18)
30-40	Less than 40	20	38	(40,38)
40-50	Less than 50	16	54	(50,54)
50-60	Less than 60	10	64	(60,64)
60-70	Less than 70	08	72	(70,72)

70-80	Less than 80	05	77	(80,77)

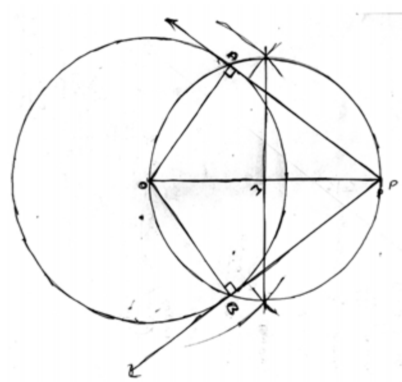
x = axis upper limit 1cm = 10 units
y=axis L.C.F 1 cm = 10 units



(Or)

b) Given radius of circle 6cm

and a point 10cm away from centre



Steps

1) Draw a circle with centre 'O' and radius 6cm
2) Take a point P out side the circle such that OP = 10cm . Join OP
3) Draw the perpendicular bisector to OP which bisects it at M.
4) Taking M as centre and PM or MO as radius draw a circle, Let the circle intersects the given circle at A and B
5) Join P to A and B
6) PA and PB are the required tangents of length 8cm each

Verification:

In ΔOAP

$OA^2 + AP^2 = 6^2 + 8^2$

$= 36+64=100$

$OP^2 = 10^2 = 100$

$\therefore OA^2 + AP^2 = OP^2$

Hence AP is a tangent similarly BP is a tangent

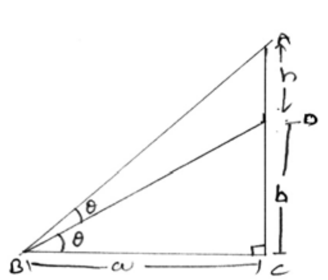
33. a) Given height of tower CD = ‘b’ m

given distance between a point foot of tower BC = ‘a’ m

Let the length of flag staff be AD = ‘h ’ m

now In Δ BCD

$\tan B = \dfrac{CD}{BC}$



$\tan\theta = \dfrac{b}{a} \Rightarrow \tan^2\theta = \dfrac{b^2}{a^2}$

now in ΔABC

$\tan B = \dfrac{AC}{BC}$

$\tan(2\theta) = \dfrac{AD+DC}{BC}$

$\tan 2\theta = \dfrac{b+h}{a}$

$\dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{b+h}{a}$

$\dfrac{2\times\dfrac{b}{a}}{1-\dfrac{b^2}{a^2}} = \dfrac{b+h}{a}$

$\dfrac{\dfrac{2b}{a}}{\dfrac{a^2-b^2}{a^2}} = \dfrac{b+h}{a}$

$\dfrac{2ab}{a^2-b^2} = \dfrac{b+h}{a}$

$\dfrac{2a^2b}{a^2-b^2} = b+h$

$h = \dfrac{2a^2b}{a^2-b^2} - b$

$$h = \frac{2a^2b - a^2b + b^3}{a^2 - b^2}$$

$$h = \frac{a^2b + b^3}{a^2 - b^2}$$

$$h = \frac{b\left(a^2 + b^2\right)}{a^2 - b^2}$$

$\therefore$ length of flag staff is $\dfrac{b\left(a^2 + b^2\right)}{a^2 - b^2}$

(Or)

b)

Class interval	Frequency	C.f
0-10	5	*5*
10-20	x	*5+x*
20-30	20	*25+x*
30-40	15	*40+x*
40-50	y	*40+x+y*
50-60	5	*45+x+y*

Median = $l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) \times h$

It is given that $\sum f = n = 60$

So, 45+x+y = 60 $\Rightarrow$ x+y=60-45

x+y = 15 __ (1)

The median is 28.5 which lies between 20 and 30.

$\therefore$ median class = 20 -30

Lower boundary of the median class 'l' = 20

No. of observations , n = 60 $\dfrac{n}{20} = \dfrac{60}{2} = 30$

Cf= cumulative frequency = 5 + x

h = size of class interval = 10

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$28.5 = 20 + \left(\frac{30 - 5 - x}{20}\right) \times 10$$

$$28.5 - 20 = \frac{25 - x}{2}$$

$$8.5 = \frac{25 - x}{2}$$

$17 = 25 - x$

$x = 25 - 17$

$x = 8$

from $\Rightarrow x + y = 15$

$8 + y = 15$

$y = 15 - 8$

$= 7$

$\therefore x = 8, y = 7$